Convex Sets and their Applications

TED BISZTRICZKY (University of Calgary)
PAUL GOODEY (University of Oklahoma)
PETER GRITZMANN (Technische Universität München)
MARTIN HENK (Universität Magdeburg)
DAVID LARMAN (UC London)

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1 Introduction

BIRS-REPORT, T. BISZTRICZKY

The main objective of this Workshop was to bring together in Banff eminent and emerging researchers from the three main branches of Convex Geometry: Discrete, Analytical and Applied. There has not been such a unifying conference in the past fifteen years. The organizers believe that this objective was met during the week of March 4 - 9. First, of the thirty-nine participants, one third represented the current group of emerging researchers in the field; furthermore, five of these thirteen participants were graduate students. Specifically, three (Langi, Naszodi and Papez) from the University of Calgary, one (Jimenez) from the University of Alberta, and one (Garcia-Colin) from the University College, London.

Next, a common feature of many of the lectures was an expository component. This reflected the acknowledgement and approval of the participants of the unifying aspect of the Workshop. The prevailing intent of the lectures was to present the major problems and recent advances of their particular branch of Convexity. Of particular note were the expository lectures on the combinatorics of polytopes, the lectures introducing some of the current topics of interests in linear and convex optimization, and the lectures concerning the various measures associated with convex bodies.

Finally, the consensus of the participants was that such a unifying convexity workshop was not only timely but also overdue. Their enthusiasm for the meeting is well evidenced by the full program of thirty - six lectures, and by a very faithful attendance at these lectures. The smallest number of listeners at any lecture was thirty, and that number was attained only at the last lecture on Thursday.

2 Abstracts

Iskander Aliev

A sharp lower bound for the Frobenius number

Ferdinand Georg Frobenius (1849–1917) raised the following problem: given N positive integers $a_1, \dots, a_N$ with $\gcd(a_1, \dots, a_N) = 1$, find the largest natural number $g_N = g_N(a_1, \dots, a_N)$ (called the Frobenius number) such that g_N has no representation as a non-negative integer combination of $a_1, \dots, a_N$.

In the present talk, after a short historical overview, we discuss a geometric approach to the Frobenius problem, based on results of Ravi Kannan, Peter Gruber and Andrzej Schinzel. The introduced technique allows us to give an optimal lower bound for the Frobenius number g_N in terms of the absolute inhomogeneous minimum of the standard $(N-1)$-simplex.

Margaret Bayer

Flag vectors of polytopes: an overview

For a d-dimensional polytope P, and $S = \{s_1, s_2, \ldots, s_k\} \subseteq \{0, 1, \ldots, d-1\}$, $f_S(P)$ is the number of chains of faces $\emptyset \subset F_1 \subset F_2 \subset \cdots \subset F_k \subset P$ with $\dim F_i = s_i$. The *flag vector* of P is the length 2^d vector $(f_S(P))_{S\subset\{0,1,\ldots,d-1\}}$. This lecture gives a historical overview of the study of flag vectors of polytopes.

The flag vector is an extension of the face vector, or f-vector, which has been the subject of research since Euler. In the cases of 3-dimensional polytopes and simplicial d-polytopes, characterizations of f-vectors are known, and in these cases, the flag vector is determined linearly by the f-vector.

Richard Stanley (1979) studied flag vectors of Cohen-Macaulay posets, a class that contains face lattices of convex polytopes. Bayer and Billera (1985) proved the generalized Dehn-Sommerville equations, the complete set of linear equations satisfied by the flag vectors of all convex polytopes. Kalai (1987) used rigidity theory to show the inequality $f_{02} - 3f_2 + f_1 - df_0 + \binom{d+1}{2} \geq 0$. The flag vectors of 4-dimensional polytopes were studied by Bayer (1987), but a complete characterization of flag vectors of 4-polytopes continues to elude us to this day.

A crucial ingredient in the characterization of f-vectors of simplicial polytopes was the connection with toric varieties. In the nonsimplicial case, the middle perversity intersection homology of the toric variety gives an h-vector, linearly dependent on the flag vector. Results from algebraic geometry translate into linear inequalities on the flag vector (Stanley 1987).

Another main source of linear inequalities is the cd-index of a polytope, discovered by Jonathan Fine (1985). The cd-index is a vector linearly equivalent to the flag vector; it can be viewed as a reduction of the flag vector by the generalized Dehn-Sommerville equations. Stanley (1994) proved the nonnegativity of the cd-index for convex polytopes. Billera and Ehrenborg (2000) strengthened the result by showing that among d-polytopes the cd-index is minimized by that of the d-simplex. This depends on a co-algebra approach to the cd-index developed by Ehrenborg and Readdy (1998).

Two separate techniques enable one to generate new linear inequalities on flag vectors from old. The convolution operation was introduced by Kalai (1988); he also used this to demonstrate a particularly nice basis for the flag vectors of polytopes. Ehrenborg (2005) gives a lifting technique that applies to inequalities on the cd-index.

We are still, apparently, far from a characterization of flag vectors of polytopes. In fact, we do not even know if the closed convex cone of flag vectors is finitely generated. Special classes of polytopes, such as cubical polytopes and zonotopes, have been studied. In addition there are some results on more general classes of partially ordered sets: general graded posets, Eulerian posets, and Gorenstein* lattices.

Károly Bezdek

Short Billiards

The talk is a survey talk on periodic billiards centered around the following theorem and conjecture of the author.

DEFINITION. We say that $\mathbf{b}$ is a *k-sided billiard arc* of the convex body $\mathbf{K} \subset \mathbf{E}^n, n \geq 2, k \geq 1$ if $\mathbf{b}$ is a k-sided polygonal arc in $\mathbf{E}^n$ whose vertices lie on the boundary of $\mathbf{K}$ and whose each angle bisector is perpendicular to a supporting hyperplane of $\mathbf{K}$ passing through the corresponding vertex of $\mathbf{b}$ and finally, whose first (resp., last) segment is perpendicular to a supporting hyperplane of $\mathbf{K}$ passing through the corresponding endpoint of $\mathbf{b}$.

THEOREM. *If the minimum width of the convex body $\mathbf{K} \subset \mathbf{E}^n, n \geq 2$ is at least 1, then the length of any billiard arc of $\mathbf{K}$ is at least 1.*

COROLLARY. Let $X \subset \mathbf{E}^n, n \geq 2$ be a (finite) set of diameter at most 1. Then the length of any billiard arc of $\mathbf{B}[X] := \bigcap_{x \in X} \mathbf{B}^n[x]$ is at least 1, where $\mathbf{B}^n[x] \subset \mathbf{E}^n$ stands for the closed n-dimensional unit ball centered at x.

