Time-frequency analysis and nonstationary filtering

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1 Objectives

The primary objective of this workshop was to bring together both theoretical researchers and the more applied practitioners in time-frequency analysis for a constructive exchange of ideas. There are many very advanced concepts in the recent theoretical publications in this field but most of these have had little impact to date upon applications to real world signals. The organizers invited some of the top theoreticians in time-frequency analysis to interact with mathematical physicists and engineers, particularly such as those in geophysics and communications engineering where nonstationary filtering is a fundamental tool. The workshop provided a format with time for formal presentations as well as unstructured time for interaction and collaboration.

This workshop served as a the capstone for the special semester "Modern Methods of Time-Frequency Analysis which was held at the Erwin Schrodinger Institute in Vienna during spring 2005. The ESI session brought together a wide spectrum of scientists from Europe, while the following BIRS workshop involved these top researchers with the North American contingent. The ESI special semester was organized by Feichtinger, Gröchenig, and Benedetto; two of whom are also organizers for this proposal. More information on the ESI workshop is available at: **http://www.univie.ac.at/NuHAG/ESI05/index.html**

A secondary objective is to encourage long-term collaboration between the theoreticians and the applied researchers. While the former often have a deeper understanding of the potential of time-frequency analysis, the latter have access to physical data and are in touch with practical necessities such as computational limitations.

2 Overview of the Field

Time-frequency analysis finds its roots in Fourier analysis, where a signal in time can be analyzed in the frequency domain as a sum of sines and cosines. Originally developed by Fourier to solve an open problem in heat flow on a plate, the techniques of Fourier analysis have had wide application in such diverse areas as partial differential equations and mathematical physics, signal processing and electrical engineering, geometry and Sturm-Liouville problems, probability theory and Brownian motion, to name just a few.

Ironically, Fourier's new ideas and techniques were radical enough that they forced a reassessment of the Calculus, and ushered in a new mathematical era of analysis that could properly deal with infinite sums, convergence issues, unusual functions that are continuous but differentiable nowhere, and other important difficulties. So while his work lead directly to the theoretical areas of Fourier and Harmonic analysis, indirectly Fourier's ideas are responsible for the development of the flourishing areas of real and complex analysis, measure theory, functional analysis, and more.

The development of Fourier analysis and exploitation of the factorization of translation-invariant linear systems (convolution integrals) by the Fourier transform has lead to a rich field with many practical applications, particularly in mathematical physics (PDEs) and engineering (signal processing, vibration analysis, and systems theory). However, there is growing recognition that the ever more complex applications absolutely necessitate the inclusion of nonstationary systems in analysis and filtering techniques. Extensions of Fouriers concepts to the nonstationary setting are numerous and include: the Gabor transform, the wavelet transform, the Wigner transform, pseudodifferential operators, Fourier integral operators, and more. While most of these extensions have origins within quantum theory, it is now true that applications abound in many other fields such as geophysics and engineering.

The essential idea in all these extensions is that rather than analyzing signals only in the time domain, or only in the frequency domain, we instead can make a joint representation of the signal in a time-frequency domain. A musical score is a excellent analogy of this analysis: an entire piece of music (signal) can be represented as a collection of notes (frequencies) that are played at particular instances of time. In this domain, it is elementary to identify particular notes, modify them, remove them, or even rearrange them. In real applications such as medical imaging or cellular communications, these time-frequency components may be identified as noise to be removed, features to be identified and enhanced, or encodings of complex data messages to be transmitted and received. Modifying the signal is a filtering operations; since the effects of the filter changes with time and frequency, this is called a nonstationary filter.

Essential in all applications is choosing a suitable time-frequency representation of a signal, whether that be through a short time Fourier transform, a Gabor transform, a wavelet transform, or via a pseudodifferential operator. Then choosing an appropriate operator to modify the signal, be that a Gabor multipier, a pseudodifferential operator, or some other form of time varying linear operator. Questions of achievability, stability, and computational speed are all critical issues.

