Becoming Mathematicians: Women and Students of Color Choosing and Leaving Doctoral Mathematics

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Few women and even fewer African Americans, Latinos, and Native Americans complete doctoral degrees in mathematics in the United States. This article proposes a framework for understanding the small numbers of women and students of color who persist in doctoral mathematics based on the notion that academic and social integration are critical to persistence and that integration develops through particular types of participation in the communities of practice of graduate school. An integrated summary of previous research on attrition and persistence of doctoral students identifies particular obstacles faced by women and students of color in doctoral mathematics and directs attention to ways in which faculty and others involved in doctoral education can work to improve the persistence rates, experiences, and diversity of their doctoral students.

KEYWORDS: doctoral students, integration, mathematics, participation, women.

The shortage of students trained in science, mathematics, and engineering has been an ongoing international concern (Burton, 2001; Maslen, 2001; National Science Foundation, 2000a; Seymour & Hewitt, 1997). Despite the growing scientific and quantitative needs of the 21st century, relatively small numbers of U.S. students develop and maintain an interest in studying science and mathematics. According to the Committee on Women in Science and Engineering of the National Research Council, increasing numbers of U.S. students initially expressing an interest in science and engineering careers switch out of those fields as undergraduates or after having attained bachelor’s degrees (Matyas & Dix, 1992).

In mathematics, the professional community has been particularly concerned with the state of doctoral education (Bass, 2003; Chan, 2003; Madison & Hart, 1990; National Research Council, 1992).

Many doctoral students are not prepared to meet undergraduate teaching needs, establish productive research careers, or apply what they have learned in business and industry. This inadequate preparation, continuing high attrition, and the declining interest of domestic students, the inadequate interest of women students, and the near-absent interest of students from underrepresented minorities in doctoral study are problems that transcend the current difficult job market. (National Research Council, 1992, p. 1)
The ratio of new doctorates in mathematics to new doctorates in chemistry, physics, biology, and engineering declined substantially over the period 1960–1985, and, among undergraduate mathematics majors earning doctoral degrees in science-related fields, the proportion earning doctorates in mathematics decreased as well (Madison & Hart, 1990). While doctoral students in the physical sciences and mathematics have higher completion rates than students in the arts and humanities, about 35% of the former do not complete their degrees (Bowen & Rudenstine, 1992; Nerad & Cerny, 1993). The percentage of doctoral students in mathematics who do not complete their PhDs has not been accurately measured, with estimates in individual departments ranging from 30% to 70% (Bowen & Rudenstine, 1992; Cooper, 2000; National Research Council, 1992; Zwick, 1991).

Over all fields of study, women have higher estimated attrition rates from graduate school than men, although this may be due to women’s tendency to enroll disproportionately in fields that have higher attrition rates overall, such as the social sciences and humanities (Bowen & Rudenstine, 1992; California Postsecondary Education Commission, 1990; Nerad & Cerny, 1993). Some studies have revealed higher attrition rates among women than among men specifically in the sciences (Berg & Ferber, 1983; Nerad & Cerny, 1993; Zwick, 1991). While the overall attrition rate in the physical sciences is lower than that in the humanities or social sciences (Bowen & Rudenstine, 1992; California Postsecondary Education Commission, 1990; Nerad & Cerny, 1993; Zwick, 1991), historically the difference between attrition rates among women and men has been highest in the physical sciences (Sells, 1973). More recently, other authors have found that gender is not a factor in doctoral degree progress (Girves & Wemmerus, 1988) or in doctoral student attrition (Bair & Haworth, 1999), although these studies do not specifically describe the situation in mathematics. Bowen and Rudenstine (1992) provided data from 10 institutions indicating that the gender gap in completion rates for mathematics doctorates closed in the 1970s, although women continued to have longer average times to degree completion.

In mathematics in particular, the participation of women in the field decreases as they progress to higher educational and professional levels. Girls and boys take similar amounts of mathematics in high school (National Center for Education Statistics, 1997). In 1996, women received 46% of bachelor’s degrees awarded in mathematics in the United States (National Science Foundation, 2002), but in the fall of 1996, women represented only 33% of full-time first-year graduate students in mathematics (Davis, 1997). Two years later, in 1998, women received 42% of master’s degrees in mathematics (National Science Foundation, 2002), and, in 2001–2002, they received 31% of doctoral degrees (Loftsgaarden, Maxwell, & Priestly, 2003). In the fall of 2002, 22% of newly hired mathematics faculty at U.S. institutions were female (Kirkman, Maxwell, & Priestly, 2003). Similar patterns are evident among African Americans, Native Americans, and Latinos; of the 465 U.S. citizens and permanent residents who received mathematics PhDs in this country in 2001–2002, only 9 (2%; 7 male and 2 female) were Latino, 17 (4%; 10 male and 7 female) were Black, and 2 (0.4%; both male) were Native American (Loftsgaarden et al., 2003). While these figures represent an increase in the numbers of women and students of color enrolling in and graduating from doctoral mathematics in recent years, the proportions of students from these groups enrolled in graduate mathematics is still quite low (National Science Foundation, 2002).
In 1998, women constituted about one third of all graduate students enrolled in mathematics (Davis, Maxwell, & Remick, 1999); in that same year, they earned 38% of the graduate degrees awarded (25% of doctorates and 42% of master's degrees; National Science Foundation, 2002). However, it is not known whether the women receiving master's degrees entered graduate school with the intent to earn a master's or whether they represent attrition from doctoral programs. The difference between 42% of master's degrees and 25% of doctoral degrees being earned by women implies a substantial attrition of women before completion of doctoral study. This post-master's loss of women from mathematics contributes to the small absolute numbers of women who complete their doctoral studies in mathematics.