DEFINITION. We say that $\mathbf{b}$ is a *k-sided billiard polygon* of the convex body $\mathbf{K} \subset \mathbf{E}^n, n \geq 2, k \geq 2$ if $\mathbf{b}$ is a k-sided polygon in $\mathbf{E}^n$ whose vertices lie on the boundary of $\mathbf{K}$ and whose each angle bisector is perpendicular to a supporting hyperplane of $\mathbf{K}$ passing through the corresponding vertex of $\mathbf{b}$.

CONJECTURE. Let $X \subset \mathbf{E}^n, n \geq 2$ be a (finite) set of diameter at most 1. Then the length of any billiard polygon of $\mathbf{B}[X] := \bigcap_{x \in X} \mathbf{B}^n[x]$ is at least 2.

REMARK. The above theorem and conjecture for $n = 2$ follow from a theorem of the author and R. Connelly (1989).

Károly Böröczky, Jr.

Convex bodies of minimal volume, surface area and mean width with respect to thin shells

Given $r > 1$, let us consider convex bodies in E^n that contain a fixed unit ball, and whose extreme points are of distance at least r from the centre of the unit ball, and we investigate how well these convex bodies approximate the unit ball in terms of volume, surface area and mean width. The main results joint with K. Böröczky, C. Schütt and G. Wintsche are as follows: As r tends to one, there are asymptotic formulae for the error of the approximation, and asymptotically the whole boundary of the extremal bodies are covered by faces that are asymptotically regular triangles in E^3.

René Brandenberg

Minimal containment under homothetics

(joint work with Lucia Roth)

Minimal containment problems arise in a variety of applications, such as shape fitting problems, data clustering, pattern recognition or medical surgery. Typical examples are norm maximization, computing the circumball, circumcylinder or the width of a given body or minimal enclosing boxes or ellipsoids. A possible general framework gives the following definition

MINIMAL CONTAINMENT PROBLEM (MCP):

Input: $d \in \mathbb{N}$, $K \subset \mathbb{R}^d$ convex body.
Task: $\min \varphi_d(C)$, such that $K \subset C \in \mathcal{C}_d$,

where $\mathcal{C}_d$ usually is the orbit of a given convex body under a group of transformations like homothetics, similarities or affine mappings and φ_d a monotone functional such as the volume or the dilatation factor of C.

In this talk we focus on the MCP under homothetics (MCP$_{Hom}$), which itself has a lot of applications but is also needed as an important subroutine in solving lots of other MCP problems. Besides some negative complexity results the following was shown by Gritzmann and Klee: if C is given by a strong separation oracle and if K is a $\mathcal{V}$-polytope then MCP$_{Hom}$ can be solved in polynomial time using the ellipsoid method.

Because of the bad practical performance of the ellipsoid algorithm much effort has been spend to find better solutions, at least when C is the Euclidean ball. One recent idea are so called core set algorithms. Here the approximation of the circumball of a point set P is reduced to the computation of the circumball of a small subset of P, where 'small' means independent of the size and the dimension of P.

We present a new and easy to implement cutting plane method, based on linear programming, which is dual in nature to the core set idea and very easy to implement. It solves the general MCP$_{Hom}$ up to any given accuracy and because of its adaptive character it also has a good practical performance.

Finally we point out some relations to well known theoretical problems in convex geometry, which play a substantial role not only in the analysis of our method but also in the task to generalize the core set method to non-euclidean containers.

David Bremner

Approaches to facet enumeration under symmetry

Well known theorems of Minkowski and Weyl tell us that every convex polytope is the convex hull of a finite set of points and the bounded intersection of a set of (facet defining) halfspaces. In practice transforming from one representation to the other is often of interest, and usually difficult. One of the obvious difficulties is that the output may be huge with respect to the input size; on the other hand there is typically a symmetry group acting on the polytope, and the practioner may only be interested in equivalence classes of the output under this group.

I will start by giving a brief survey of the state of the art of facet enumeration, including some idea of what kind of inputs on which the known techniques face difficulties.

I will then describe some preliminary experience with a pivoting technique for generating equivalence classes of facets of a convex polytope under the action of an isometry group. I describe connections with previously studied "adjacency decomposition" methods, as well as some of details of invariants, isometry testing for bases, and pruning the search. I discuss the performance of the pivoting method, which depends not just on the degeneracy of the polytope, but on how the symmetry group acts on bases (of facets) of the polytope. This work is joint with Achill Schürmann and Frank Vallentin.

Time permitting, I will mention some work of David Avis that applies the "extend and canonicalize" techniques of Read, McKay, and others to enumerate the entire face lattice up to symmetry.

Jesús A. De Loera

Transportation Polytopes: a twenty-year update

A transportation polytope consists of all multidimensional arrays of nonnegative numbers that satisfy certain sum conditions on subsets of the entries. They arise naturally in optimization and statistics and have also interest for pure mathematics due to the appearance of permutation matrices, latin squares, magic squares, as lattice points of these polytopes. In this talk we present recent advances on the understanding of the combinatorics and geometry of these polyhedra. In particular, we try to give a complete report on the status of a long list of open questions last collected in the 1984 monograph by Yemelichev-Kovalev-Kravtsov and the 1986 survey paper of Vlach.

Richard Ehrenborg

The cd-index, polytopes and Gorenstein* lattices

The f-vector enumerates the number of faces of a polytope according to dimension, that is, f_i is the number of faces of dimension i. The flag f-vector is a refinement of the f-vector which counts flags of faces in the polytope. There are linear relations between the entries of the flag f-vector known as the generalized Dehn-Sommerville relations. Hence it would be interesting to have an explicit basis for the subspace spanned by these relations.

The $\mathbf{cd}$-index, conjectured by Fine and proved by Bayer and Klapper, gives such a basis. It offers an efficient way to encode the flag f-vector of a polytope. In fact, Stanley showed that the $\mathbf{cd}$-index exists for Eulerian poset, namely a poset where each interval satisfies the Euler-Poincaré relation.

Very little is known about the $\mathbf{cd}$-index of a general polytope. Fine conjectured that the $\mathbf{cd}$-index of a polytope has non-negative coefficients. This conjecture was proven by Stanley, in fact, he proved that the cd-index is non-negative for spherical-shellable (S-shellable) complexes.

A poset is Gorenstein* if it is Eulerian and the associated chain complex is Cohen-Macaulay. The most natural example of a Gorenstein* poset is the face lattice of a convex polytope. For Gorenstein* posets Stanley stated two conjectures: (1) The $\mathbf{cd}$-index for Gorenstein* poset is non-negative. (2) The $\mathbf{cd}$-index for Gorenstein* lattice is coefficientwise minimized by the $\mathbf{cd}$-index of the simplex of the same dimension.

A partial step toward Stanley's second conjecture was taken by Billera and Ehrenborg. They proved the $\mathbf{cd}$-index of a polytope is coefficientwise minimized by the simplex of the same dimension. Their proof uses the geometric fact that polytopes are shellable.

Kalle Karu using techniques from algebraic geometry proved Stanley's first conjecture, that the $\mathbf{cd}$-index of a Gorenstein* posets.