Theoretical work includes identifying appropriate function spaces that represent signals well in the time-frequency domain (the modulation spaces), identifying mapping properties of operators on these spaces, questions about choices of bases and frames for such spaces, and many of the analogous results that are common in general Banach space theory and its applications.

3 Recent Developments and Open Problems

This is an exciting moment in time-frequency analysis as the theory is evolving rapidly while new applications are also constantly emerging. Similar to the trend from linear to nonlinear problems, the move from stationary to nonstationary leads to a richer solution set but at the expense of greater mathematical and computational complexity. Stationary filtering has been an important signal processing tool in industry for many years but today we have an emerging understanding of nonstationary filtering that promises to have a immense impact on signal processing as well as the associated modelling of the real world. The rapid increase of available computing power makes the implementation of complex nonstationary filters possible today where they were only concepts a short while ago.

Examples of recent concrete applications in nonstationary filter theory include the development of Gabor deconvolution and Gabor wavefield extrapolation for seismic imaging, nonstationary filtering in cell phone networks, nonstationary noise reduction, modelling of spatially variable quantum systems, coherent state techniques, and filtering and analysis in commercial music production. In addition, any physical system that can be modeled as a variable coefficient partial differential equation can be re-expressed as an equivalent nonstationary filter problem.

Many of the open problems are deep questions in the analysis of functions, including such things as optimal choices of base functions for frames, linear independence of time-frequency translates of base functions, properties of modulation spaces and linear operators on these spaces, and the representation of linear operators or nonstationary filters via pseudodifferential operators and Gabor multipliers.

4 Presentation Highlights

The talks concentrated on four or five general areas, both theoretical and applied. Research questions about frames, whether in the Gabor, wavelet, shearlet or other exotic domains, were addressed by Bodmann, Fournasier, Heil, Jorgensen, Kutyniok, Larson, and Torresani. The Gabor transform, and representing nonstationary filters through Gabor multipliers and pseudodifferential operators, were addressed by Ali, Balazs, Feichtinger, Lamoureux, Okoudjou, and Strohmer. Localization operators were addressed by Groëchnig, Oliaro and Toft. Applications considered included seismic and medical imaging, signal processing, deconvolution, and psychoacoustics; talks on applications were given by Balazs, Casazza, de Hoop, Fishman, Gibson, Hermann, Hlawatsch, Klauder, Margrave, Mitchell, Pfander, Sacchi, Shen and Stolk.

Details of individual talks are given below.

5 The Talks

SPEAKER: Syed T. Ali
TITLE: A Suggestion for a Vectorial Gabor Transform
ABSTRACT: Using some recent results on coherent states over matrix and C*-algebraic domains, a possible candidate for a vectorial Gabor transform will be presented. Such a transform is expected to have applications to signals with additional (internal) degrees of freedom. Some interesting holomorphic properties of such transforms will be discussed.

SPEAKER: Peter Balazs
TITLE: Gabor Multipliers with Application to Psychoacoustics
LINK-Preprint: http://www.kfs.oeaw.ac.at/xxl/dissertation/dissertation.pdf
ABSTRACT: In this talk the basic ideas of the PhD thesis 'Regular and Irregular Gabor Multipliers with Application To Psychoacoustic Masking' will be presented. The concept of frame multiplier will be introduced. Frame multipliers are a generalization of Gabor multipliers to frames without further structure. Basic results, like the dependency of the operator on the symbol, are proved. Furthermore irregular Gabor frames are investigated. In particular some results on irregular Gabor multipliers are proved like the continuous dependency of Gabor multipliers on the symbol, the lattice and the windows. Finally a concept is presented how to implement a masking filter, which approximates the simultaneous and temporal masking known in psychoacoustics. As the linear frequency scale (in Hz) is not very well adapted to human perception, another is chosen (Bark), this filtering can be seen as an irregular Gabor multiplier with adaptable mask.

