

**THE BIRS FIVE DAY WORKSHOP
MODULAR FORMS AND STRING DUALITY
JUNE 3–6, 2006
ABSTRACTS**

JUNE 4, 2006

9:00am–10:00am **Don Zagier** (MPIM Bonn and College de France)

Introduction: Modular Forms and Differential Equations

10:15am–11:15am **Ron Livné** (Hebrew and IAS Princeton)

Modularity of Galois Representations: Overview

Given a two dimensional Galois representation of geometric origin, over a field, it is conjectured, and often known, that it arises from a modular form for $GL(2)$. Even when general theorems do not exist (yet), it is sometimes possible to prove a given instance of the conjecture by making a guess and verifying it. In this survey talk we will give a miscellany of results in these directions.

11:30am–12:30pm **Bong H. Lian** (Brandeis)

An Introduction to Mirror Symmetry

This will be a mathematical survey of both the history and development of mirror symmetry beginning with the early suggestions from physics. Topics may include

- 0) Two dimensional super conformal field theory
- 1) Basic examples of mirror manifold constructions
- 2) Mathematical predictions of mirror symmetry
- 3) Fourier-Mukai transforms

3:00pm–4:00pm **Jan Stienstra** (Utrecht)

From multi-grid to multi-helix; remarkable geometries from AdS/CFT

A 1-grid with parameters a, b, c is the system of parallel equidistant lines in the plane described by the equation $ax + by + c = \text{integer}$. An N -grid is a system of N 1-grids. The 'dual' of an N -grid can be presented as a tessellation of the plane by rhombi. We are interested in grids with integral (a, b) -parameters. These give rise to periodic rhombus tilings. Moreover, viewing (a, b) as a vector the tiles can be equipped with extra pictures. There are simple 'braiding rules' to transform a given rhombus tessellation into a new one. It turns out that by repeated application of the braiding rules one can reach a tiling in which the extra pictures form an interesting pattern, like the projection of a link with a spanning Seifert surface and embedded into the Seifert surface is a bi-partite graph. In the talk this will be illustrated with many pictures. We will also indicate how this relates to the AdS/CFT correspondence, Mirror Symmetry and the McKay correspondence.

4:15pm–5:15pm **Emanuel Scheidegger** (TU Vienna)

Topological Strings on K3 Fibrations and Modular Forms

We explain that motivated by heterotic-type II duality, certain Gopakumar-Vafa invariants (and hence, conjecturally, Gromow-Witten invariants) for Calabi-Yau manifolds that admit a K3 fibration can be collected in a generating function. This function is in general an automorphic form determined by the topology of the fibration. In the class of K3 fibrations in toric varieties in which the Picard lattice of the fiber has rank one, we show how this automorphic form can be determined explicitly.

5:30pm–6:30pm **Helena Verrill** (Louisiana State)

The Batyrev–Borisov Construction of Toric Calabi–Yau Varieties

This talk will give an overview of the Batyrev-Borisov construction of toric Calabi-Yau hypersurfaces and complete intersections. It will start with a quick summary of the definition of toric varieties and reflexive polytopes, discuss properties of toric Calabi-Yaus, and Kreuzer and Skarke’s method of enumerating reflexive polyhedra. Some examples will be discussed, which have been used to find new modular Calabi-Yau threefolds, though the topic of modularity will be left to another speaker. This talk will not introduce new results, and is particularly aimed at graduate students.

JUNE 5, 2006

9:00am–10:00am **Don Zagier** (MPIM Bonn and College de France)

Something about Modular Forms and Their Applications to Physics

10:15am–11:15am **Masanobu Kaneko** (Kyushu)

Modular and Quasimodular Forms and their Applications

I shall review works with Don Zagier on ”mirror symmetry in dimension one” and with Masao Koike on modular and quasimodular solutions of certain differential equation, with a brief mention to their possible connection to conformal field theory.