Recently, Ehrenborg and Karu proved Stanley's second conjecture. I will end the talk by outlining the proof and where it differs from the earlier proof for polytopes.

This is joint work with Kalle Karu.

Ferenc Fodor

Geometric transversals in low and high dimensions

This talk contains results that were achieved jointly with Ted Bisztriczky (Calgary) and Deborah Oliveros (Mexico City), and with Gergely Ambrus (Szeged, Auburn) and András Bezdek (Auburn).

Let $\mathcal{F}$ denote a family of ovals in the Euclidean plane. A line is a *transversal* to a family $\mathcal{K}$ if it intersects every member of $\mathcal{K}$. $\mathcal{K}$ has the property T if it has a transversal. $\mathcal{K}$ has the property $T(k)$ if every at most k-membered subfamily of $\mathcal{K}$ has a transversal. $\mathcal{K}$ has the property $T - k$ if there is a line that meets all members of $\mathcal{K}$ with the possible exception of at most k of them.

In 1989, Tverberg proved that $T(5) \Rightarrow T$ for a disjoint family of translates of an oval, a conjecture of Grünbaum (1958). In general, we know that neither $T(3)$ nor $T(4)$ is enough to guarantee the same. Katchalski and Lewis (1980) proved that there exists a universal constant k_3 such that $T(3) \Rightarrow T - k_3$ for any finite family of disjoint translates of an arbitrary oval. They estimated $k_3 \leq 192\pi$ and conjectured that $k_3 = 2$. It was shown, using a construction with unit disks, by A. Bezdek (1991) that $k_3 \geq 2$. The upper estimate on k_3 was improved by Tverberg (1991) and later by Holmsen (2000). The currently known best upper bound for k_3 is 22, established by Holmsen (2000). Holmsen (2000) constructed examples which show that $k_3 \geq 4$. Holmsen also showed that $k_3 = 4$ for finite families of unit squares whose sides are parallel to the coordinate axes.

Danzer (1963) proved that $T(5) \Rightarrow T$ for a pairwise disjoint family of unit disks. Kaiser (2002) showed that $k_3 \leq 12$ for such a family. Finally, Heppes settled the question in 2004 by proving that $T(3) \Rightarrow T-2$ for unit disks. An example of Aronov, Goodman, Pollack, and Wenger (2000) showed that $T(4) \not\Rightarrow T$ for unit disks. It was proved by T. Bisztriczky, D. Oliveros and F. F. in 2005 that if $\mathcal{F}$ is a finite family of mutually disjoint unit disks with the property $T(4)$, then $\mathcal{F}$ has the property $T - 1$.

A family of balls in $\mathbf{R}^d$ is *thinly distributed* if the distance between the centres of any two balls is at least twice the sum of their radii. Hadwiger (1957) proved that for any family of thinly distributed balls in $\mathbf{R}^d$, $T(d^2) \Rightarrow T$. Grünbaum (1960) improved Hadwiger's statement by proving that $T(2d-1) \Rightarrow T$. Holmsen, Katchalski and Lewis (2003) showed that there exists a constant $n_0 \leq 46$ such that $T(n_0) \Rightarrow T$ for any family of pairwise disjoint unit balls in $\mathbf{R}^3$. The constant n_0 was improved subsequently by Cheong, Goaoc and Holmsen (2004) to 11.

G. Ambrus, A. Bezdek and F. F. (2005) improved on the distance condition in Hadwiger's (1960) theorem proving that if $\mathcal{F}$ is a family of unit balls in $\mathbf{R}^d$ with the property that the mutual distances of the centres are at least $2\sqrt{2+\sqrt{2}}$ then $T(d^2) \Rightarrow T$.

We note that Cheong, Goaoc, Holmsen and Petitjean (2005) very recently proved that $T(4d-1) \Rightarrow T$ for disjoint unit balls in $\mathbf{R}^d$.

Natalia Garcia-Colin

On a generalization of a problem of McMullen
regarding the neighborliness in convex polytopes

McMullen proposed the following question. Determine the largest integer $n = f(d)$ such that any set of n points in general position in the affine $d - space$ $\mathbb{R}^d$ can be mapped by a projective transformation on to the vertices of a convex polytope. It is known that

$$2d + 1 \leq f(d) < 2d + \left\lceil \frac{d+1}{2} \right\rceil$$

.

In the paper where Larman proved the lowed bound, he also proved that the lower bound is sharp in the cases where d=1,2 and 3. The upper bound was proved by Ramirez-Alfonsin by constructing a family of Lawrence Oriented Matroids where every of its members can be made cyclic by reorienting one element.

Using the techniques developed by Ramirez-Alfonsin, in his paper, we construct a family of Lawrence Oriented matroids that can always be made cyclic by reorienting a subset $S \in X$ of the ground set X (of vertices) with cardinality at most k. This construction gives an upper bound for the following problem:

Determine the largest integer $n = f(d, k)$ such that any set of n points in general position in the affine $d-space$ $\mathbb{R}^d$ can be mapped by a permissible projective transformation on to the vertices of a $k-neighborly$ convex polytope. Namely:

$$d + \left\lfloor \frac{d}{k+1} \right\rfloor + 1 \leq f(d, k) < 2d - k + 1$$

.

Finally, we prove the following related problem in the plane using purely geometric methods:

In $\mathbb{R}^2$ let X be a subset of n point in general position. Let $g(X)$ the largest k such that there exists a subdivision A, B of X such that $conv(A \backslash \{x_1, x_2, ..., x_k\}) \cap conv(B \backslash \{x_1, x_2, ..., x_k\}) \neq \emptyset$. If

$$g(n) = \max_{X \in |X| = n} g(X), \qquad then \qquad \lim_{n \to \infty} \frac{g(n)}{n} = \frac{1}{2}.$$

Paolo Gronchi

Shadow systems

Shadow systems were introduced in 1958 by Rogers and Shephard [6] as families of convex hulls of a given set of points moving with constant speed along a fixed direction. Rogers and Shephard showed that the volume a shadow system is a convex function of the time-like parameter t.

Shephard [7] noted that the elements of such a family can be defined as the projections of a higher dimensional convex body along the direction $z + tv$ onto the hyperplane $z^\perp$. This fact enables us to construct shadow systems and also to extend the convexity property of the volume to different quantities. Precisely, a first consequence is that projections, Minkowski sums and convex hulls of shadow systems are still shadow systems. Hence, the brightness along a fixed direction is a convex function of t and, via Cauchy's formula, also the surface area is convex in t. Similarly, taking projections onto 1-dimensional subspaces, we infer that the mean width and the diameter are convex functions of the parameter t. By the same argument, Shephard [7] proved that quermassintegrals and mixed volumes of shadow systems are convex functions of t.