SPEAKER: Bernhard Bodmann
TITLE: Frame paths and error bounds for sigma-delta quantization
ABSTRACT: We study the performance of finite frames for the encoding of vectors by applying first-order sigma-delta quantization to the frame coefficients. Our discussion is restricted to uniform tight frames, given by N vectors in a d-dimensional Hilbert space, and mostly concerns the use of quantizers that assume only integer multiples of a step-size δ (mid-tread). We prove upper and lower bounds for the maximal reconstruction error in terms of geometric quantities of a path interpolating the sequence of frame vectors. We calculate these bounds for various known frame families and introduce the so-called d-circles and semicircles frames. The latter give a slight improvement in the upper bound over the harmonic frames. The maximal error for all of these families is asymptotically of the order $\delta d^{3/2}/N$, with numerical constants that are comparable to that of coordinatewise application of the sigma-delta algorithm.

SPEAKER: Pete Casazza
TITLE: Pure Mathematics, Applied Mathematics and Engineering: A common thread
LINK-Preprint: http://www.math.missouri.edu/˜pete/
ABSTRACT: We will see that the famous 1959 Kadison-Singer Problem is equivalent to fundamental unsolved problems in a dozen areas of research in both mathematics and engineering, including problems in signal reconstruction, internet coding, reconstruction from sparse representations and much more. This gives all these areas of research common ground on which to interact and helps to explain why each area has

volumes of literature on their respective problems without a satisfactory resolution.

SPEAKER: Maarten de Hoop
TITLE: Analysis of ‘wave-equation’ imaging of reflection seismic data with curvelets
ABSTRACT: In this presentation we discuss how techniques from dyadic parabolic decomposition of phase space and curvelet frames can be exploited in representing and analyzing the process of ‘wave-equation’ seismic imaging. The approach aids in the fundamental understanding of the notion of scale in the data and how it is coupled through imaging to scale in - and regularity of - the medium. Furthermore, the use of curvelets admits a rigorous treatment of the concept of controlled illumination.

SPEAKER: Hans Feichtinger
TITLE: What do we know about Gabor multipliers?

SPEAKER: Lou Fishman
TITLE: Phase Space and Path Integral Methods in Seismic Wave Propagation Modeling and Imaging
ABSTRACT: Seismic wave propagation modeling and imaging are complicated by the large-scale and rapidly-varying environments encountered in the earth. Traditionally, these classical problems have been addressed by

1. direct approximations on the wave field (e.g., asymptotic ray theory, Gaussian beams, spectral representations),

2. the application of approximate wave equations (e.g., formal one-way wave equations), and

3. the direct application of computational pde methods (e.g., finite differences, finite elements, spectral methods).

This talk will survey the application to seismic wave propagation modeling and imaging of what is loosely referred to as “phase space and path integral methods.” These methods were originally developed in the quantum physics and theoretical pde communities, and include the Feynman path integral constructions for the Schrdinger equation, and the theories of pseudodifferential and Fourier integral operators, for example. For fixed-frequency modeling, the primary aims of this approach are

1. to incorporate well-posed, one-way methods into the inherently two-way global formulations,

2. to exploit the correspondences between the classical wave propagation problem, quantum physics, and modern mathematical asymptotics (microlocal analysis), and

3. to effectively extend Fourier-analysis-based constructions to inhomogeneous environments.

It will be seen that the explicit, exact, well-posed, one-way reformulation of ”elliptic wave propagation” problems (e.g., the scalar Helmholtz equation) in phase space provides an explicit mathematical framework for wave-equation-based seismic migration, both unifying the diverse approximations (e.g., wide-angle parabolic modeling, generalized phase screens, generalized phase shift plus interpolation (GPSPI)), and systematically extending the physically based GPSPI algorithm. These developments result in improved seismic imaging algorithms.

