ABSTRACTS

M. Asakura. On dlog image of K_2 of elliptic surface minus singular fibers

Let U be a nonsinular (not necessarily complete) variety over an algebraically closed field of characteristic zero. There are the dlog maps $K_2(U) \to H^2(U)$ to Betti etc. In this talk we study the rank of dlog image (which does not depend on cohomology theories), when U is an elliptic surface minus singular fibers. I introduce an arithmetic method to give an upper bound. This also allows us to construct indecomposable parts in Bloch's $\mathrm{CH}^2(X,1)$ in special examples.

S. Bloch. Motives associated to graphs

P. Brosnan. Jumps in Archimedean height (joint with Greg Pearlstein)

Let $f : X \to S$ be a family of varieties of relative dimension n smooth over the complement of a normal crossing divisor D in S. Let A and B be two families of cycles in X such that $\dim A_s + \dim B_s + 1 = n$. Suppose further that A_s does not meet B_s when s is not in D. To this data, one associates a height function $h(s) =< A_s, B_s >: S - D \to R$ whose properties have been studied extensively by R. Hain. We say that the height jumps on D if it cannot be written as $\log |f|$ for f meromorphic. (Actually, the true definition is more technical, but this white lie is acceptable for an abstract.) Using Pearlstein's generalization of Schmid's orbit theorems to admissible variations of mixed Hodge structure, we give a criterion for h to jump. Moreover, it turns out that the "jump" is a computable quantity which is in some ways more intrinsic than the height itself as it only depends on the normal functions associated to A and B. By computing the jump, we are able to compute the asymptotics of the height associated by Hain to the Ceresa cycle as one tends to the boundary in the moduli space of curves in a multi-parameter family.

Rob de Jeu. Numerical verification of a conjecture of Perrin-Riou for number fields

Perrin-Riou proposed a conjecture about relations between p-adic regulators and p-adic L-functions that, in special cases, one can interpret as a p-adic analogue of Beilinson's conjecture. I want to report on joint work with Amnon Besser, Paul Buckingham and Xavier-Franqis Roblot concerning the numerical verification of this conjecture for certain totally real number fields.

H. Esnault. Feynman motives

H. Gangl. Multiple polylogarithms, polygons and algebraic cycles (joint work with A.B. Goncharov and A. Levin) We give a DGA on polygons which can be mapped to Bloch's DGA on algebraic cycles and to Goncharov's motivic iterated integrals. The Hodge realization of the associated algebraic cycle is also described via the combinatorics of polygons, its main part giving a multiple polylogarithm.

T. Geisser. An integral version of Kato's conjecture

I will give an integral version of a conjecture of Kato on the exactness of the Kato complex over a finite field. The integral version is equivalent to Kato's conjecture and the vanishing of certain rational higher Chow groups. We also give an application to class field theory over finite fields.

H. Gillet. Motivic cohomology and arithmetic Chow groups

A few year ago Goncharov gave a construction of a regulator map for Bloch's higher Chow groups, and gave a definition of "arithmetic higher Chow groups". I will outline a related costruction for motivic cohomology. The motivation for this is to have a purely sheaf theoretic construction of the arithmetic Chow groups and their product structure. There main difficulty to be overcome is analytic - determining a suitable complex of forms "with mild singularities" which is both the target of a regulator map, and can be used to compute Dolbeault cohomology. However the relationship between these groups and the usual arithmetic Chow groups of a variety over the integers depends on knowing that motivic cohomology computes the usual Chow groups – which still seems to be open.

M. Hanamura. Mixed motivic sheaves

I explain (1) the construction of the triangulated category of mixed motivic sheaves over a base variety (generalizing my previous work of mixed motives over a field), and (2) the degeneration theory of motives (motivic version of the degeneration theory of Steenbrink).

K. Kimura. Murre's conjectures for certain product varieties

We consider Murre's conjectures on Chow groups for a fourfold which is a product of two curves and a surface. We give a result which concerns Conjecture D: the kernel of a certain projector is equal to the homologically trivial part of the Chow group. We also give a proof of Conjecture B for a product of two surfaces. math.AG/0509249

A. Langer. Displays and crystalline cohomology

For a ring R such that a given prime p is nilpotent we construct a category of higher displays which generalizes Zink's category of displays classifying p-divisible groups. Using the relative de Rham Witt-complex we impose a display structure on the crystalline cohomology of certain smooth schemes over R which are close to abelian or Calabi-Yau manifolds. This gives rise to a relative Dieudonne-theory and generalizes earlier work of Fontaine and Kato.