**Why Should We Care?**

One might argue that postsecondary students are free to choose their field of study and that those who leave mathematics are expressing their freedom in a rational way. The presumption here is that individuals can enter any area of study that meets their needs, maximizes their potential, and suits their interests. Alternatively, it is possible that a field of study's epistemology and culture affect students' choices and success. For example, a community of practice imposes certain cultural standards and implicit expectations of students (Lave & Wenger, 1991; Wenger; 1998); the community of practice of mathematics sets expectations that students may be unable or unwilling to meet. It is therefore possible that the people who succeed in mathematics are those who are able or willing to adapt themselves to these cultural practices; that is, they learn, or are self-selected, to work within the existing structure and to play by the existing rules (Stage & Maple, 1996). Individuals whose talents, values, skills, or interests make it difficult or undesirable for them to adapt to that structure may not be able to successfully negotiate the educational and professional systems necessary to allow them to continue to participate in mathematics. Thus, research on attrition may be confounded by the loss and self-selection of those students without a predisposition to accept the epistemological and methodological practices of the discipline. In a study comparing the careers of female and male scientists, Sonnert and Holton (1995) found little evidence that women in science follow or believe in a radically different epistemology or methodology that some feminist theorists of science have suggested. It may, of course, be proposed that women (and men) with alternative methodological and epistemological approaches do not flourish or survive in the science pipeline for very long, so that the scientists who are reasonably successful under the current system of science are predisposed to it, or at least have learned to accept it. (p. 156)

Thus, it is not clear whether students who leave mathematics do so because they are rejecting the intellectual content of mathematics, or whether they are rejecting other parts of the sociocultural practices associated with becoming a mathematician. Losing people from mathematics because they do not “fit in” might result in less diversity of mathematical thought among those who stay in mathematics. Systems that fail to diversify are often unstable and vulnerable; the strategy of using diversity to ensure long-term vitality has served well in a variety of natural, social, and economic systems (Wilson, 1992). By broadening the focus of mathematics to include a more diverse range of scholars, the discipline of mathematics would likely be
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enriched by an expanded range of mathematical thought. “New entrants bring questions, fresh ideas, new and different perspectives on old problems, new energies, and new skills. They are not blinded by the familiar. The experience they bring enlarges the repertoire of strategies that can be employed” (Wilson, 1992, p. 4). Increasing the diversity of those who participate in mathematics can help the profession flexibly meet the challenges posed by the growing quantitative sophistication of economic and political structures in the 21st century.

The preceding argument is based on a notion of equity as “enlightened self-interest” (Secada, 1989). An alternative perspective is to consider the rights and opportunities of individuals previously excluded from the enterprise of mathematics. All members of a democratic society have a reasonable expectation of equitable opportunities. Mathematics serves as an important gateway to many careers and disciplines of study. If more people participate in mathematics, these individuals will have increased opportunities to avail themselves of those benefits (Secada, 1989).

In a related argument, science and mathematics have prestige as tools that help us understand and manipulate our natural world. Concentrating this prestige in the hands of a subset of the population allows members of that subset to maintain hegemony over mathematical development, which affects the types of mathematics that are valued and pursued. Alternatively, if more individuals were to be included as full participants in mathematics, then the groups that those individuals are seen to represent would share in the development of mathematical thought, which might have equity implications reaching beyond the domain of mathematics. In addition, if large numbers of people perceive that they are outcasts from mathematics and science, they are less likely to support critical societal investments in mathematical and scientific development (Secada, 1989). Finally, with growing national concerns about the lack of a quantitatively literate populace, pre-college students might benefit from being able to emulate a broader range of appropriate role models in mathematics. For all of these reasons, loss of diverse students from mathematics is a critical concern.

Focusing on Doctoral Students

This concern is magnified at the graduate level in mathematics, where high attrition rates continue to erode an already increasingly homogeneous student population. However, a large part of the research on attrition from science, technology, engineering, and mathematics (STEM) investigates undergraduate students who have declared majors in STEM disciplines and then change their majors (e.g., Holland & Eisenhart, 1990; Leitze, 1996; Seymour & Hewitt, 1997; Strenta, Elliot, Adair, Matier, & Scott, 1994). These studies probably include at least some people with only a casual initial interest in science or mathematics who leave STEM because they find that their initial choice of major is not a good match for their emerging interests, talents, and goals. It might be more appropriate to consider people who initially select mathematics because they are genuinely interested and capable in mathematics—people who want to do mathematics—and later find that the reality of being enrolled in mathematics does not meet with their expectations; such students exist in doctoral programs. Doctoral students are committed to their interest in mathematics, and they are at least talented and knowledgeable enough in mathematics to qualify for admission to a graduate program. Many of them may have a suffi-
cient interest in and commitment to mathematics that they have explored it in various ways outside of their formal classroom experiences. They should be more reflective than non-mathematics students about their experiences within mathematics.

Leaving graduate school without a degree can be a discouraging and sometimes devastating experience (Etzkowitz, Kemelgor, & Uzzi, 2000; Hinchey & Kimmel, 2000; Lovitts, 2001). Furthermore, the large number of doctoral students who leave education after investing several years of their lives, as well as the long interval often required to complete the degree for those who stay, represents an inefficient use of resources for universities and funding agencies (Bowen & Rudenstine, 1992; National Science Foundation, 1998).

To understand why students in some groups do not persist in mathematics, we need to start with the question “What does it take to succeed in advanced mathematics study?” In this article, I describe a framework for understanding doctoral student persistence in mathematics based on Tinto’s (1993) notion that integration into the academic and social communities of graduate school is critical for persistence.

I argue that students become integrated through certain forms of participation in the communities of practice of their programs and departments (Lave & Wenger, 1991; Rogoff, 1995; Rogoff, Matusov, & White, 1996; Wenger, 1998). In addition, I present a synthesis of the research literature on doctoral student persistence and attrition as it applies to mathematics. This synthesis identifies some of the obstacles to participation and integration faced by doctoral students in mathematics. All students face certain obstacles, but some are particularly challenging for women and members of other underrepresented groups.

A Framework for Understanding Doctoral Student Persistence in Mathematics

Building students’ sense of belongingness in mathematics has been proposed as a critical feature of an equitable K–12 education (Allexsaht-Snider & Hart, 2001; Ladson-Billings, 1997; National Council of Teachers of Mathematics, 2000; Tate, 1995). A similar construct has been proposed at the doctoral level, with several authors arguing that students’ involvement or integration in the communities of their departments is an important factor in their persistence (Girves & Wemmerus, 1988; Herzig, 2002; Lovitts, 2001; National Research Council, 1992; National Science Foundation, 1998; Tinto, 1993). In this section, I first discuss the integration of doctoral students and then turn to the question of the process by which doctoral students become integrated through particular forms of participation in the communities of practice of their graduate programs. I subsequently pull these ideas together to develop a framework for understanding doctoral students’ persistence in mathematics and for understanding the particular obstacles faced by women and students of color.

Social and Academic Integration

Student involvement in departmental, institutional, and professional activities contributes favorably to retention and completion (Bair & Haworth, 1999). For example, students supported as research assistants have greater opportunities to learn the norms of the department and, hence, to become integrated (Girves & Wemmerus, 1988; Lovitts, 2001; National Science Foundation, 1998; Tinto, 1993). Girves and Wemmerus (1988) built a quantitative model based on survey responses
of 486 graduate students and reported that the factor with the strongest direct effect on degree progress was student involvement in the department.

