

Speaker: Bojko Bakalov

Title: Lie Pseudoalgebras

Abstract: One of the algebraic structures that has emerged recently in the study of the operator product expansions of chiral fields in conformal field theory is that of a Lie conformal algebra. A Lie pseudoalgebra is a "higher-dimensional" generalization of the notion of a Lie conformal algebra. On the other hand, Lie pseudoalgebras can be viewed as Lie algebras in certain pseudo-tensor categories.

I will review the classification of finite simple Lie pseudoalgebras, and I will discuss their relationship to solutions of the classical Yang-Baxter equation and to linear Poisson brackets. I will also describe the irreducible representations of the Lie pseudoalgebra $W(\mathfrak{d})$, which is closely related to the Lie-Cartan algebra W_N of vector fields, where $N = \dim \mathfrak{d}$. (Based on a joint work with A. D'Andrea and V. G. Kac.)

Speaker: Jon Brundan

Yangians, Whittaker modules and cyclotomic Hecke algebras.

There has recently been some progress in understanding some algebras introduced originally by Kostant in 1978. These algebras can be viewed as quantizations of the Slodowy slice associated to a nilpotent orbit in a semisimple Lie algebra. In type A, it turns out that these quantizations of the Slodowy slice are closely related to the Yangian of the Lie algebra \mathfrak{gl}_n . Actually, they are generalizations of the Yangians which we call shifted Yangians.

In recent work with A. Kleshchev, we have worked out the combinatorics of the finite dimensional representations of shifted Yangians. The approach uses in an essential way a theorem of Skryabin relating representations of these algebras to certain categories of generalized Whittaker modules. In particular, we are able to reprove and generalize the known results about representations of Yangians, all as a direct application of the Kazhdan-Lusztig conjecture.

There is also a close connection between shifted Yangians and the degenerate cyclotomic Hecke algebras, thanks to a Schur-Weyl duality which interpolates between the classical Schur-Weyl duality and

Drinfeld's affine analogue of it. This leads to a natural representation theoretic construction of some higher level Fock spaces for the Lie algebra \mathfrak{gl}_∞ , complete with their dual canonical bases.

Speaker: Jacob Greenstein

An application of free Lie algebras to current algebras

We realize the current algebra of a Kac-Moody algebra as a quotient of a semi-direct product of the Kac-Moody Lie algebra and the free Lie algebra of the Kac-Moody algebra. We use this realization to study the representations of the current algebra. In particular we see that every ad -invariant ideal in the symmetric algebra of the Kac-Moody algebra gives rise in a canonical way to a representation of the current algebra. These representations include certain well-known families of representations of the current algebra of a simple Lie algebra. Another family of examples, which are the classical limits of the Kirillov-Reshetikhin modules, are also obtained explicitly by using a construction of Kostant. Finally we study extensions in the category of finite dimensional modules of the current algebra of a simple Lie algebra.

Speaker: Mark Haiman

A combinatorial formula for Macdonald polynomials

I'll explain recent joint work with Jim Haglund and Nick Loehr, in which we prove a combinatorial formula for the Macdonald polynomial $\tilde{H}_{\mu}(x; q, t)$ which had been conjectured by Haglund. Such a combinatorial formula had been sought ever since Macdonald introduced his polynomials in 1988.

The new formula has various pleasant consequences, including the expansion of Macdonald polynomials in terms of LLT polynomials, a new proof of the charge formula of Lascoux and Schützenberger for Hall-Littlewood polynomials, and a new proof (and more general version) of Knop and Sahi's combinatorial formula for Jack polynomials.

In general, our formula doesn't yet give a new proof of the positivity theorem for Macdonald polynomials, because it expresses them in terms

of monomials, rather than Schur functions. However, it does yield a new combinatorial expression for the Schur function expansion when the partition μ has parts ≤ 2 , and there is hope to extend this result.

Speaker: David Hernandez

The Kirillov-Reshetikhin conjecture and solutions of T-systems.

In this talk we present a proof of the Kirillov-Reshetikhin conjecture for all untwisted quantum affine algebras : we prove that the characters of Kirillov-Reshetikhin modules solve the Q-system, and so we get explicit formulas for the characters of their tensor products. Moreover we establish exact sequences involving tensor products of Kirillov-Reshetikhin modules and prove that their q-characters solve the T-system. For simply-laced cases these results were first obtained by Nakajima with geometric arguments which are not available in general. The proof we present is different and purely algebraic, and so can be extended uniformly to non simply-laced cases.

Speaker: Seok-Jin Kang

Title: Combinatorics of Young walls and crystal bases.

We will discuss the construction of irreducible highest weight crystals using Young walls. We will also discuss the possible connection between modular representation theory and crystal bases.