More recently, Campi, Colesanti and Gronchi [1] proved that the Sylvester functional (i.e., the expected value of the volume of a random polytope from a convex body) is a convex function of the parameter of parallel chord movements, a particular kind of shadow systems. Campi and Gronchi [2], [4] proved the same convexity property for the volume of the L^p-centroid bodies and the L^p-zonotopes. Furthermore, they showed [3] that the reciprocal of the volume of the polar body of an origin-symmetric shadow system is a convex function of the parameter. Meyer and Reisner [5] extended such a result to the non symmetric case.

The convexity of a functional along parallel chord movements can be used, via Steiner symmetrization, to characterize ellipsoids as minimizers. The same property leads also to maximizers in special classes. Namely, triangles among two-dimensional convex sets and parallelograms in the symmetric case, parallelotopes among zonoids [4], simplices among d-dimensional polytopes with at most $d + 3$ vertices [5].

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María A. Hernández Cifre

On the minimal annulus of a convex body: some optimization problems

(joint work with P. Herrero)

Let K be a convex body (i.e., a compact convex set) in the Euclidean plane. Associated with K are a number of well-known functionals: the area A, the perimeter p, the diameter D, the minimal width ω, the circumradius R_K and the inradius r_K.

Another interesting functional to be considered for a convex body K is the thick of its *minimal annulus*. The minimal annulus of the body K is the annulus (the closed set consisting of the points lying between two concentric discs –concentric n-balls in $\mathbb{R}^n$) with minimal difference of radii that contains the boundary of K. Of course, the minimal annulus is uniquely determined (Bonnesen, 1929, in $\mathbb{R}^2$ and $\mathbb{R}^3$, and Bárány, 1988, in higher dimension). This object and its properties were studied originally by Bonnesen for planar convex sets in order to sharp the isoperimetric inequality in $\mathbb{R}^2$.

In this talk we intend to present how the minimal annulus of a planar convex body K is related with the six classic geometric measures associated with it. First, we obtain all the possible bounds (upper and lower bounds) for the measures A, p, D, ω, R_K and r_K of a convex body K with fixed minimal annulus. Then, we deal with the relation between the minimal annulus and, either the circumradius, or the inradius of K: we study some properties relating the minimal annulus with both measures, and then we solve the problem of maximizing and minimizing the remaining geometric measures when, either the circumradius and the minimal annulus, or the inradius and the minimal annulus, are given. We prove the optimal inequalities for each of those problems, determining also its corresponding extremal sets.

Markus Kiderlen

Spherical liftings and projections in convex geometry

Let K be a lower dimensional convex body in d-dimensional space containing the origin. The support function of K can be obtained from its support function relative to its affine hull by applying a linear transformation, which we call a spherical lifting. Starting from this motivation, we will introduce more general spherical liftings. Spherical liftings map positive finite measures on the unit sphere of a linear space L to measures on the unit sphere in d-dimensional space. The dual operators, the so-called spherical projections, will also be introduced. We will show that many geometric operations, like projections or translative integrals can conveniently be expressed using spherical liftings and projections. One central result will be that spherical projections preserve convexity, implying in particular a directed version of the observation that the 1-st projection function of a convex body is a support function.

We will then turn to averages of spherical lifted projections, where averaging is understood with respect to the invariant probability measure on all k-dimensional subspaces L. We discuss in how far a convex body is determined by one or several of these averages.

Alexander Koldobsky

Inequalities of Khinchin type and sections of L_p-balls, $p > -2$.

We extend Khinchin type inequalities to the case $p > -2$. As an application we verify the slicing problem for the unit balls of finite-dimensional spaces that embed in L_p, $p > -2$.

David Larman

Determining properties of convex bodies from information about certain sections

The talk centred around partial results to three problems:

- Let K be a convex body in E^d and let p be a point of $\operatorname{int} K$ such that every two section of K through p has a projective centre different from p. Is K an ellipsoid?
- Let K, L be convex bodies in E^d with $L \subset \operatorname{int} K$. Suppose we know the $(d-1)$-volume of every $d-1$ section of K which touches L. Does this determine K uniquely?
- Let K, L be convex bodies in E^d with $L \subset \operatorname{int} K$. Suppose that every $(d-1)$ section of K that touches L is centrally symmetric. Is K an ellipsoid?

Carl Lee

Multiple views of h-vectors

I will give a brief survey of several different ways of looking at h-vectors of polytopes, including combinatorial views (winding numbers, shellings, bistellar operations), and algebraic (the face-ring, stress, weights, the "volume ring"). I will offer reminders of several interesting open problems.

Zsolt Lángi

Isoperimetric inequalities for k_g-polygons

(joint work with Balázs Csikós and Márton Naszódi)

The discrete isoperimetric problem is to determine the maximal area polygon with at most k vertices and of a given perimeter. It is a classical fact that the unique optimal polygon on the Euclidean plane is the regular one. The same statement for the hyperbolic plane was proved by Károly Bezdek and on the sphere by László Fejes Tóth. In the present paper we extend the discrete isoperimetric inequality in the following way.

Let $\Gamma \subset \mathbf{M}$ be a simple closed polygon in $\mathbf{M}$ and let $k_g \geq 0$ be fixed. If $\mathbf{M} = \mathbb{S}^2$, we assume that Γ is contained in an open hemisphere. Take the closed curve P obtained by joining consecutive vertices of Γ by curves of geodesic curvature k_g facing outward (resp. inward). If k_g is the geodesic curvature of a circle of radius r, then Γ is assumed to have sides of length at most $2r$ and the smooth arcs of P connecting two consecutive vertices are assumed to be shorter than or equal to a semicircle. We call P an *outer* (resp. *inner*) k_g*-polygon* with the same set of *vertices* as that of Γ. We call a k_g-polygon with perimeter l a (k_g, l)*-polygon*. An outer (resp. inner) (k_g, l)-polygon is *optimal* if its area is maximal among the areas of outer (resp. inner) (k_g, l)-polygons having the same number of vertices. We prove the following statements.

PROPOSITION. Let $\mathbf{M}$ be $\mathbb{S}^2$, $\mathbb{E}^2$ or $\mathbb{H}^2$. Let $l > 0$ and $k_g \geq 0$ be given. Then the only optimal inner (k_g, l)-polygons in $\mathbb{M}$ are the regular ones.

THEOREM. Let $\mathbf{M}$ be $\mathbb{S}^2$, $\mathbb{E}^2$ or $\mathbb{H}^2$. Let $k_g \geq 0$, $l > 0$ and n be given with the above restrictions. If l is not equal to the perimeter of the circle of geodesic curvature k_g, then the only optimal outer (k_g, l)-polygons in $\mathbf{M}$ are the regular ones. If l is equal to the perimeter of the circle of geodesic curvature k_g, then a (k_g, l)-polygon is optimal if and only if its underlying polygon Γ is inscribed in a circle of geodesic curvature k_g.