SPEAKER: Massimo Fornasier
TITLE: Frame adaptive methods for signal processing and operator equations
ABSTRACT: We illustrate several adaptive algorithms for the solutions of bi-infinite singular linear systems. Such algorithms are realized from exact iterative schemes (e.g., Richardson, steepest-descent methods, matching pursuit) by finite dimensional approximations of each iteration, performed with a greedy approach. We show that these algorithms are convergent and optimal with respect to certain sparseness classes of vectors as soon as the system matrix has sufficient off-diagonal decay. Bi-infinite linear systems of this type typically arise in the solution of functional operator equations (e.g., integral and differential equations) by discretization with respect to frames, i.e., stable, complete, and redundant expansions. We present applications in signal processing/transmission and PDE.

SPEAKER: Peter Gibson
TITLE: Gabor deconvolution of one-dimensional seismic data
ABSTRACT: The last several years have seen a new technique for deconvolution based on the Gabor transform incorporated into industrial seismic image processing, as a replacement for so-called Wiener deconvolution coupled with certain corrections. The Gabor methods are nonstationary, and are thus much better suited to the extraction of reflectivity, of which the data is a nonstationary combination. The real nonstationarity stems from the relationship of the reflectivity to the Green's function of the standard model for a layered earth. In this sense Gabor deconvolution can be viewed as a technique for solving a nonlinear inverse problem while simultaneously removing the effects of a non-Dirac source signal. In this talk we will describe in detail the theory and implementation of Gabor deconvolution as it is applied to actual seismic data.

SPEAKER: Karlheinz Gr ochenig
TITLE: Mapping properties of localization operators
LINK-Author: http://ibb.gsf.de/homepage/karlheinz.groechenig/
ABSTRACT: We will discuss the mapping properties of localization operators, which are a version of non-stationary filters. Planned topics:

1. Boundedness of localization operators on modulation spaces,
2. What happens when the symbols are rough?
3. Composition of localization operators
4. The range of a localization operator.

SPEAKER: Christopher Heil
TITLE: The Homogeneous Approximation Property for Wavelet Frames
LINK-Author: www.math.gatech.edu/˜heil
ABSTRACT: The Homogeneous Approximation Property is a key property of Gabor systems which leads to necessary conditions for Gabor frames in terms of the Beurling density of the associated sequence of time-frequency shifts of the generator. We show that, with some restrictions, wavelet frames and wavelet Schauder bases also satisfy an analogue of the Homogeneous Approximation Property with respect to the affine group, and that this leads to necessary conditions for existence in terms of an affine Beurling density. However, in stark contrast to the Gabor case, we show that the density depends on the generator, and there is no Nyquist density. This is joint work with Gitta Kutyniok.

SPEAKER: Felix Herrmann
TITLE: Non-linear seismic data regularization and separation with directional curvelet frames
LINK-Preprint: zoozoo.eos.ubc.ca/ felix/BIRS_prep.pdf
ABSTRACT: In this paper, directional frames known as curvelets are applied to solve two important tasks in seismic data processing namely data interpolation and primary-multiple separation. We show that by extending the Fast Discrete Curvelet Transform (from CurveLab at www.curvelet.org) to include non-uniform Fourier Transforms (from NFFT www.math.mu-luebeck.de/potts/nfft/) a new directional frame is defined which is particular suitable to solve non-parametric seismic data interpolation problems. We show that minimizing the ℓ^1-norm as part of inverting the frame synthesis operator gives the sparsest set of curvelet coefficients that explain the unstructured data. Hitting this set with the regularly sampled synthesis operator gives the interpolated result. This approach is a practical application of recent ideas on robust uncertainty principles.

The second topic involves using curvelets to separate two signal components – the primaries and multiples with the multiples constituting those events that include a bounce at the surface. The aim is to separate the multiples from the primaries in the presence of noise and given an inaccurate prediction for the multiples. The main distinction of this signal separation problem is that the two signal components are sparse in the same frame as opposed to the signal components in Morphological Component Analysis. We show that we arrive at an viable alternative to match filtering approaches by formulating this signal-separation problem in terms of a weighted ℓ^1 optimization problem with the weights defined by the predicted multiples.