Speaker: Rinat Kedem

Title: Constructions of affine Lie algebra modules via graded tensor products via generalized Kostka polynomials.

The graded tensor product is a tensor product of finite-dimensional \mathfrak{g} -modules, endowed with a \mathfrak{g} -equivariant grading. This grading is related to the action of the loop algebra on the "fusion product" of representations of conformal field theory, and was originally defined

by Feigin and Loktev. A conjecture, which has been proven in some special cases is that the graded multiplicity of an irreducible \mathfrak{g} -module in the graded tensor product is related to the Kostka polynomial or one of its generalized or level-restricted versions.

I will discuss how this graded tensor product allows us to construct integrable modules in two very different ways. One is in terms of the inductive limit of the graded tensor product of an infinite number of \mathfrak{g} -modules. The other is a generalization of the semi-infinite construction of Feigin and Stoyanovksy, which allows us to compute the characters of arbitrary highest weight integrable modules. This last requires use of the inverse of the matrix of generalized Kostka polynomials, and hence gives an interesting alternating sum expression for characters corresponding to non-rectangular highest weights in terms of rectangular ones.

Speaker: Sergei Loktev

Title: Weyl modules over \mathfrak{sl}_r -valued currents

Abstract: We discuss Weyl modules over \mathfrak{sl}_r -valued currents in one and two variable.

For one-dimensional currents a construction of basis, proposed by V.Chari and the speaker, will be described. If there will be enough time, the relation to Demazure modules and fusion modules will be discussed.

For two-dimensional currents relation to the space of diagonal coinvariants and parking functions, observed by B.Feigin and the speaker, will be explained.

Speaker: Kailash Misra

Title: Perfect crystal for $D_4^{(3)}$

Abstract: The crystal base theory developed by Kashiwara and independently by Lusztig provides an important combinatorial tool to study the representations of symmetrizable Kac-Moody algebras. It is known that the crystal base for affine Kac-Moody Lie algebras can be concretely realized as a subset of the semi-infinite tensor products

of perfect crystals. In this talk we will present a perfect crystal for the integrable highest weight $sl_4(3)$ -module of level $k > 0$. This is a joint work with Kashiwara, Okado and Yamada.

Speaker: Adriano A Moura

Title: Blocks of Finite Dimensional Representations of Classical and Quantum Affine Algebras.

Abstract: It is well known that the category of finite dimensional representations of classical or quantum affine algebras is not semisimple. To understand its block decomposition in the quantum case, P. Etingof and the speaker introduced the notion of Elliptic Characters. However, the original definition using analytic properties of the R-matrix imposed some un-natural restrictions to the problem ($|q|$ should be different from 1). In particular, it was unclear how to compute the classical limit of the block decomposition. In this talk based on joint work with V. Chari we present a definition of Elliptic Characters from the point of view of the Braid Group action and the theory of q -Characters. This allow us to obtain the block decomposition for generic q as well as for $q=1$.

SPEAKER: Evgeny Mukhin

TITLE: Multiple orthogonal polynomials in Bethe Ansatz.

ABSTRACT: We show that the Bethe Ansatz equation for the non-homogeneous sl_n Gaudin model and two finite dimensional representations one of which is a symmetric power of vector representation, is solved in term of zeroes of multiple orthogonal Jacobi-Pi\~neiro polynomials. Equivalently, the spaces of polynomials with two finite ramification points with special exponents at one of the points have a basis explicitly given via multiple orthogonal Jacobi-Pi\~neiro polynomials. In a similar way, multiple orthogonal Laguerre polynomials appear in the Bethe Ansatz related to the trigonometric Gaudin model and multiple orthogonal little q -Jacobi polynomials in the Bethe Ansatz related to the XXZ model.

This is a joint work with A. Varchenko.

Speaker: Alejandra Premat
Monomial Bases for Demazure Modules

Abstract:

We will discuss certain monomial bases of quantum Demazure modules for the algebra $U_q(\text{affine-}\mathfrak{sl}_n)$ and show how to compute them using a description of the crystal graphs by Young diagrams. We will also see that the transition matrices from these bases to the Global bases are upper triangular with ones in the diagonal.