J. Lewis. Motivic normal functions

We propose an explicit description of a candidate Bloch-Beilinson filtration on Bloch's higher Chow groups for projective algebraic manifolds, in terms of a variational version of the Abel-Jacobi map, namely an arithmetical normal function. The ideas presented here reflect the speaker's take on joint work in progress with Matt Kerr, who independently arrived at similar results.

Paulo Lima-Filho. From Motivic to Deligne to Bredon: realizations for real varieties

In this talk we will briefly explain how the $RO(Z/2)$-graded Bredon cohomology of real varieties is a natural realization of motivic cohomology, and why this realization should factor through a form of Deligne cohomology for real varieties. Roughly speaking, Bredon is to motivic cohomology as the more classical Borel cohomology is to etale-motivic cohomology. We present, as a concrete example, the computation of the bigraded Bredon cohomology RING of real quadrics with coefficients in the Mackey functor Z. The result is similar to Yagita's computations of motivic cohomology with finite coefficients.

V. Maillot. Special values of logarithmic derivatives of Artin L-functions (joint work with D. Roessler)

We formulate several variants of a conjecture relating the arithmetic degree of certain hermitian fibre bundles with the values of the logarithmic derivative of Artin's L-functions at negative integers. This generalizes conjectures by Colmez and Gross-Deligne and complements Beilinson's conjectures for the Artin motives. We announce several results in the direction of these statements.

A. Rosenschon. Rigidity

Let F be a cohomology theory with torsion values which is defined for smooth schemes over fields. Given an extension K/k of fields, we ask when one has rigidity in the sense that the map $F(X) \to F(X_K)$ is an isomorphism. For example, this is known to hold in case K/k is an extension of algebraically closed fields, and F is etale cohomology or algebraic K-theory (with finite coefficients). We prove the analogous result for other types of field extensions (not necessarily algebraically closed) in case

F belongs to a certain class of theories including, for example, algebraic K-theory, motivic cohomology, and étale cohomology (with finite coefficients). We outline how to prove a similar result for algebraic cobordism. This is joint work with Østvær.

S. Saito. Homology theory of Kato type and motivic cohomology of arithmetic schemes

The objective of the talk is the étale cycle map:

$$\rho_X^{r,q} \;:\; \mathrm{CH}^r(X,q;\mathbb{Z}/n\mathbb{Z}) \to H^{2r-q}_{\text{ét}}(X,\mathbb{Z}/n\mathbb{Z}(r)).$$

where X is a smooth projective variety over a finite field k (geometric case) or a regular proper flat scheme over the ring $\mathcal{O}_k$ of integers in k, a number field or a local field (arithmetic case). In the latter case we assume that the reduced part of any special fiber of $X \to \mathrm{Spec}(\mathcal{O}_k)$ is a simple normal crossing divisor. The left hand side of the map is the higher Chow group with finite coefficients (S. Bloch and M. Levine), and the right hand side denotes the étale cohomology group with a suitable coefficient. An intriguing question is how close it is to an isomorphism. Thanks to the finiteness results on étale cohomology, it would be a way to approach the finiteness problem for higher Chow groups of arithmetic schemes.

In case $r = d := \dim(X)$ and $q = 0$, $\mathrm{CH}^d(X,0;\mathbb{Z}/n\mathbb{Z}) = \mathrm{CH}^d(X)/n = \mathrm{CH}_0(X)/n$ with $\mathrm{CH}_0(X)$, the Chow group of zero-cycles modulo rational equivalence on X, and the above map is known to be an isomorphism by higher dimensional class field theory (Kato-Saito, Colliot-Thélène-Sansuc-Soulé). In this talk we report some new results extending the result to other cases obtained by joint works with Uwe Jannsen and Kanetomo Sato.

Theorem 0.1. *(Jannsen-Saito) Let X be as above. Let $d = \dim(X)$. Fix a prime p. In arithmetic case assume $d \leq 3$ or X has good reduction over every point of $\mathrm{Spec}(\mathcal{O}_k)$ of characteristic p. Then cycle map*

$$\rho_X^{d,1} \;:\; \mathrm{CH}^d(X,1;\mathbb{Z}/p^n\mathbb{Z}) \to H^{2d-1}_{ét}(X,\mathbb{Z}/p^n\mathbb{Z}(d)),$$

is an isomorphism if $\mathbf{BK}_3(/k,\mathbb{Z}/p\mathbb{Z})$ (see below) holds.

In particular $\mathrm{CH}^d(X,1;\mathbb{Z}/p^n\mathbb{Z})$ is finite under the assumption.

For a perfect field F and an integer $q \geq 1$ we introduce the following condition:

$\mathbf{BK}_q(/F,\mathbb{Z}/p\mathbb{Z})$: For any finitely generated field L over F the Galois symbol map

$$h^q_{L,\mathbb{Z}/p\mathbb{Z}} : K_q^M(L) \to H^q(\mathrm{Spec}(L)_{\text{ét}},\mathbb{Z}/p\mathbb{Z}(q))$$

is surjective, where $K_q^M(L)$ is the Milnor K-group of a field.