SPEAKER: Franz Hlawatsch
TITLE Linear Methods for Time-Frequency Filtering (joint work with Gerald Matz)
ABSTRACT: Time-frequency (TF) filters are linear time-varying (LTV) systems whose filter characteristics (gain/attenuation, pass/stop) are specified in the TF domain. Such a TF filter specification is convenient and intuitive in many applications. In this talk, we discuss various linear TF filter methods that can be grouped into the following two broad classes.

- Explicit filter design: The LTV filter is calculated such that a TF representation ("symbol") of the filter is equal to, or optimally approximates, a given TF weight function. A variation of this principle using an orthogonal projector side constraint results in "time-frequency projection filters" with very sharp time-frequency pass/stop selectivity.
- Implicit filter design: The LTV filter is implemented as an analysis-weighting-synthesis scheme based on a linear TF representation (an example of such a TF filter is provided by a Gabor multiplier). Thus, the filter is designed implicitly during the filtering process.

We also explain the connections of TF filters with the theory of underspread operators and TF transfer functions. The performance and application of the TF filters presented is demonstrated through numerical simulation.

SPEAKER: Palle Jorgensen
TITLE: Computation of wavelet coefficients in generalized multiresolution systems
LINK-Preprint: http://arxiv.org/abs/math.CA/0405301
ABSTRACT: We consider wavelets in $L^2(R^d)$ which have generalized multiresolutions. This means that the initial resolution subspace V_0 in $L^2(R^d)$ is not singly generated. As a result, the representation of the integer lattice Z^d restricted to V_0 has a nontrivial multiplicity function. We show how the corresponding analysis and synthesis for these wavelets can be understood in terms of unitary-matrix-valued functions on a torus acting on a certain vector bundle. Specifically, we show how the wavelet functions on R^d can be constructed directly from the generalized wavelet filters.

SPEAKER: John Klauder
TITLE: Signal Transmission in Passive Media
ABSTRACT: Under rather general assumptions, and in a relatively simple and straightforward manner, it is shown that the characteristics of signals which travel through homogeneous, as well as inhomogeneous, passive media have the principal features usually associated with the phenomena of precursors, as generally follows from more elaborate studies. The simplicity of the present arguments permit analytic studies to be made for a greater variety of media than is normally the case.

SPEAKER: Gitta Kutyniok
TITLE: Shearlets: Sparse Directional Representations of Images within a Multiresolution Analysis Structure
LINK-Preprint: http://www.math.uni-giessen.de/Numerik/gittak/publications.html
ABSTRACT: In this talk we describe a new class of multidimensional representation systems, called shearlets. They are obtained by applying the actions of dilation, shear transformation and translation to a fixed function, and exhibit the geometric and mathematical properties, e.g., directionality, elongated shapes, scales, oscillations, recently advocated by many authors for sparse image processing applications. In contrast to other approaches these systems can be studied within the framework of a generalized multiresolution analysis, which leads to a recursive algorithm for the implementation of these systems, that generalizes the classical cascade algorithm. This is joint work with Demetrio Labate.

SPEAKER: Michael Lamoureux
TITLE: The Rotation Algebra in Time-Frequency Analysis
ABSTRACT: The translation and modulation operators that appear in the Gabor transform generate a representation of a well-studied family of operators on Hilbert space, known as the rotation algebras. These algerbas arise naturally in physics in the study of Bloch electrons, and mathematically are noncommutative

generalizations of a two torus. We will present some of the basic properties of this field of algebras and their connection with Gabor theory.