Jospeh M. Ling

Non-linear inequalities for 4-dimensional convex polytopes

In this talk, we consider the characterization problem for the f-vectors and the flag f-vectors for 4-polytopes. Four new (infinite) lists of quadratic inequalities for the flag f-vectors of 4-polytopes are presented. These inequalities extend the four inequalities obtained by M. Bayer in 1984. Four cubic inequalities for the flag f-vectors are also presented. Furthermore, the projections of the newly found inequalities onto the f-vectors yields new (infinite) lists of quadratic inequalities for the f-vectors. An application of these include an estimate of the number of edges in terms of the number of vertices and the number of facets.

Alexander Litvak

On the vertex index of convex bodies

We introduce the vertex index of a given d-dimensional centrally symmetric convex body, which, in a sense, measures how well the body can be inscribed into a convex polytope with small number of vertices. This index is closely connected to the illumination parameter of a body, introduced earlier by Karoly Bezdek, and, thus, related to the famous conjecture in Convex Geometry about covering of a d-dimensional body by 2^d smaller positively homothetic copies. We provide asymptotically sharp (up to logarithmic terms) estimates of this index in the general case and discuss extremal cases. More precisely, we show that the vertex index varies between $cd/\sqrt{\ln 2d}$ and $Cd^{3/2}\ln(2d)$, where c and C are absolute positive constants. Here, the lower estimate is sharp (up to a logarithmic term) for crosspolytopes and the upper estimate is sharp (again, up to a logarithmic term) for ellipsoids. Also, we provide precise estimates in dimensions 2 and 3. We conjecture that the vertex index of a d-dimensional Euclidean ball is $2d\sqrt{d}$. We prove this conjecture in dimensions two and three.

Monika Ludwig

Elementary moves on triangulations

(joint work with Matthias Reitzner)

Let P be an n-dimensional polyhedron in $\mathbb{R}^N$, that is, a finite union of n-dimensional convex polytopes. A finite set of n-simplices αP is a *triangulation* of P if no pair of simplices intersects in a set of dimension n and if their union equals P. An *elementary move* applied to αP is one of the two following operations: a simplex $T \in \alpha P$ is dissected into two n-simplices T_1, T_2 by a hyperplane containing an $(n-2)$-dimensional face of T; or the reverse, that is, two simplices $T_1, T_2 \in \alpha P$ are replaced by $T = 3DT_1 \cup T_2$ if T is again a simplex. Triangulations αP and βP are equivalent by elementary moves, $\alpha P \sim \beta P$, if there are finitely many elementary moves that transform αP into βP.

THEOREM. If αP and βP are triangulations of the n-polyhedron P, then $\alpha P \sim \beta P$.

This result is a metric version of the Alexander-Newman theorem for simplicial complexes. As an application the following extension result is obtained.

THEOREM. Every valuation on simplices in $\mathbb{R}^n$ has a unique extension to a valuation on polyhedra in $\mathbb{R}^n$.

Efren Morales-Amaya

A Characterization of ellipsoids

(joint work with J. Jeronimo)

Motivated by a theorem due to Rogers [3], we give a characterization of ellipsoid in the spirit of the Höbinger Problem [1], [2]. Namely, we proved that if $K \subset \mathbb{R}^n$ is a convex body, $n \geq 3$, and for every three for every three parallel hyperplanes A, D and E there exists point $p \in \mathbb{R}^n$ with the following property: for every line l passing through p, the central projections K_A and K_D of K from $l \cap A$ and $l \cap D$, respectively, onto E are homothetics, then K is an ellipsoid.

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Márton Naszódi
Touching Homothetic Bodies and Antipodality

According to Klee's definition (1960), an *antipodal set* in Euclidean d-space is a set X with the property that, through any two points of X, there is a pair of parallel hyperplanes supporting X. In this talk, I present two research topics that are related by the idea of antipodality.

In the first part of the talk, the maximum number of touching positive homothetic copies of a convex body in Euclidean d-space is discussed. According to a conjecture of Károly Bezdek and János Pach, this number is 2^d; which bound, if it holds, is sharp as it is attained by cubes. The previously known bound was 3^d, I improved it to 2^{d+1}. I present the proof of this recent result.

The second part of the talk focuses on the extension of the concept of antipodality to hyperbolic d-space. This is a joint work with Károly Bezdek and Deborah Oliveros. We define antipodality in three different ways, as follows.

Following Klee, we say that a set X in hyperbolic d-space is *p-antipodal* if, through any two points of X, there is a pair of parallel hyperbolic hyperplanes supporting X.

Following Erdős' concept of antipodality (1957), a set X in hyperbolic d-space is *a-antipodal*, if the angle determined by any three points of X is acute.

Finally, an *h-antipodal* set in hyperbolic d-space is a set X with the property that for any $x_1, x_2 \in X$, the set X is contained in the intersection of the horoballs H_1 and H_2, where H_1 is the horoball bounded by the horosphere that passes through x_1, contains x_2 inside and is perpendicular to the hyperbolic line $\overline{x_1 x_2}$, and H_2 is defined similarily.

We find upper bounds on the cardinality of an antipodal set in hyperbolic d-space, according to the different definitions.

Shmuel Onn
Multiway polytopes: universality and convex optimization
(slides are available at `http://ie.technion.ac.il/~onn/Talks/multiway_polytopes.pdf`)

A k-way (transportation) polytope is the set of all $m_1 \times \cdots \times m_k$ nonnegative arrays $x = (x_{i_1,\dots,i_k})$ such that the sums of the entries over some of their lower dimensional sub-arrays (margins) are specified. More precisely, for any tuple $(i_1, \dots, i_k)$ with $i_j \in \{1, \dots, m_j\} \cup \{+\}$, the corresponding *margin* $x_{i_1,\dots,i_k}$ is the sum of entries of x over all coordinates j with $i_j = +$. The *support* of $(i_1, \dots, i_k)$ and of $x_{i_1,\dots,i_k}$ is the set $\mathrm{supp}(i_1, \dots, i_k) := \{j : i_j \neq +\}$ of non-summed coordinates. For instance, if x is a $4 \times 5 \times 3 \times 2$ array then it has 12 margins with support $\{1, 3\}$ such as $x_{3,+,2,+} = \sum_{i_2=1}^{5} \sum_{i_4=1}^{2} x_{3,i_2,2,i_4}$. Given a family $\mathcal{F}$ of subsets of $\{1, \dots, k\}$ and margin values $u_{i_1,\dots,i_k}$ for all tuples with support in $\mathcal{F}$, the corresponding k-way polytope is the set of nonnegative arrays with these margins,

$$T_{\mathcal{F}} \;=\; \left\{ x \in \mathbb{R}_+^{m_1 \times \cdots \times m_k} \;:\; x_{i_1,\dots,i_k} \;=\; u_{i_1,\dots,i_k}\,,\;\; \mathrm{supp}(i_1, \dots, i_k) \in \mathcal{F} \right\} .$$