A frequent cause of doctoral students leaving graduate school is a feeling of isolation or too little contact between faculty and students (Herzig, 2002; Lovitts, 2001; Stage & Maple, 1996); this is true of women in particular (Etzkowitz et al., 2000; Herzig, in press). Students in departments with impersonal environments that do not provide professional support to students are more likely to leave; disciplines with higher attrition rates and longer times to degree completion have been characterized by individual learning, solitary research, and library (as opposed to laboratory) work (Nerad & Cerny, 1993).

Tinto (1993) proposed a model for studying doctoral student persistence as a function of social and academic integration within the student and faculty communities that reside in the local department or program. Tinto based this model of attrition on Durkheim’s (1951; cited in Tinto, 1993) model of suicide; both models represent a form of voluntary withdrawal from local communities that is as much a reflection of the community as it is of the individual who withdraws. Moreover, each can be seen to signal somewhat similar forms of rejection of conventional norms regarding the value of persisting in those communities. (Tinto, 1993, p. 99)

In Durkheim’s model, “egotistical” suicide occurs when an individual does not become socially and intellectually integrated within societal communities. Tinto argues, by analogy, that students who do not become integrated in the social and academic communities of their graduate programs are more likely to voluntarily withdraw from graduate school. Experiences that enhance students’ membership in the social and academic communities of their programs strengthen their goals and commitments, which in turn increase the likelihood that they will persist. Students who are not well integrated into their departmental communities and cultures have been found to be more likely to leave graduate school for other reasons; for example, poorly integrated students are less likely to tolerate financial hardship (Lovitts, 2001). Thus, other reasons implicated in attrition actually can mask an underlying issue of integration.

In doctoral study, intellectual and social integration are interrelated (Tinto, 1993). While intellectual integration involves sharing values with the community into which a student is integrated, social interactions within the program, both with other students and with faculty, are important parts of membership in academic communities. “Social membership within one’s program becomes part and parcel of academic membership, and social interaction with one’s peers and faculty becomes closely linked not only to one’s intellectual development, but also to the development of important skills required for doctoral completion” (Tinto, 1993, p. 232). That is, doctoral students need to do more than simply learn the content of the mathematics taught in classes; perhaps even more important, they need to learn to participate in social and cultural practices. “A person’s intentions to learn are engaged and the meaning of learning is configured through the process of becoming a full participant in a sociocultural practice. This social process includes, indeed it subsumes, the learning of knowledgeable skills” (Lave & Wenger, 1991, p. 29).

Becoming integrated, then, is a process of becoming a “full participant” (Lave & Wenger, 1991) in the community defined by a particular sociocultural practice.
To become integrated, a doctoral student needs to master the unspoken cultural rules or "tacit knowledge" of her discipline (Gerholm, 1990), including knowledge of the multiple formal and informal discourses of the field and when to use each of these discourses. Gerholm argues that graduate students develop this knowledge through contact with more experienced researchers. Disciplines in which it is difficult to develop tacit knowledge favor particular types of students:

The combination of disciplines with much tacit knowledge and few possibilities in graduate education to acquire it would seem to favor students endowed with large amounts of... "cultural capital," i.e. a stock of knowledge, a frame of reference and a capacity to make the proper judgments called "taste." (Gerholm, 1990, p. 269)

Herzig (2002) found that what differentiated mathematics doctoral students who persisted from those who left was that those who stayed were more likely to have had family members who were involved in mathematics, to have participated in research experiences as undergraduates, or to have been committed to mathematics from a very young age. Consequently, doctoral students who persisted were more likely to have entered graduate school already in possession of important forms of cultural capital that facilitated their integration into mathematics.

In Tinto's (1993) model, persistence through the point of completing coursework and qualifying examinations reflects not only a student's abilities but also her interactions with faculty members outside of the classroom, which in turn shape these faculty members' judgments about a student's competence.

Attainment of candidacy can be viewed as a form of social initiation into a group whose members have a vested interest in maintaining the norms of that group. And to the degree that that initiation requires prescribed social behaviors and beliefs, it can also be argued that successful completion of a doctoral degree calls for the successful performance of a social role called "graduate student." (Tinto, 1993, p. 255)

Thus, a doctoral student also needs to adopt the identity of a mathematician, or at least of a mathematics graduate student. Students who do not fit faculty expectations for graduate students in a given field—those who do not master the tacit knowledge and dominant discourses of the field—may not be judged to be competent in that field (Etzkowitz et al., 2000).

How, then, do students come to be integrated in the academic and social communities of their departments, programs, and disciplines? This is the subject of the next section.

The Process of Becoming Integrated

Tinto (1993) discussed three stages of persistence toward a doctoral degree. In the first stage, students adjust to the academic and social communities within graduate school and make judgments about the relevance of the program to their career goals and the desirability of membership in the community. In the second stage, students develop the knowledge and skills, or "competence," deemed necessary for doctoral research, culminating in comprehensive exams. The third stage of persistence is completion of a dissertation. According to Tinto, student persistence through the first two stages reflects not only individual characteristics but also interactions between
students and faculty in the department and program. These interactions play a role in developing competence and affect the judgments others make about these competencies; faculty judgments of student competence within the classroom also are shaped by social judgments arising from interactions outside of the classroom.

In the dissertation stage, a student's experiences closely reflect her interactions with a small number of faculty in the department. A faculty member "serves as a role model and becomes the primary socializing agent in the department . . . . It is the number of faculty members a student comes to know as professional colleagues that is associated with involvement in the doctoral program," which in turn is "directly related to doctoral degree progress" (Girves & Wemmerus, 1988, p. 185). The extent and nature of graduate students' interactions with faculty members are critical means by which they become integrated into departmental communities.

The underlying process of these interactions between doctoral students and faculty members can be conceptualized as legitimate peripheral participation (Lave & Wenger, 1991); students participate in authentic ways at the periphery of legitimate mathematical practice, which, in time, moves them to more full participation in the community of practice. "The important point concerning learning is one of access to practice as [a] resource for learning, rather than to instruction" (Lave & Wenger, 1991, p. 85). Established faculty members or more advanced graduate students set the stage for the activity of the newcomers. The activity of the community provides a "curriculum" for those students who have legitimate access to that activity; that is, students learn through their participation in the academic community. "Participation in the cultural practice in which any knowledge exists is an epistemological principle of learning. The social structure of this practice, its power relations, and its conditions for legitimacy define possibilities for learning" (Lave & Wenger, 1991, p. 98).