SPEAKER: David Larson
TITLE: Frames and Operator Theory
ABSTRACT: A few years ago the speaker and his collaborators developed an operator-interpolation approach to wavelets and frames using the local commutant (i.e. commutant at a point) of a unitary system. This is really an abstract application of the theory of operator algebras to wavelet and frame theory. The concrete applications of operator-interpolation to wavelet theory include results obtained using specially constructed families of wavelet sets. Our methods include the construction of certain groups of measure preserving transformations, and groups and algebras of operators, with special algebraic properties. Other results include applications of a theory of projection decompositions of positive operators, and a theory of operator-valued frames. We will discuss some unpublished and partially published results, and some brand new results, that are due to this speaker and his former and current students, and other collaborators.
Note this talk was cancelled due to travel delays.

SPEAKER: Gary Margrave
TITLE: A stable, explicit nonstationary filter for wavefield extrapolation
ABSTRACT: We present a new approach to the design of stable and accurate wavefield extrapolation operators needed for explicit depth migration. We split the theoretical operator into two component operators, one a forward operator that controls the phase accuracy and the other an inverse operator, designed as a Wiener filter that stabilizes the first operator. Both component operators are designed to have a specific fixed length and the final operator is formed as the convolution of the components. We utilize this operator design method to build an explicit, wavefield extrapolation method based on the migration of individual source records. Two other features of our method are the use of dual operator tables, with high and low levels of evanescent filtering, and frequency-dependent spatial down sampling. Both of these features improve the accuracy and efficiency of the overall method. Empirical testing shows that our method has a similar performance to the time-migration method called phase shift, meaning it scales as NlogN. We illustrate the method with tests on the Marmousi synthetic dataset. We call our method FOCI which is an acronym for forward operator conjugate inverse.

SPEAKER: Ross Mitchell
TITLE: Time/Frequency Applications in Medical Imaging
ABSTRACT: Medical imaging research at the Hotchkiss Brain Institute, University of Calgary, is focused on the application of mathematics, computer science, physics and engineering to help understand, diagnose, treat and monitor neurological disease. Several multi-disciplinary research teams, consisting of both basic scientists and clinicians, have been deployed within Foothills Medical Center. This seminar will provide an overview of the Fourier-based medical imaging modalities of computerized tomography and magnetic resonance imaging. It will then cover several neurological applications of time/frequency analysis. In particular, our team is using time/frequency techniques to investigate signals and images from patients suffering from stroke, brain cancer, multiple sclerosis, and epilepsy. We believe that time/frequency techniques have tremendous potential to advance the science of medical imaging, and improve outcomes for patients.
Note: this presentation will be targeted towards a non-medical audience. Nevertheless, it may contain some graphic images.

SPEAKER: Kasso Okoudjou
TITLE: On some Fourier multipliers for modulation spaces
LINK-Author: http://www.math.cornell.edu/~kasso
ABSTRACT: In this talk, I will use some time-frequency analysis techniques to study the continuity propeprties of a class of Fourier multipliers on the modulation spaces. It must be pointed out that, in general, these Fourier multipliers are known to be unbounded on Lebesgue spaces.

SPEAKER: Alessandro Oliaro

TITLE: Continuity of localization operators in L^p spaces
ABSTRACT: We study some properties of two-wavelet localization operators, i.e., operators which depend on a symbol and two different windows. In the case when the symbol F belongs to $L^p(\mathbb{R}^{2n})$, we give results on $L^q(\mathbb{R}^n)$ boundedness, non-boundedness and compactness of the corresponding operator.

SPEAKER: Goetz Pfander
TITLE: Sampling of operators and channel measurements

SPEAKER: Mauricio Sacchi
TITLE: On the Regularization of the Local Radon Transform - Applications to Seismic Coherent Noise Atenuation

SPEAKER: Zuowei Shen
TITLE: Deconvolution: A wavelet frame approach.

SPEAKER: Chris Stolk
TITLE: Combining finite elements and geometric wave propagation in 1-D
ABSTRACT: We consider the initial value problem for a strictly hyperbolic partial differential equation on the circle. We transform the equation to an operator valued ODE $du/dt = R(t)u$, where $R(t)$ is bounded. The transformation involves application of differential operators, solving an elliptic differential equation, and applying a coordinate transformation involving the characteristics, which can be done at cost $O(N)$. The resulting ODE is solved using a multiscale time-stepping method, which results in an $O(N)$ algorithm for the original hyperbolic equation.