Speaker: D. Sagaki - S. Naito

Crystal of Lakshmibai-Seshadri paths associated
to a level-zero integral weight for an affine Lie algebra

Let $\lambda = \sum_{i \in I_0} m_i \varpi_i$, with $m_i \in \mathbb{Z}_{\geq 0}$, be an integral weight of level zero that is a sum of level-zero fundamental weights ϖ_i , $i \in I_0$, for an affine Lie algebra \mathfrak{g} . We study a certain crystal $\mathbb{B}(\lambda)_{cl}$, which is (modulo the null root of \mathfrak{g}) the crystal of all Lakshmibai-Seshadri paths of shape λ , and prove that the $\mathbb{B}(\lambda)_{cl}$ is isomorphic as a crystal to the tensor product $\bigotimes_{i \in I_0} \mathbb{B}(\varpi_i)_{cl}^{\otimes m_i}$ of the crystals $\mathbb{B}(\varpi_i)_{cl}$, $i \in I_0$. Here we note that for each $i \in I_0$, the $\mathbb{B}(\varpi_i)_{cl}$ turns out to be isomorphic as a crystal to the crystal base of the level-zero fundamental module $W(\varpi_i)$ over the quantum affine algebra $U_q^{\prime}(\mathfrak{g})$.

Speaker: Anne Schilling

Title: Crystal structure on rigged configurations

Abstract: Rigged configurations label the Bethe vectors of a given spin model. According to a bijection by Kirillov and Reshetikhin (generalized by Kirillov, S., Shimozono) rigged configurations correspond to highest weight crystal paths. The natural question

arises whether there exist "unrestricted" rigged configurations corresponding to any crystal path, not necessarily highest weight. In this talk we define unrestricted rigged configurations and describe the crystal structure on this set.

Speaker: Mark Shimozono

Title: Schubert calculus on the affine Grassmannian

Abstract: We present a generalization of the Robinson-Schensted-Knuth correspondence which conjecturally realizes the Cauchy identity that gives the perfect pairing between the Schubert bases of cohomology and homology of the affine Grassmannian of type $A_{n-1}^{(1)}/A_{n-1}$. This involves two kinds of tableaux that are defined using respectively the weak and strong Bruhat orders on the affine Weyl group. When n goes to infinity the bijection converges to the usual RSK map. We state a Pieri rule for the multiplication in cohomology, which uniquely determines the basis.

We are also investigating the properties of a jeu de taquin algorithm on weak order tableaux which may lead to a rule for the structure constants for homology. These constants generalize the fusion Littlewood-Richardson coefficients that come from the tensor product of representations at a given level.

This is ongoing joint work with Thomas Lam, Luc Lapointe, and Jennifer Morse.

Speaker: Catharina Stroppel

Title: The classification of projective functors for Kac-Moody Lie algebras

We consider the Bernstein-Bernstein category \mathcal{O} attached to a semisimple complex Lie algebra. Projective functors are the direct summands of the functors given by tensoring with finite dimensional representations. These functors were classified by Bernstein and Gelfand. We want to give an alternative approach to this classification using deformation theory. We will explain how this alternative proof can be generalized to the Kac Moody situation giving rise to a classification of projective functors. As an explanation we briefly mention the connection to knot and tangle invariants.

Speaker: S. Viswanath

Dynkin diagram sequences and tensor product stabilization

In this talk, we will consider sequences of Dynkin diagrams Z_k of the form $X \cdots Y$ where X and Y are two fixed dynkin diagrams and k is the number of intermediate nodes. The classical series A_k, B_k, C_k, D_k are all of this form and we can construct many more such series of indefinite Kac-Moody algebras as well (e.g $E_n, G_n, (E-E)_n, \dots$).

Our goal will be to show that for the Z_k , multiplicities of irreducible representations in tensor product decompositions exhibit a stabilization behavior as $k \rightarrow \infty$. This parallels the situation for the series A_k where this result is implied directly by the Littlewood-Richardson rule. We'll use Littelmann's path model to do this.

The stable values of these multiplicities can be used as structure constants to define a "stable tensor product" operation on a space $\mathcal{R}(X|Y)$ that could be called the "stable representation ring". We'll show that this multiplication operation is indeed associative, making $\mathcal{R}(X|Y)$ a bonafide C algebra that captures tensor products in the limit $k \rightarrow \infty$.

Speaker: Milen Yakimov

General finiteness of the fusion tensor product

Kazhdan and Lusztig proved a finiteness result for the fusion tensor product for smooth modules over an affine Kac-Moody algebra which can be viewed as an analog of the fact that the product of finite dimensional modules over a simple Lie algebra is finite dimensional. In the classical situation Kostant's theorem from the late 70's provides a much more general finiteness: for any subalgebra k of a complex simple Lie algebra g which is reductive in g , the category of finite length, admissible (g,k) -modules is stable under tensoring with finite dimensional g -modules (with applications to category O , Harish-Chandra modules, etc.). We will

describe a proof of an analog of this theorem for the fusion tensor product of smooth affine modules, based on an approach different from the one of Kazhdan and Lusztig.