According to sociocultural views of education, learning is intertwined with, and indeed inseparable from, students' participation in the communities of practice that reside in their graduate programs (Boaler, 2002; Rogoff, 1994). This participation might take many different forms, including such activities as listening to lectures and taking notes, working with other students to solve problems, reading texts, attending seminars, teaching and grading papers, studying for exams, and conducting research. Each of these activities constitutes a different type of learning opportunity. Participation in a community of practice is not a distinct educational activity but, rather, a lens through which we might analyze any of the educational activities in which students engage. Focusing on learning as participation, rather than as acquisition or transmission of knowledge, draws attention to particular aspects of the learning process (Rogoff, 1994). In analyzing the structure of doctoral education, we need to examine the particular practices and activities in which students participate, the nature of their participation, and what they learn from that participation.

Rogoff and colleagues (Rogoff, 1995; Rogoff, Baker-Sennett, Lacasa, & Goldsmith, 1995) have identified three "planes" on which participation in a community of practice can be viewed: the individual, interpersonal, and community planes, which Rogoff labels participatory appropriation, guided participation, and apprenticeship, respectively. These three aspects of a learning situation are not distinct but are always present in interactions and merely represent different grains of focus for analysis of the activities of the community of practice as a whole and of students' work within it. Next, I briefly describe the participation
of students in doctoral programs as viewed on each of these three planes of focus. I then use this perspective as part of a framework for understanding the experiences of women and students of color in doctoral mathematics.

**Participatory Appropriation in Doctoral Mathematics**

The typical model for doctoral education in mathematics, at least in the first years of graduate school, is one in which faculty lecture and students take notes and study extensively outside of class, with most faculty-student interactions taking place as faculty grade assignments and exams (National Research Council, 1992). This "transmission" model of teaching poses teachers as knowledgeable and active (their role is to tell students all they need to know) and students as ignorant and passive (their role is to absorb the information provided to them) (Rogoff, 1994). Rogoff (1994) has drawn attention to an alternative view of learning in which students and teachers are viewed as co-participants in communities of learners. On the individual plane of participation in such communities, the notion of participatory appropriation models how individuals change through their involvement with an activity and, in the process, become prepared for subsequent involvement in related activities. "This is a process of becoming, rather than acquisition" (Rogoff, 1995, p. 143).

Bass (2003) distinguishes between mathematics as a discipline and as a profession, and argues that while mathematics doctoral programs in the United States provide strong disciplinary training in the core areas of mathematical scholarship, they need to do a better job of preparing students for all aspects of work within the profession of mathematics, including serious professional development for teaching, uses of technology, exposition, developing and pursuing a research program, participation in the local and broader mathematical communities, and development of a "cultural awareness in students of the significance of their discipline in the larger worlds of science and society and of the expectation that they will serve as emissaries of their discipline in the outside world" (p. 775). These categories of professional development call for students to appropriate a range of important skills for functioning as mathematicians, including acquiring mathematical knowledge, developing fluency in the practices of mathematics, and developing identities as mathematicians (Boaler, 2002). However, given that students spend the first 2 or 3 years of their graduate training isolated from the community of practice of research mathematics, the things they learn—what they acquire through their participation—are specific to the experiences they have. For example, they appropriate skills for taking courses and exams and, in some cases, for working as teaching assistants. The nature of the activities in which these students participate gives them only limited opportunities to develop the knowledge, practices, and identities of research mathematicians.

**Guided Participation in Doctoral Mathematics**

On the interpersonal plane, students and other community members engage in "guided participation" (Rogoff, 1995), in which they communicate and interact while they participate in the activities that are valued within the community. This level of detail focuses on the interactions that take place among students and between students and faculty members as they work on course material, exams, and the other activities of the community.
In the course-taking stage of graduate study in mathematics, students tend to work either alone or in collaboration with other students as they struggle with common assignments and prepare for exams (Cooper, 2000; Herzig, 2002; National Research Council, 1992). Most of their communication and interactions are with other students; their interactions with faculty members are primarily restricted to listening to lectures in class and occasionally speaking with faculty during office hours. The guidance that students receive at this stage primarily comes through their work with each other.

When students pass their qualifying exams and courses, they finally gain access to the research community of mathematicians. At this point, they begin to participate in that community through their interactions with established faculty members and more advanced graduate students. Unfortunately, mathematics is a highly specialized discipline, and many graduate students have few peers with whom to collaborate (National Research Council, 1992). Consequently, the communities of practice in which they participate can be very small, and the student-advisor relationship becomes crucial (Tinto, 1993).

Apprenticeship in Doctoral Mathematics

On the community plane, student learning can be conceptualized as an apprenticeship that "focuses on a system of interpersonal involvements and arrangements in which people engage in culturally organized activity in which apprentices become more responsible participants" (Rogoff, 1995, p. 143). This notion is similar to Lave and Wenger’s (1991) description of apprenticeship as legitimate peripheral participation in a community of practice. Viewed at this level of detail, mathematics students begin their graduate programs by participating in long-established structures of coursework and exams that are far removed from the authentic work of practicing mathematicians.

In a case study of two mathematics doctoral students, Wiles (1999) identified two communities of practice in which graduate students participate: the course-taking community and the research community. During their first years in graduate school, most of mathematics students’ activity centers on coursework, preparation for qualifying examinations (which Tinto, 1993, called developing and demonstrating competence) and, for some, work as teaching assistants. Their activities at the stage of course taking are far from the activities of practicing mathematicians; although the work of mathematicians involves doing research, often collaboratively, graduate students’ early experiences have little to do with research and are often individual. "To the extent that the community of practice routinely sequesters newcomers, ... these newcomers are prevented from peripheral participation" (Lave & Wenger, 1991, p. 104).

Apprentices [can be] put to work in ways that deny them access to activities in the arenas of mature practice. Gaining legitimacy may be so difficult that some fail to learn until considerable time has passed. ... Gaining legitimacy is also a problem when masters prevent learning by acting in effect as pedagogical authoritarians, viewing apprentices as novices who “should be instructed” rather than as peripheral participants in a community engaged in its own reproduction. (Lave & Wenger, 1991, p. 76)

Lave and Wenger (1991) argued that learners need “access to peripherality” in addition to legitimate participation, and they pointed to “the crucial character of broad, and broadly legitimate, peripheral participation in a community of practice
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as central for increasing understanding and identity” (p. 85). If students are to become full participants in the community of practice of mathematicians, they need access to all of the means of membership. Since the activities required of mathematics graduate students restrict their access to that community, they have little or no opportunities to participate in the community, and therefore their abilities to become integrated into the community are inhibited. Instead, they are participating in a distinct course-taking community of practice, and their persistence at this stage depends on the degree to which they become integrated into this community. It is not clear to what extent this participation prepares them to participate in the community of practice of mathematics research they will encounter later.