In order to extend 0.1 to $\rho_X^{d,1}$ for $q > 1$, we need a condition $(\mathbf{RES})_{\mathbf{q}}$ on resolution of embedded singularities for subvarieties of dimension$\leq q$ in a smooth variety endowed with a simple normal crossing divisor as a boundary.

Theorem 0.2. *(Jannsen-Saito) Let X be a smooth projective variety of dimension d over a finite field F. For an integer $q \geq 1$, the cycle map*

$$\rho_X^{d,q} \;:\; \mathrm{CH}^d(X, q; \mathbb{Z}/p^n\mathbb{Z}) \to H_{\acute{e}t}^{2d-q}(X, \mathbb{Z}/p^n\mathbb{Z}(d)),$$

is an isomorphism if $\mathbf{BK}_{q+2}(/F, \mathbb{Z}/p\mathbb{Z})$ *and* $(\mathbf{RES})_\mathbf{q}$ *hold.*

In particular $\mathrm{CH}^d(X, q; \mathbb{Z}/p^n\mathbb{Z})$ *is finite under the assumption.*

$(\mathbf{RES})_\mathbf{q}$ is shown to hold for $q = 1$ so that 0.1 in geometric case is a special case of 0.2.

Concerning 1-cycles on an arithmetic scheme over the ring of integers in a local field, we have the following result.

Theorem 0.3. *(Saito-Sato) Let X be a regular projective flat scheme over the ring $\mathcal{O}_k$ of integers in a local field k such that the reduced part of the special fiber is a normal crossing divisor on X. Let n be an integer invertible in $\mathcal{O}_k$. Let $d = \dim(X)$. Then the cycle map*

$$\rho_X^{d-1,0} \;:\; \mathrm{CH}^{d-1}(X)/n \to H_{\acute{e}t}^{2d-2}(X, \mathbb{Z}/n\mathbb{Z}(d-1)),$$

is an isomorphism. In particular $\mathrm{CH}^{d-1}(X)/n = CH_1(X)/n$ *is finite.*

The basic idea of the proof is to reduce it to showing the *global* vanishing of E^2 terms of the niveau spectral sequence associated by the method of Bloch-Ogus to étale homology theory on a suitable category of schemes. The reduction depends on the result of Suslin-Voevodsky and Geisser-Levine that the Bloch-Kato conjecture implies the Beilinson-Lichtenbaum conjecture. This kind of vanishing result was first studied by K. Kato as an application of higher dimensional class field theory. We study the problem in a more general context by introducing a notion of a homology theory of Kato type which is a homology theory on a category of schemes satisfying certain axioms and then show the vanishing result for E^2 terms of the associated Bloch-Ogus spectral sequence.

V. Snaith. Stark's Conjecture and new Stickelberger phenomena

I will introduce a new conjecture concerning the construction of elements in the annihilator ideal associated to a Galois action on the higher-dimensional algebraic K-groups of rings of integers in number fields. The conjecture is motivic in the sense that it involves the (transcendental) Borel regulator as well as being related to l-adic étale cohomology. In addition, the conjecture generalises the well-known Coates-Sinnott conjecture. For example, for a totally real extension when $r = -2, -4, -6, \ldots$ the Coates-Sinnott conjecture merely predicts that zero annihilates K_{-2r} of the ring of S-integers while our conjecture predicts a non-trivial annihilator. By way of supporting evidence, we prove the corresponding (conjecturally equivalent) conjecture for the Galois action on the étale cohomology of the cyclotomic extensions of the rationals.

If time permits I shall also describe work in progress on the analogous conjecture for elliptic curves over number fields.

Z. Wojtkowiak. On Galois action on torsors of paths

We are studying representations obtain from actions of Galois groups on torsors of paths on a projective line minus a finite number of points. Using these actions on torsors of paths we construct geometrically representations of Galois groups which realize ℓ-adically the associated graded Lie algebra of the fundamental group of the tannakien category of mixed Tate motives over $\operatorname{Spec}\mathbb{Z}$, $\operatorname{Spec}\mathbb{Z}[\frac{1}{q}]$, $\operatorname{Spec}\mathcal{O}_{\mathbb{Q}(\sqrt{-q})}$ for any prime number q ($q \neq 2$ in the last case) and over $\operatorname{Spec}\mathcal{O}_{\mathbb{Q}(\sqrt{-q})}[\frac{1}{q}]$ for any prime number q congruent to 3 modulo 4.