In this talk we present the following two remarkable contrasting statements regarding multiway polytopes and discuss some of their many applications:

UNIVERSALITY THEOREM: Every rational polytope $P = \{y \in \mathbb{R}_+^m : Ay = b\}$ is polynomial time representable as an $r \times c \times 3$ multiway polytope of line-sums, that is, with $\mathcal{F} = \{\{1,2\}, \{1,3\}, \{2,3\}\}$,

$$T \quad = \quad \{\, x \in \mathbb{R}_+^{r \times c \times 3} \;:\; \sum_i x_{i,j,k} = w_{j,k}\,,\; \sum_j x_{i,j,k} = v_{i,k}\,,\; \sum_k x_{i,j,k} = u_{i,j} \,\} \,.$$

OPTIMIZATION THEOREM: For any fixed $d, k, m_1, \dots, m_{k-1}$, and family $\mathcal{F}$ of subsets of $\{1, \dots, k\}$, there is a polynomial oracle-time algorithm that, given n, arrays $w_1, \dots, w_d \in \mathbb{Z}^{m_1 \times \cdots \times m_{k-1} \times n}$, margin values $u_{i_1,\dots,i_k}$ for all tuples $(i_1, \dots, i_k)$ with support in $\mathcal{F}$, and convex $c : \mathbb{R}^d \longrightarrow \mathbb{R}$ presented by evaluation oracle, solves the corresponding convex integer multiway programming problem,

$$\max\{\, c(w_1 x, \dots, w_d x) \;:\; x \in \mathbb{N}^{m_1 \times \cdots \times m_{k-1} \times n}\,,\;\; x_{i_1,\dots,i_k} \;=\; u_{i_1,\dots,i_k}\,,\;\; \mathrm{supp}(i_1, \dots, i_k) \in \mathcal{F} \,\} \,.$$

These results, their consequences, applications and extensions appear in several recent papers joint with various coauthors among J. De Loera, R. Hemmecke, U. Rothblum and R. Weismantel, including *Convex combinatorial optimization* (Disc. Comp. Geom. 32:549–566, 2004), *The complexity of three-way statistical tables* (SIAM J. Comp. 33:819–836, 2004), *All rational polytopes are transportation polytopes and all polytopal integer sets are contingency tables* (IPCO 10, LNCS 3064:338–351, 2004), *Markov bases of three-way tables are arbitrarily complicated* (J. Symb. Comp. 41:173–181, 2006), *N-fold integer programming* (submitted), and *Convex integer programming* (in preparation).

Peter Papez

Ball-Polyhedra

This talk outlines the results of a joint paper written by K. Bezdek, Z. Lángi, M. Naszódi P. Papez. The main goal of this paper is to study the geometry of intersections of finitely many unit balls from the point of view of discrete geometry in Euclidean space. We call these sets *ball-polyhedra*. They have been studied in the past, in particular Reuleaux polygons; although the name ball-polyhedra seems to be a new terminology for this special class of linearly convex sets. In fact, there is a special kind of convexity entering along with ball-polyhedra which we call *lens-convexity* and study as well. This paper is not a survey on ball-polyhedra, instead it lays a rather broad ground work for future study of ball-polyhedra by proving several new properties of them and raising open research problems as well.

In this talk, I first define ball-polyhedra and supporting spheres. The supporting spheres are the objects that play the role of supporting hyperplanes in the theory of polyhedra. Next, we examine a special class of ball-polyhedra called standard ball-polyhedra. This is the family of ball-polyhedra for which the Eüler-Poincare formula holds. We also examine Steinitz' Theorem for the edge graph of standard ball-polyhedra. The talk concludes with a survey of results from diverse areas of geometry related to ball-polyhedra.

Carla Peri

Uniqueness and stability results in geometric tomography

Geometric tomography concerns the retrieval of information about a geometric object via measurements of its sections or projections.

In this talk we consider two types of data for line sections, namely parallel or point X-rays. After reviewing some of the main results on continuous parallel or point X- rays and discrete parallel X-rays, we present recent uniqueness results for discrete point X-rays (see [1]). The discussion will show that, somewhat surprisingly, non- uniqueness results hinge on the existence of arbitrary long arithmetic progressions of relative prime numbers, and on the existence of some geometric incidence structures.

The final part of the talk will concern recent progress in stability estimates (see [2]).

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Rolf Schneider

Intersections of balls in normed spaces

(joint work with José Pedro Moreno)

Let $\|\cdot\|$ be a norm on $\mathbb{R}^d$ and B its unit ball. Any positive homothet of B is called a *ball*. For $K \in \mathcal{K}$ (the system of nonempty compact convex subsets of $\mathbb{R}^d$), let $\beta(K)$ denote the intersection of all balls containing K. Thus, $\mathcal{B} := \{K \in \mathcal{K} : \beta(K) = K\}$ is the system of all intersections of balls. The system $\mathcal{M}$ of Mazur sets is defined by the property that $K \in \mathcal{K}$ belongs to $\mathcal{M}$ if and only if to every hyperplane H with $K \cap H = \emptyset$

there exists a ball B' with $K \subset B'$ and $B' \cap H = \emptyset$. Motivated by questions and observations in two papers by Granero, Moreno and Phelps from 2004, we first give examples of norms for which (a) $\mathcal{B}$ is not ball stable (i.e., not closed under the addition of balls), (b) $\mathcal{B}$ is not closed, hence the ball hull map β is not continuous. For polyhedral norms, we show that β is Lipschitz continuous, and we give complete characterizations of the norms having one of the following properties: (a) $\mathcal{B}$ is closed under Minkowski addition, (b) $\mathcal{B}$ is closed under addition of balls, (c) $\mathcal{M} = \mathcal{B}$, (d) $\mathcal{M}$ contains only balls and one-pointed sets.

Carsten Schütt

On the minimum of several random variables

(joint work with Y. Gordon, A. Litvak, and E. Werner)

Let f_i, $i = 1, \ldots, n$, be symmetric, identically distributed random variables. We investigate expectations

$$\int_\Omega \left\| \sum_{i=1}^n x_i f_i(\omega) \right\|_M d\mathbb{P}(\omega)$$

where $\| \ \|_M$ is an Orlicz norm. We find out that these expressions are maximal if the random variables are in addition required to be independent.

In case the random variables are independent we get quite precise estimates for the above expectations. In particular, for independent Gauß variables we have for all $x \in \mathbb{R}^n$

$$c_1 \|x\|_M \le \int_\Omega \max_{1 \le i \le n} |x_i f_i(\omega)| d\mathbb{P}(\omega) \le c_2 \|x\|_M$$

where the Orlicz function is $M(t) = e^{-\frac{1}{t^2}}$. This case is of particular interest to us. In a paper on generalized zonotopes these estimates are applied to obtain estimates for volumes of certain convex bodies.