The Persistence Framework

Combining these ideas of participation and integration leads to a framework in which, as students participate in the communities of practice of their departments and programs, viewed on the individual, interpersonal, and community planes, they become integrated into those communities, which increases the likelihood they will persist (see Figure 1). Experiences that enhance students’ participation lead to increased integration, increasing the likelihood that they will persist; experiences that inhibit their participation make it more difficult for them to become integrated, making it less likely that they will persist. Both the nature and the extent of students’ participation determine how likely they are to become integrated and to persist in mathematics. In mathematics, students have to negotiate two sequential communities of practice, and they must participate and become integrated in each (see Figure 2). Participating and becoming integrated in the course-taking community may have little to do with whether a student will succeed in becoming integrated in the research community.

Factors Affecting the Success of Doctoral Students in Mathematics

In this section, I review the literature on doctoral student persistence and attrition, with a particular focus on identifying the factors that inhibit or enhance diverse students’ opportunities to participate in the communities of practice of mathematics and, ultimately, to develop a sense of integration in those communities. To par-

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**FIGURE 1.** Persistence framework: As students participate in the communities of practice of graduate study, they become more integrated and are more likely to persist. Their participation can be viewed on the individual, interpersonal, and community planes.
FIGURE 2. Communities of practice: Doctoral students need to participate and become integrated into two sequential but disjoint communities of practice. Obstacles and enhancers affect their participation and, consequently, their opportunities to become integrated.

Although the persistence framework, these factors are presented in three categories: features of individual graduate students, aspects of relationships among members of the communities of practice of doctoral study, and features of the broader community of doctoral study. As will become clear in this analysis, while there are obstacles to participation and integration that all graduate students face, women and students of color face additional obstacles that limit their opportunities to participate fully and effectively in mathematics practice.

Studies Included in the Review

Persistence and attrition in undergraduate STEM, including choosing a mathematics or science major, have been studied broadly (Hilton & Lee, 1988; Holland & Eisenhart, 1990; Leitze, 1996; Linn & Kessel, 1996; Maple & Stage, 1991; McJamerson, 1990; Seymour & Hewitt, 1997; Strenta et al., 1994; Tinto, 1993; Ware, Steckler, & Leserman, 1995). Certainly, persistence in undergraduate mathematics and science education has an important impact on which students enroll in doctoral study, and perhaps on their expectations when they do. However, because of the differences between the nature of graduate and undergraduate education (Tinto, 1993) and the scope of the research literature, findings from undergraduate STEM are not included in this review.

The nature of graduate study makes attrition at the doctoral level difficult to define and measure. Students change enrollment status by transferring to other departments or taking leaves of absence. In some programs, students are initially admitted into a master's program and are considered doctoral candidates only after completing some milestone, such as qualifying exams or a master's thesis. Some
students leave a doctoral program with a master’s degree as a “consolation prize” (Douglas, 1997). Others may remain enrolled for a long period, making it difficult to determine whether or not they intend to complete their degrees. Students can stay involved with their departments even when they are nominally enrolled, or they may leave emotionally before they leave physically (Golde, 1996). These ambiguities make it difficult to determine which students have left and which are persisting. Furthermore, as Etzkowitz, Kemelgor, Neuschatz, and Uzzi (1992) argued in the case of women in science, the risk of studying only those women who leave is that we may then overlook the discrimination faced and overcome by women who have persisted. Thus, it is important also to consider ways in which women might be at a disadvantage during their graduate training. In order to understand issues regarding students’ participation in and decisions about their graduate training and the obstacles they face, we have to look not only at people who leave but also at the experiences of all students as they proceed through various stages of their graduate education. Consequently, studies were included in this review if they presented empirical data addressing the experiences of graduate students; reports describing graduate study from other perspectives were not considered germane to the model developed here.

Causes of the small numbers of women and students of certain racial and ethnic groups have seldom been studied in specific STEM disciplines; instead, these issues have often been investigated in broad categories such as engineering or science (which may include mathematics). However, every discipline has a distinct culture, and cultures vary locally as well (Becher, 1989). Mathematics exhibits some particular differences from the other STEM disciplines with which it is often categorized, for example its lack of both laboratory work and empirical data collection as a basis for future developments. The differing nature of knowledge in each discipline carries with it different social characteristics for its production, involving different questions, methods, and cultures (Becher, 1989), which in turn are likely to have distinct effects on students’ choices about entering and persisting within disciplines. A useful model of graduate student retention needs to accommodate these disciplinary differences, which is one goal of the persistence framework.

Only a small number of studies address doctoral student experiences specifically in mathematics. While I augment the findings from mathematics with research from other disciplines, the differences among these disciplines may limit their helpfulness in understanding the situation in mathematics. Throughout the review, I specifically analyze aspects of these structures and cultures to infer ways in which this broader literature can apply specifically to mathematics. Further research on the doctoral student experience needs to focus on these experiences in particular disciplinary contexts.

The studies included in the review are listed in the Appendix. Two categories of studies were included. First, any study that specifically reported on the experiences of doctoral students in mathematics was included; there were 11 such studies. Of these 11, 9 reported only on mathematics (Carlson, 1999; Committee on the Participation of Women, 2003; Cooper, 2000; Herzig, 2002, in press; Manzo, 1994; Mathematical Association of America and National Association of Mathematicians, 1997; National Research Council, 1992; Stage & Maple, 1996); 2 others included mathematics combined with physics (Hollenshead, Younce, & Wenzel, 1994) or computer science (Becker, 1984, 1990).
Studies in the second group did not deal with mathematics specifically but were included because they illuminated aspects of the persistence framework; in particular, these studies provide important perspectives concerning students’ opportunities to participate and become integrated into the communities of practice of their programs. Four studies investigated women graduate students or faculty members in the sciences (Etzkowitz et al., 1992, 2000; Grant, Kennelly, & Ward, 2000; Sonnert & Holton, 1995), and seven studies addressed students across a range of disciplines in the humanities, social sciences, and physical sciences (Bair & Haworth, 1999; Berg & Ferber, 1983; California Postsecondary Education Commission, 1990; Girves & Wemmerus, 1988; Golde, 1996; Lovitts, 2001; Nerad & Cerny, 1993). Two additional studies contained helpful statistical information that provided context for the arguments being made (Bowen & Rudenstine, 1992; Zwick, 1991).

A brief description of each study’s focus and methods is also included in the Appendix. These studies use a mix of qualitative, quantitative, and mixed-method analyses, drawing their data from surveys, institutional databases, and interviews; most use a combination of methodologies and data sources. Many of the studies are based on small samples of students, involving interview and focus group methods in small numbers of departments and institutions, often sampling from only one department or several departments at one or two institutions. Because the numbers of women and students of color in doctoral mathematics are so small overall, their numbers in specific doctoral programs are generally quite small as well. While these qualitative studies are very valuable for the insights they provide to the lived experiences of graduate students, the considerable differences among departments and institutions make it difficult to generalize from these studies to the overall population of graduate students. The consistency of findings across these studies is compelling, but future research needs to include larger scale studies to validate and extend the findings from these investigations.