11:30am–12:30pm **Johannes Walcher** (IAS Princeton)

Opening Mirror Symmetry on the Quintic

Aided by mirror symmetry, we determine the number of holomorphic disks ending on the real Lagrangian in the quintic threefold. The tension of the domainwall between the two vacua on the brane, which is the generating function for the open Gromov–Witten invariants, satisfies a certain extension of the Picard–Fuchs equation governing periods of the mirror quintic. We verify consistency of the monodromies under analytic continuation of the superpotential over the entire moduli space. We reproduce the first few instanton numbers by a localization computation directly in the A -model, and check Ooguri–Vafa integrality. This is the first exact result on open string mirror symmetry for a compact Calabi–Yau manifold.

3:00pm–4:00pm **Gert Almkvist** (Lund)

Apéry-like limits connected with Calabi-Yau differential equations

Let $\sum_{n=0}^{\infty} A(n)x^n$ be the analytic solution of a Calabi-Yau differential equation (4-th order). Then $A(n)$ satisfies a recursion formula with polynomial coefficients with starting values $A(n) = 0$ for $n < 0$ and $A(0) = 1$. Let $B(n)$ satisfy the same recursion with $B(n) = 0$ for $n \leq 0$ and $B(1) = 1$. Very often the limit of $B(n)/A(n)$ exists when $n \rightarrow \infty$. Usually the limit is a rational linear combination of values of L -functions at $s = 2$ or $s = 3$. This is a joint work with van Straten and Zudilin.

4:15pm–5:15pm **Christopher Herzog** (Washington)

How Exceptional Collections Stack Up

I would attempt to give a broad overview of my two recent papers with Robert Karp, hep-th/0507175 and hep-th/0605177. The papers advance the program of using exceptional collections of objects in the derived category of coherent sheaves to understand the low energy gauge theory description of a D-brane probing a Calabi-Yau singularity.

5:30pm–6:30pm **Nam-Hoon Lee** (KIAS)

Constructing Calabi–Yau Manifolds

A smoothing theorem for normal crossings to Calabi-Yau manifolds was proved by Y. Kawamata and Y. Namikawa. This talk is about a study of the observation that the Picard groups and Chern classes of these Calabi-Yau manifolds are constructible from the normal crossings in such smoothings. Various applications will be discussed, including the construction of many new examples of Calabi-Yau 3-folds with Picard number one. With this construction as a starting point, I hope to convince audience that smoothing normal crossings is a promising method of constructing Calabi-Yau manifolds. This talk is based on my recent preprint (math.AG/0604596).

8:00pm–9:00pm **Ling Long** (Iowa State)

Modular Forms for Noncongruence Subgroups

Majority of finite index subgroups of the modular groups are noncongruence subgroups. In the 1970, Atkin and Swinnerton-Dyer have pioneered the investigation on modular forms for noncongruence subgroups. Some of their important observations have been verified by Scholl. Despite that, modular forms for noncongruence subgroups still remains to be very mysterious. In this talk, we will discuss some recent results on the arithmetic properties of modular forms for noncongruence groups.

JUNE 6, 2006

9:00am–10:00am **Adrian Clingher** (Stanford)

Geometry underlying the F-Theory/Heterotic String Duality in Eight Dimensions

One of the dualities in string theory, the F-theory/heterotic string duality in eight dimensions, predicts an interesting correspondence between two seemingly disparate geometrical objects. On one side of the duality there are elliptically fibered K3 surfaces with section. On the other side, one finds elliptic curves endowed with certain flat connections. I will discuss the basic Hodge theoretic framework underlying the duality as well as its consequences in algebraic geometry and number theory.

10:15am–11:15am **Sergei Gukov** (CalTech)

Strings, Fields, and Arithmetic

String theory and quantum field theory are known to have many deep connections and applications to geometry and topology. In recent years, new connections between string/field theory and number theory started to emerge. Examples include a relation between elliptic genera and Jacobi forms, complex multiplication and black hole attractors/RCFTs, etc. In this talk, I will review some the known relations and in the end present new ones.

11:30am–12:30pm **Noriko Yui** (Queen's)

Motives, Mirror Symmetry and Modularity

We consider certain families of Calabi–Yau orbifolds and their mirror partners constructed from Fermat hypersurfaces in weighted projective spaces. We use Fermat motives to interpret the topological mirror symmetry phenomenon. These Calabi–Yau orbifolds are defined over \mathbf{Q} , and we can discuss the modularity of the associated Galois representations. We address the modularity question at motivic level. We give some examples of modular Fermat motives. We then formulate a modularity conjecture about rank 4 Fermat motives that there exist Siegel modular forms on some congruence subgroups of $Sp(4, \mathbf{Z})$.