For a given sequence of real numbers $a_1, \ldots, a_n$ we denote the k-th smallest one by

$$k\text{-}\min_{1 \le i \le n} a_i.$$

Let $\mathcal{A}$ be a class of random variables satisfying certain distribution conditions (the class contains $N(0,1)$ Gaussian random variables). We show that there exist two absolute positive constants c and C such that for every sequence of positive real numbers $x_1, \ldots, x_n$ and every $k \le n$ one has

$$c \max_{1 \le j \le k} \frac{k+1-j}{\sum_{i=j}^n 1/x_i} \le \mathbb{E}\ k\text{-}\min_{1 \le i \le n} |x_i \xi_i| \le C\ \ln(k+1) \max_{1 \le j \le k} \frac{k+1-j}{\sum_{i=j}^n 1/x_i},$$

where $\xi_1, \ldots, \xi_n$ are independent random variables from the class $\mathcal{A}$. Moreover, if $k = 1$ then the left hand side estimate does not require independence of the ξ_i's. We provide similar estimates for the moments of $k\text{-}\min_{1 \le i \le n} |x_i \xi_i|$ as well.

Grzegorz Sójka

Minor illuminations and and the determination of convex bodies by values of $\pm\infty$-chord functions

(joint work with David Larman)

The notion of illuminations is strongly connected with the famous Hadwiger Illumination Conjecture. It says that it should be possible to cover boundary of arbitrary n-dimensional convex body by at most 2^n translates of its interior.

The second notion mentioned in the title are chord functions. In 1998 A. Soranzo generalized the definition of i-chord functions to the case $i = 3D \pm \infty$. For arbitrary convex body K and non-zero vector u he used the following formulae:

$$\rho_{-\infty,K}(u) = 3D \min \{\rho_K(u), \rho_{=}K(-u)\};$$

$$\rho_{+\infty,K}(u) = 3D \max\{\rho_K(u), \rho_=K(-u)\},$$

where ρ_K denotes the radial function of K.

In 2004 D. Larman and Grzegorz Sójka found a link between this two subjects. They generalized the notion of illuminations to the case when the source is some internal point of the convex body considered. During this presentation we will speak about their observation and related results.

Valeriu Soltan

Homothety classes of convex sets

Let A_H denote the homothety class (i. e., the family of positive homothetic copies) generated by a closed convex set $A \subset \mathbb{R}^n$. We study the conditions under which the Minkowski sum, the Minkowski difference, and the binary intersection, defined, respectively, by

$$B_H + C_H = \{B' + C' \mid B' \in B_H,\ C' \in C_H\},$$
$$B_H \underset{n}{\sim} C_H = \{B' \sim C' \mid B' \in B_H,\ C' \in C_H,\ \dim(B' \sim C') = n\},$$
$$B_H \underset{n}{\cap} C_H = \{B' \cap C' \mid B' \in B_H,\ C' \in C_H,\ \dim(B' \cap C') = n\}$$

belong to a unique homothety class generated by a closed convex set of dimension n in $\mathbb{R}^n$ (more generally, belong to the union of countably many homothety classes generated by closed convex sets in $\mathbb{R}^n$).

We also study planar sections and projections of homothetic convex sets in $\mathbb{R}^n$. In particular, closed convex sets $B, C \subset \mathbb{R}^n$ (not necessarily compact) are homothetic if and only if either of the following conditions holds: (a) the orthogonal projections of B and C on each 3-dimensional plane of $\mathbb{R}^n$ are homothetic, where similarity ratio may depend on the projection plane, (b) there are points $p \in B$ and $q \in C$ such that such that for every pair of parallel 3-dimensional planes L and M through p and q, respectively, the sections $B \cap L$ and $C \cap M$ are homothetic.

Jozsef Solymosi

Additive Discrete Geometry

One of the most important results in discrete geometry,a theorem of Szemerédi and Trotter [2], gives a sharp bound on the maximum number of incidences between points and lines in the Euclidean plane. In particular it says that n lines and n points determine at most $O(n^{4/3})$ incidences. Let us suppose that an arrangement of n lines and n points defines $cn^{4/3}$ incidences, for a given positive c. It is widely believed that such arrangements, where the number of incidences is close to the maximum, have special structure. However no results are known in this direction. There are numerous proofs of the Szemerédi-Trotter theorem (the most elegant is Székely's [3]) but none of them gives information about the structure of arrangements with many incidences. In this talk we mentioned that if n is large enough and the number of incidences is at least $cn^{4/3}$ then the arrangement contains a triangle. This seemingly obvious statement is quite difficult to prove, the only known proof uses Szemerédi's Regularity Lemma [1]. We gavefurther examples how to analyze extremal point-line arrangements using methods from algebra and number theory.

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Alina Stancu

From a characterization of ellipsoids to the p-affine surface area

Characterizing the ellipsoids as the only convex bodies with sufficiently regular boundary which are homothetic to their illumination, or p-illumination, bodies will lead naturally to an interpretation of the p-affine surface area. As consequences we will discuss a couple of inequalities relating these affine quantities to volumes.

Konrad J. Swanepoel

Explicit upper bounds for edge-antipodal polytopes

A d-polytope P is *edge-antipodal* if for any two vertices x and y joined by an edge there exist two parallel hyperplanes, one through x and one through y, such that P is contained in the closed slab bounded by the two hyperplanes. This notion was introduced by Talata (1999), who conjectured that the number of vertices of an edge-antipodal 3-polytope is bounded above by a constant. Csikós (2003) proved an upper bound of 12, and K. Bezdek, Bisztriczky and Böröczky (2005) gave the sharp upper bound of 8. Pór (200?) proved that the number of vertices of an edge-antipodal d-polytope is bounded above by a function of d. However, his proof is existential, with no information on the size of the upper bound. Our main result is an explicit bound.

THEOREM Let $d \geq 2$. Then the number of vertices of an edge-antipodal d-polytope is bounded above by $(\frac{d}{2}+1)^d$.

This theorem is proved by considering a metric relative of edge-antipodal polytopes that we call subequilateral polytopes. For more detail, as well as references to the literature, see `http://arxiv.org/math.MG/0601638`

Roman Vershynin

New convex geometry problems in linear programming

The Simplex Method is the oldest and easiest algorithm in Linear Programming. Nevertheless, it puts the theory of computing in an awkward position. This is not a polynomial time algorithm (counterexamples are known), but in practice it runs in polynomial time. To theoretically explain the strange behavior, Spielman and Teng introduced the notion of the Smoothed Analysis of Algorithms. There, one ”smoothes” an input by a small random perturbation, in hope that this models ”most” practice problems. Spielman and Teng showed that the smoothed complexity of the simplex method is polynomial. Their analysis brings up a variety of new problems in convex geometry. We go one step further to show that the number of steps in the smoothed simplex algorithm is actually polylogarithmic, rather than polynomial, in the number of constraints of the linear program.