Two studies use extensive databases of information about students who received prestigious fellowships (Bowen & Rudenstine, 1992; Sonnert & Holton, 1995). While these data address issues faced by students who are among the highest achievers, overlooked are the experiences of the vast majority of students who are not in that category. Again, while these studies are the best that are available, and they do provide some very important and useful perspectives on the graduate student experience, more focused studies that reflect the experiences of a broader population of students also are needed.

Some studies focus on attrition of doctoral students, while others focus on their persistence or other aspects of their experience in graduate school. Collectively, this body of research provides important insights into the opportunities that doctoral students—particularly women and students of color—have to participate in practices and to become integrated into the mathematical communities of graduate school. Throughout the review, I discuss specific concerns involved with the theories and interpretations some of these studies present.

In subsequent sections, I synthesize the previous research on doctoral student attrition and persistence as it applies to women and students of color in mathematics. I discuss these findings through the lens of the framework described earlier, interpreting the body of research in terms of the opportunities women and students of color have to participate in authentic mathematical activity viewed on the individual, interpersonal, and community planes; obstacles to that participation; and the
TABLE 1
Features of graduate students, relationships, and communities that have been reported to affect doctoral student persistence

<table>
<thead>
<tr>
<th>Individual graduate students</th>
<th>Relationships among community members</th>
<th>Communities of graduate study</th>
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<tbody>
<tr>
<td>Independence and autonomy</td>
<td>Pedagogy</td>
<td>Department and program structure</td>
</tr>
<tr>
<td>Achievement</td>
<td>Moral support from significant others</td>
<td>Financial support</td>
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<tr>
<td>Confidence</td>
<td>Mentoring and advising</td>
<td>Support for students with families</td>
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<tr>
<td>Family responsibilities</td>
<td>Feelings of isolation</td>
<td>Epistemology</td>
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<td></td>
<td>Perceived competition</td>
<td>The job market</td>
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ways in which participation helps or hinders these students in becoming integrated and persisting in doctoral study in mathematics. As mentioned, to facilitate this analysis, I present the research in three categories: features of individual graduate students, features of relationships among community members, and features of the communities as a whole (see Table 1). Of course, these three planes of focus on a community of practice—individual, interpersonal, and community—are intertwined and represent different units of analysis for the same whole. As I review each of these in turn, there necessarily are overlaps. In a final section, I explicitly tie the elements of this review to the framework and argue that policymakers need to pay careful attention to the opportunities students have to participate and consequently to become integrated in their departments and programs, the obstacles to their participation and integration, and the unique obstacles faced by women and students of color.

Features of Individual Graduate Students

Mathematics is generally regarded as an objective field of knowledge in which mathematicians work to discover truths about the natural world (Maddy, 1990; Steen, 1999). This presumed objectivity of mathematics leads to a cultural “blindness” to personal issues in which students who do not correspond to the cultural norm (male, White, childless, self-assured) are at a disadvantage (Hinchey & Kimmel, 2000). “Academic science presumes a taken-for-granted male model of social organization that takes little or no account of non-work related roles or social relationships” (Etzkowitz et al., 1992, p. 161). In this section, I discuss personal factors in the persistence and attrition of women and people of color from doctoral mathematics: independence and autonomy, achievement, confidence, and family responsibilities. These factors focus on the individual plane of analysis, but, of course, they interact in important ways with students’ relationships with other students and faculty and with their participation in the apprenticeship structures of their programs.

Independence and Autonomy

Women graduate students in science and mathematics have been stereotyped as less capable and uncompetitive; and as a result they may not be taken seriously by faculty (Becker, 1990; Committee on the Participation of Women, 2003; Etzkowitz
Herzig et al., 2000; Stage & Maple, 1996). Starting from a young age, women’s socialization leads them to look for interaction, attention, and reinforcement rather than to be autonomous and independent learners (Etzkowitz et al., 2000; Fennema & Peterson, 1985). This pattern of socialization can work against them in the eyes of their advisors, especially in a disciplinary culture such as that of mathematics, wherein work is expected to be individualistic and independent. Because of their socialization, female graduate students’ styles of interaction may be different than those expected by male faculty; these behaviors may be misinterpreted as inferior rather than different.

In [one] department a female academic model based on inter-personal relationships, affiliation and nurturance had become accepted as legitimate and had even become the departmental norm. This was in strong contrast to another research site where the expression by women of a need for these characteristics in the laboratory environment was derided as a desire for dependence and emotionality by the adherents of the patriarchal system that was in place. (Etzkowitz et al., 1992, p. 174)

Etzkowitz et al. (2000) argue that students can act independently only if they feel safe and accepted. Students who do not feel that they fit in may have more difficulty acting autonomously. In effect, autonomy and independence are double-edged swords for women in science.

Isolated and without interpersonal connection, a woman’s ability to be playfully creative is impeded. . . . A gendered “apartheid system” exists in which many male advisors offer support to male students, but leave women to figure things out for themselves. With no support or connection with an advisor, taking risks in the lab becomes threatening. People only take risks when they feel safe to do so. In contrast, there is sufficient support and acceptance, by way of informal interactions with male advisors and peers, for male students to enjoy the freedom to be innovative. (Etzkowitz et al., 2000, p. 86)

That is, male science students have enhanced relationships with faculty (which will become more visible on the interpersonal plane, discussed in the next section) that provide them with increased opportunities to develop a sense of belonging. This feeling of acceptance is a prerequisite for independent and autonomous work. Denied the same degree of relationships with faculty, female students in science have a more difficult time acting independently.

Furthermore, the forms of communication used by women and some students of color may make it particularly difficult for their attempts at mathematical communication to be accepted (Orr, 1997; Rosser, 1995), which may affect their participation or integration within mathematics.

I illustrate subsequently the importance of faculty mentoring for doctoral students. A student who is believed to be less capable—even if this belief results from stereotyping—may have a difficult time finding an advisor and other mentors, which will make it more difficult for her to complete her degree. This type of difference in behavior and perception may be critical to the experiences and persistence of students of color as well.

Achievement

Faculty in mathematics graduate programs have cited the poor preparation of graduate students as a problem in their programs (Conference Board of the Math-
emathematical Sciences, 1987, cited in Madison & Hart, 1990; Herzig, 2002; National Research Council, 1992). According to Bair and Haworth’s (1999) review of research on doctoral student attrition, a majority of studies found that traditional measures of achievement, such as graduate and undergraduate grades and standardized test scores, are unreliable predictors of doctoral student persistence. Of the small number of studies that indicated positive correlations between achievement measures and persistence, none focused specifically on mathematics, and most were published more than two decades ago.