9:00am–10:00am **Albrecht Klemm** (Wisconsin)

Modular, Quasimodular Forms and Gromov–Witten Invariants

10:15am–11:15am **Ron Livné** (Hebrew and IAS Princeton)

Explicit descriptions of universal $K3$ families over Shimura curves

It is quite hard to give explicit algebraic description of abelian varieties of dimension > 1 . However, the Kummer surfaces of abelian surfaces - and sometimes related $K3$ surfaces - do have useful projective models. In joint work with A. Besser we give some instances where this can be done universally over Shimura curves over \mathbf{Q} . While the universal families of abelian surfaces exist only when rather high level is added to the moduli problem, our universal $K3$ fibrations exist over very low level Shimura curves - often level < 1 , allowing particularly simple descriptions.

11:30am–12:30pm **Chuck Doran** (Washington)

Algebraic Topology of Calabi–Yau Threefolds in Toric Varieties

We compute the integral homology (including torsion), the topological K-theory, and the Hodge structure on cohomology of Calabi-Yau threefold hypersurfaces and complete intersections in Gorenstein toric Fano varieties. The methods are purely topological. This is joint work with John Morgan.

3:00pm–4:00pm **Marco Aldi** (Northwestern)

Twisted homogeneous coordinate rings of abelian surfaces via Mirror Symmetry

Seidel’s mirror map reconstructs the homogeneous coordinate ring of a given projective CY or Fano variety in terms of Lagrangian intersection data on its mirror. We discuss the computation of Seidel’s mirror map for abelian and Kummer surfaces and related work on integrality and noncommutative geometry.

4:15pm–5:15pm **Masahiko Saito** (Kobe)

Painlevé Property of ODEs and Deformation of Logarithmic Symplectic Varieties

We will analyze movable branching points of algebraic ordinary differential equations, and give a necessary condition for Painlevé property by means of geometry of logarithmic symplectic varieties. The result establishes the reason why the condition of Okamoto-Painlevé pairs is necessary for Painlevé equations. Furthermore, we can give a very simple proof of a result of Fuchs’, Poincaré, Malmquist and M. Matuda

5:30pm–6:30pm **Rolf Schimmrigk** (Indiana, South Bend)

String Modular Ω –Motives and Aspect of Mirror Symmetry

The purpose of this talk is to describe some new extensions of recent string modularity results for elliptic curves and $K3$ surfaces to higher dimensional varieties of both Calabi-Yau and Fano type. The resulting examples establish that it is possible to construct Calabi-Yau varieties from the physics of the string worldsheet in all physically interesting dimensions. These constructions also provide arithmetic checks for ideas in mirror symmetry. Implications for the mirrors of rigid Calabi-Yau threefolds, as well as elliptic curves, will be discussed. The results concerning special types of Fano varieties provide checks for a conjecture of Serre concerning the type of modular forms associated to generalized Calabi-Yau Hodge structures.

8:00pm–9:00pm **Ahmed Sebber** (Bordeaux)

Differential theta relations and Galois theory for Riccati equations

JUNE 8, 2006

9:00am–10:00am **Andrey Todorov** (UC Santa Cruz)

Regularized Determinants of CY Metrics and Applications to Mirror Symmetry

We will prove the existence of the analogue of the Dedekind Eta Function for CY threefolds and K3 surfaces with unimodular Picard Group. We will discuss the combinatorial properties of the Generalized Dedekind Eta function in the A and B model and its relations with Harvey–Moore–Borcherds Product Formulas.

10:15am–11:15am **TBA**

TBA

11:30am–12:30pm **TBA**

Terry Gannon (Alberta)

He and his wife had a twin (boy and girl) on May 28th, 2006. Accordingly, Terry got grounded, and had to cancel his talk.

The Monster, Modular Functions and RCFT

Rational conformal field theory ‘explains’ the modularity of lattice theta series, affine Kac-Moody algebra characters, Monstrous Moonshine functions, etc. We try to identify the essence of this argument, and use this to speculate on the nature of a more conceptual second proof of the Monstrous Moonshine conjectures.