SPEAKER: Thomas Strohmer
TITLE: Capacity of time-varying channels and pseudodifferential operators

SPEAKER: Joachim Toft
TITLE: Schatten properties for pseudo-differential operators and localization operators on modulation spaces of Hilbert type
ABSTRACT: Schatten-von Neumann (SN) classes are spaces of linear and continuous operators between Hilbert spaces. The largest SN class consists of of continuous operators, and all other SN classes are subsets of compact operators, where in particular the smallest SN-class is the set of trace-class operators. Consequently, by using such classes, it is possible to make a detailed study on compactness. In general it is hard to decide if an explicit operator belongs to a certain SN class or not. One is therefore forced to search embedding properties between SN classes and well-known spaces. In the past, such embeddings have been established between SN classes in context of pseudo-differential operators (psdo) acting on L^2, and Besov, Sobolev or modulation spaces. In the talk we present a non-trivial generalization of embedding between SN classes in psdo and modulation spaces, where the L^2 here above is replaced by general modulation spaces of Hilbert type. This generalization is obtained by a combination of careful use of time-frequence methods and Hilbert space techniques.

SPEAKER: Bruno Torresani
TITLE: Identifying sparse hybrid time-frequency models
ABSTRACT: Several signal families may be adequately modeled as sparse expansions with respect to unions of time-frequency bases or frames. We shall focus on probabilistic models involving several layers of randomness (sparse subset of the dictionary, coefficients of the expansion,...) and the corresponding estimation algorithms . A couple of two-steps estimation procedures will be studied and compared. Thesretical estimates as well as numericalm results will be presented.

6 Outcome of the Meeting

The best outcome of this meeting was to get the theoreticians and the applied researchers talking together. Many of the theoretical people have not been aware of the details nor the great extent to which applied researchers have been making use of time-frequency ideas in concrete applications. In fact, researchers have created commercial software and hardware in imaging (medical, seismic, etc) and telecommunications (cellphones) that take advantage of these techniques, and have developed a whole vocabulary that is distinct from the theoretical work. The applied researchers have been similarly unaware of details of extensive theoretical work that has been done on the mathematics of time-frequency analysis which will directly benefit the applications. In particular in optimal choices of frames, deconvolution work in wavelet bases, properties of localization operations which are particularly suited for rapid numerical computations, design and implementation of Gabor multipliers and other time-frequency filters all depend on good theoretical work.

It has been particularly useful to bring together the European and the North American researchers. The Vienna school is outstanding in its theoretical work and is developing a strong applied connections. The organizers at Calgary are very pleased to be learning about this theoretical work and begin applying it to their large project in the mathematics of seismic imaging. We expect further collaborations to develop from these new connections.

7 Participants

Ali, S. Twareque	Concordia University and Institut des Sciences Mathematiques
Balazs, Peter	Austrian Academy of Science
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Bodmann, Bernhard	University of Waterloo
Casazza,Peter	University of Missouri
de Hoop, Maarten	Purdue University
Drabycz, Sylvia	University of Calgary
Feichtinger, Hans	University of Vienna
Fishman, Lou	University of Calgary/MDF International
Fornasier, Massimo	University of Vienna and University of Rome, La Sapienza
Gibson, Peter	York University
Gröchenig, Karlheinz	Institute of Biomathematics and Biometry, Munich
Heil, Christopher	Georgia Tech
Herrmann, Felix	University of British Columbia
Hlawatsch, Franz	Vienna University of Technology
Hogan, Chad	University of Calgary
Jorgensen, Palle	The University of Iowa
Klauder, John	University of Florida
Kutyniok, Gitta	Justus-Liebig-University, Giessen
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Stolk, Chris	University of Twente
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Vasudevan, Kris	University of Calgary
Yedlin, Matt	University of British Columbia

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