In one older study, students who left graduate school without completing degrees had initially been rated by faculty as having the same level of promise as students who persisted (Sells, 1973). Current research is even more consistent in finding that students who leave graduate school do not have lower grades or GRE scores (Berg & Ferber, 1983; Bowen & Rudenstine, 1992; Girves & Wemmerus, 1988; Golde, 1996; Lovitts, 2001; Zwick, 1991). However, students admitted to a graduate program are already among the top performers, so the small variance they exhibit in terms of grades and standardized test scores limits the ability of these measures to predict degree completion (Tinto, 1993).

In interviews with 10 mathematicians, Herzig (2002) found that their beliefs in the importance of “talent” or “ability” led them to virtually ignore doctoral students in their first few years of the program, describing instruction as an opportunity for students to discover or prove whether they possessed that talent or ability. Some of these faculty felt that they were doing students a favor by helping them avoid wasting time if they were unlikely to complete the program.

These beliefs on the part of the faculty provide an explicit obstacle to students’ participation in the program, in that students are required to prove themselves first; only after they have proven themselves would they have opportunities to participate in meaningful ways in mathematics practice. This forms a sort of a “catch-22,” since it ignores the way that meaningful participation might enhance students’ abilities and skills at mathematics. (Herzig, 2002, p. 187)

This faculty belief in talent moved the structure of doctoral education from one of fostering students’ development as mathematicians to one in which the emphasis was on filtering out students not possessing the prerequisite talent or ability. In contrast, on the basis of a review of research on children’s motivation to learn, Ames (1992) concluded that making ability a salient feature of an educational environment interferes with students’ motivation to learn, their use of effective learning strategies, and their engagement with the content of the curriculum.

Confidence

Confidence has been described as an important feature in graduate students’ persistence toward a degree (Berg & Ferber, 1983) and as an influence on their career plans and persistence in pursuing them (Becker, 1984; Berg & Ferber, 1983). Science students have been reported to need confidence in their abilities in order to persevere when experiments fail; some students interpret the discouragement of a failed experiment to mean that they are ill suited to scientific work (Golde, 1996). Similarly, mathematics students may need patience and perseverance in solving mathematics problems, both in coursework and in research.
Men in science and mathematics have tended to appear more confident than women (Becker, 1984, 1990; Etzkowitz et al., 2000; Sonnert & Holton, 1995). Overall, relative to men, “women appear more timid, tend to set lower goals for themselves, and are likely to be given less encouragement” (Berg & Ferber, 1983, p. 631). In a survey of previously enrolled graduate students in the physical and biological sciences, Berg and Ferber (1983) found that significantly more women than men (63% vs. 37%) indicated that the ability to handle the work was an important factor in their choice of a field of study, despite the lack of significant differences in GPAs between men and women in their sample. The authors interpreted these findings to mean that women are “less likely to take their ability to do whatever they want to do for granted” (p. 635), although it may simply be the case that women are more willing than men to express their uncertainty (Burton, 1999a).

While women have reported entering graduate school with lower self-confidence than men, their confidence is often further eroded by their experiences within graduate school (Becker, 1984; Etzkowitz et al., 2000; Golde, 1996).

Confidence and ability are often intermingled in the research literature (Burton, 1999a), as researchers describe students’ confidence in their abilities and frequently the relationship between confidence and gender. However, most authors discuss confidence and ability without defining these constructs and without problematizing their often gendered connotations and implications (Burton, 1999a, 2001). Some authors have equated ability with the possession of certain forms of cultural capital; that is, students are constructed as talented not because of any measurable, definable attribute such as ability but, rather, because they possess particular types of cultural capital that are consistent with practices and expectations in schools (Herzig, 2004; Zevenbergen, 2004). Similarly, students may be described as confident when they possess the cultural capital necessary to behave in ways that are valued within the mathematical community.

Family Responsibilities

In a synthesis of 118 studies on doctoral student attrition, Bair and Haworth (1999) concluded that “the weight of evidence in this study indicates that the number of children or dependents of doctoral students is not a significant predictor of persistence or attrition” (p. 22). This is consistent with the common finding that parenthood does not have an impact on research productivity in science (Cole & Zuckerman, 1987). Citing sociologist Dorothy Smith, Grant et al. (2000) called this finding a “critical rift”: “a point at which the world we study as sociologists diverges sharply from women’s lived experiences” (p. 62).

When one of the authors presented a paper that summarized this common finding at a meeting of scientists, she was approached afterward by a well-known scientist who is also the mother of two young children. Gesturing emphatically with an oatmeal-encrusted sleeve of the sweater she had not had the time to change between feeding her five-month-old and dashing to the conference, she said, “That can’t possibly be true. Something is wrong. Go back and look at your evidence again.” (Grant et al., 2000, p. 62)

Several authors have warned about sample selectivity and other biases in studies of the effects of parenthood on the productivity of scientists (Grant et al., 2000; Sonnert & Holton, 1995). For example, scientists who faced the most serious
obstacles to combining an academic career with family life might no longer participate in science, skewing the sample of those who remain; furthermore, most studies on scientific productivity focus on elite scientists (Grant et al., 2000). While many of these studies assess the effects of family responsibilities on scientific productivity, they do not examine the reverse: the ways in which a career in science affects the decisions scientists make about marriage and family (Grant et al., 2000). There are many ways of being a parent, and the impact on scientific productivity is probably different for parents who choose to be more or less involved with their children’s daily lives.

What are obliterated by narrowly focused survey studies are the sacrifices in personal life that women scientists make to do scientific work, or the extraordinary management work they do to coordinate the demands of two greedy institutions that do not blend well with each other. . . . Most endured considerable costs, in terms of sleep loss, near complete loss of leisure time, and stressful days to make it work successfully. (Grant et al., 2000, p. 81)

To avoid conflicts with the demands of their careers, scientists (particularly women) have decided not to marry or have children, to postpone having children until their careers are more established, or to have fewer children than they had originally planned (Etzkowitz et al., 2000; Grant et al., 2000; Sonnert & Holton, 1995); these decisions have been made more commonly in the natural and physical sciences than in the social sciences. Doctoral students with dependents generally take longer to complete their degrees (Nerad & Cerny, 1993), and the impact of having dependents is substantially greater on women than on men (Lovitts, 2001; Nerad & Cerny, 1993; Sonnert & Holton, 1995), particularly in the physical sciences and mathematics.