Wolfgang Weil

Directed tomographic transforms

(based on joint work with Paul Goodey)

The basic problem in Geometric Tomography is to retrieve information about a compact (convex or star-shaped) set $K \subset \mathbb{R}^d$ from data arising from sections or projections of K. Generalizing classical results on projection or section functions for centrally symmetric bodies, we introduce directed section functions of star bodies and two different types of directed projection functions of convex bodies. These are functions on the flag manifold $\{(L,u)\}$, where L varies among the j-dimensional subspaces $L \subset \mathbb{R}^d$, $1 \leq j \leq d-1$, and u is a variable unit vector in L. These directed section resp. projection functions determine a body K uniquely (resp. uniquely up to a translation). As a more general problem, one can consider the averaged directed section and projection functions (obtained as integrals over all j-dimensional subspaces L containing the direction u) and ask whether even they determine the underlying body. In the main part of the lecture, we study certain of these averaged functions and show relations between them as well as uniqueness results. It turns out that uniqueness holds for a large range of dimensions d and j, but that there are also infinitely many pairs (j, d) where uniqueness fails. The proofs are based on the fact that the considered tomographic transforms can be described by linear operators on the unit sphere S^{d-1}, which intertwine the action of the rotation group. The injectivity properties of the operators are represented in the non-vanishing of the multipliers w.r.t. spherical harmonics. The explicit behaviour of the multipliers is complicated but recursion formulas leading to the mentioned injectivity results were obtained using Zeilberger's algorithm.

Elisabeth Werner

Spaces between polytopes and zonotopes

(joint work with Y. Gordon, A. Litvak and C. Schütt)

We study geometric parameters associated with the Banach spaces $(\mathbb{R}^n, \|\|_{k,q})$ normed by

$$\|x\|_{k,q} = \left(\sum_{1 \le i \le k} | < x, a_i > |^{*q} \right)^{1/q},$$

where $\{a_i\}_{i \le N}$ is a given sequence of N points in $\mathbb{R}^n$, $1 \le k \le N$, $1 \le q \le \infty$ and $\{\lambda_i^*\}_{i \ge 1}$ denotes the decreasing rearrangement of a sequence $\{\lambda_i\}_{i \ge 1} \subset \mathbb{R}$. In particular, we give estimates on the volume of the unit balls of these spaces.

Jörg M. Wills

On the zeros of the Ehrhart polynomial

(joint work with M.Henk)

The Ehrhart polynomial counts the number of lattice points of the integer multiples nP of a lattice polytope P in $\mathbb{Z}^d$. It can be written as a product

$$G(s, P) = \prod_{i=1}^{d} (1 + \frac{s}{\gamma_i}),$$

where $s \in \mathbb{C}$ is the complex variable and $-\gamma_i \in \mathbb{C}$ the zeros (or roots) of G. For $s \in \mathbb{N}$, $G(s, P)$ counts the lattice points of $s\,P$. The motivation of such investigation comes from the interaction between P and the zeros $-\gamma_i$, i. e., between Geometry and Algebra. In this talk we discuss two topics:

- Relations between the γ_i and Minkowski's successive minima, in particular between their arithmetic and geometric means.
- Polytopes with all zeros $-\gamma_i$ on the line Re $s = -\frac{1}{2}$. In this case the Ehrhart polynomials have some properties in common with the Riemann ζ-function, as Bump et al. (2000) and Rodriguez-Villegas (2002) proved.

We show some basic properties of these polytopes.

3 Participants

Aliev, Iskander	University of Edinburgh, I.Aliev@ed.ac.uk
Bayer, Margaret	University of Kansas (Lawrence), bayer@math.ku.edu
Bezdek, Karoly	University of Calgary, bezdek@math.ucalgary.ca
Bisztriczky, Ted	University of Calgary, tbisztri@math.ucalgary.ca
Böröczky, Jr., Karoly	Alfréd Rényi Institute of Mathematics, carlos@renyi.hu
Brandenberg, René	Technische Universitt München, brandenb@ma.tum.de
Bremner, David	University of New Brunswick, bremner@unb.ca
De Loera, Jesus	University of California (Davis), deloera@math.ucdavis.edu
Ehrenborg, Richard	University of Kentucky, jrge@ms.uky.edu
Fodor, Ferenc	University of Szeged, fodorf@math.u-szeged.hu
Garcia-Colin, Natalia	University College London, natalia@math.ucl.ac.uk
Goodey, Paul	University of Oklahoma, pgoodey@math.ou.edu
Gritzmann, Peter	Technische Universitt München, gritzman@ma.tum.de
Gronchi, Paolo	University of Florence, paolo@fi.iac.cnr.it
Henk, Martin	Universität Magdeburg, henk@math.uni-magdeburg.de
Hernandez Cifre, Maria	Universidad de Murcia, mhcifre@um.es
Jimenez, Carlos	University of Alberta, cjimenez@math.ualberta.ca
Kiderlen, Markus	University of Aarhus, kiderlen@imf.au.dk
Koldobsky, Alexander	University of Missouri (Columbia), koldobsk@math.missouri.edu
Langi, Zsolt	University of Calgary, zlangi@math.ucalgary.ca
Larman, David	UC London, d.larman@ucl.ac.uk
Lee, Carl	University of Kentucky, lee@ms.uky.edu
Ling, Joseph	University of Calgary, ling@math.ucalgary.ca
Litvak, Alexander	University of Alberta, alexandr@math.ualberta.ca
Ludwig, Monika	TU Wien, monika.ludwig@tuwien.ac.at
Morales, Efren	CIMAT Mexico, efren@cimat.mx
Naszódi, Márton	University of Calgary, nmarton@math.ucalgary.ca
Onn, Shmuel	Technion - Israel Institute of Technology, onn@ie.technion.ac.il
Papez, Peter	University of Calgary, pdpapez@math.ucalgary.ca
Peri, Carla	U.Cattolica Milan, carla.peri@unicatt.it
Schneider, Rolf	University of Freiburg, rolf.schneider@math.uni-freiburg.de
Schütt, Carsten	Universität Kiel, schuett@math.uni-kiel.de
Sójka, Grzegorz	Warsaw University of Technology, grzes@pfu.pl
Soltan, Valeriu	George Mason University, vsoltan@gmu.edu
Solymosi, Jozsef	University of British Columbia, solymosi@math.ubc.ca
Stancu, Alina	Polytechnic University of NY and UMass Lowell, astancu@poly.edu
Swanepoel, Konrad	University of South Africa, swanekj@unisa.ac.za
Vershynin, Roman	University of California (Davis), vershynin@math.ucdavis.edu
Weil, Wolfgang	Universität Karlsruhe, weil@math.uni-karlsruhe.de
Werner, Elisabeth	Case Western Reserve University, emw2@po.cwru.edu
Wills, Jörg	TU Siegen, wills@math.uni-siegen.de