Homology stability of moduli of vector bundles over a curve

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1 Overview

Arapura and Dhillon spent an intensive and productive week at BIRS, under the auspices of the Research in Teams programme. During this period, they were able to produce an outline of a research project, described below, on the moduli of bundles over a curve. Although some more work will be needed to flesh out the details, a finished paper is expected to result from this research in the near future.

2 Mathematical Details

Let $C$ be a smooth projective curve of genus $g \geq 2$ over field of complex numbers, and let $G = G_n$ be a classical group (i.e. one of $GL_n, SL_n, SO_n, Sp_n$). The research project involves the study of the moduli stack $Bun_G(C)$ (respectively moduli space $M_G(C)$) of (stable) principal $G$-bundles over $C$. When $G = GL_n$, these objects can be identified with the moduli stack or space of vector bundles. The basic goal is to understand the Hodge structure, and the underlying motive, on the cohomology of $Bun_G(C)$ and $M_G(C)$ as $C$ varies. This can be reduced to a series of subproblems:

1. Construct a relative theory of motives in the spirit of Andrè [1].
2. Use an Atiyah-Bott type isomorphism [3, 6] to “compute” the motive of $Bun_G(C)$ in terms of the motive of $C$. Apply this to the universal curve.
3. Find good estimates to relate the cohomology and motive of $Bun_G(C)$ to that of $M_G(C)$. For vector bundles, suitable estimates have been found in [2, 5]. In general, some of the basic tools are contained in [4].

Since the estimates in 3 should grow with $n$, this can be used to show that $H^*(M_{G_n}(C))$ stabilizes, as expected.

References