Historically, the traditional male academic’s role required him to choose between a family and a career in academic science (Etzkowitz et al., 2000; Wertheim, 1995). While career and family are no longer assumed to be in conflict for men, this is not the case for women. Consequently, women graduate students in science who marry or have children have been viewed as not serious about their studies, or as unreliable and not worth the investment; men who marry or have families do not face the same biases (Etzkowitz et al., 2000).

In order to accommodate family life, women in science often choose to pursue careers in industry, and those who remain in academe tend to aspire to jobs in small teaching colleges rather than research universities (Etzkowitz et al., 1992). Golde (1996) reported that both female and male graduate students in science left graduate school because of a perceived imbalance between family and work responsibilities. “The lives that were modeled for students (few children, many divorces) did not give students hope that they could lead the balanced lives they valued” (p. 209). In mathematics in particular, some women reported having left graduate studies altogether because of the perceived incompatibility of the life of a doctoral student in mathematics and a personal life outside of mathematics (Stage & Maple, 1996). These perceived conflicts for women, between life in and out of mathematics and science, may explain why female graduate students have been reported to be less likely to be married and have families than male graduate students (Berg & Ferber, 1983; California Postsecondary Education Commission, 1990; Nerad & Cerny, 1993).
Some graduate students and scientists face the so-called “two-body problem” that can prevent graduate students and faculty from pursuing professional opportunities. Women are more likely to make career compromises to follow a spouse than are men, and are more affected by the concurrent timing of the graduate school, tenure, and childbearing clocks than men (Etzkowitz et al., 2000; Grant et al., 2000; Sonnert & Holton, 1995). What makes women faculty members attractive as role models for female graduate students might not be their professional success as much as their ability to combine a successful career with a family life (Etzkowitz et al., 2000; Golde, 1996; Sonnert & Holton, 1995). An additional obstacle facing students with family responsibilities is family-unfriendly scheduling, which limits the ability of some students to participate in department practices and activities (Committee on the Participation of Women, 2003).

**Features of Relationships Among Community Members**

Moving to the interpersonal plane of analysis, I now consider aspects of the doctoral student experience that concern students’ interactions with other members of the communities of practice of doctoral study in mathematics. These factors include pedagogy, moral support from significant others, mentoring and advising, feelings of isolation, and perceived competition.

**Pedagogy**

Given the important role of coursework in mathematics doctoral programs in the United States, pedagogical considerations are likely to play an important role in the recruitment and retention of doctoral students. However, the effects of pedagogy on the graduate student experience have been largely unexplored. Students in one doctoral program in mathematics complained about the teaching in their department, including few feedback mechanisms in their courses, “the lack of interaction between the instructor and the students, difficulty discerning the important information, incomprehensible lectures, non-English textbooks, and the lack of motivation or connections among mathematical ideas and the mathematical ‘big picture’ ” (Herzig, 2002, p. 190).

Despite common conceptions of mathematics as objective and nonemotional, mathematicians often talk about their emotional responses to their work (Burton, 1999b; Herzig, 2002). However, this emotional side of mathematical work contrasts sharply with the image of mathematics in most classrooms, where it is often presented devoid of any context, mystery, or excitement. “My participants, as teachers, are not exploiting their experiences. Nor are most of them giving learners a sense of the fun, excitement, [and] challenge which holds them in the discipline” (Burton, 1999b, p. 139). Burton’s conversations with mathematicians indicated how little they thought about their teaching and “especially how little they attempted to convey the struggle and the pleasure which they had described to me of doing mathematics” (p. 140). Similarly, the mathematicians interviewed by Herzig (2002) described their goal for instruction as being to communicate important mathematical ideas rather than to share their excitement or vision about mathematics.

Doctoral coursework in mathematics seems to focus primarily on a “transmission” (Rogoff, 1994) or “banking” (Freire, 1970) model of education in which students are assumed to be passive and their minds empty, as teachers work to tell them all they need to know about mathematics. However, this view of teaching and
learning is contradicted by mathematicians’ belief in the importance of talent or ability in students’ success and their consequent description of the purposes of early doctoral coursework and qualifying exams as opportunities for students to prove or discover whether they possess those attributes. In this discourse of talent, faculty members have limited responsibility for student success; their role as instructors is to provide a situation in which students can exert exclusive responsibility for their learning as they prove or discover whether or not they have the talent to succeed (Herzig, 2002).

Moral Support From Significant Others

Encouragement from people influential in their lives plays an important role in students’ decisions to enroll and persist in graduate studies. Both men and women have acknowledged the importance of encouragement and moral support from family members (Becker, 1990; Berg & Ferber, 1983; Carlson, 1999; Hollenshead et al., 1994; Sonnert & Holton, 1995; Stage & Maple, 1996). However, more women than men report receiving moral support (Becker, 1984; Berg & Ferber, 1983; Lovitts, 2001; Sonnert & Holton, 1995). Becker (1984, 1990) found that women in mathematics and computer science recalled both support and discouragement, while men recalled little encouragement and no discouragement: “The encouragement seemed really helpful to the women in their decisions to attend graduate school. The men seemed to be able to decide without ostensible support” (Becker, 1990, p. 127). Women may be less likely than men to embark on graduate studies when these types of support are unavailable, or, alternatively, women may be more aware of or more willing to acknowledge support (Becker, 1990; Berg & Ferber, 1983; Sonnert & Holton, 1995). Few mathematics graduate students report having received moral support or encouragement from faculty or mentors within their departments or programs, although, when such encouragement was offered, it made a significant difference to them (Herzig, 2002, in press; Mathematical Association of America and National Association of Mathematicians, 1997).

Mutual support among graduate students is important as they learn and complete assignments together, share information, and provide important moral support to each other (Golde, 1996; Stage & Maple, 1996). This can be particularly important for students who are members of underrepresented groups; Cooper (2000) reported that African American doctoral students in the mathematics doctoral program at the University of Maryland highly valued the academic and social support of other Black students.

Mentoring and Advising

Encouragement from mentors, in graduate school, college, and even high school, plays an important role in students’ decisions to enroll and persist in graduate studies in mathematics (Carlson, 1999; Cooper, 2000; Herzig, 2002; Hollenshead et al., 1994; Manzo, 1994; National Research Council, 1992; Sonnert & Holton, 1995; Stage & Maple, 1996). The overall norms and expectations of the departments and the quality of relationships with faculty are important factors in predicting degree progress (Girves & Wemmerus, 1988). Mentors within graduate school can be particularly valuable in providing moral support and encouragement (Cooper, 2000; Herzig, 2002; Hollenshead et al., 1994) and advice on how to negotiate the system (Etzkowitz et al., 2000). Women mathematics students have reported that when
they had doubts about continuing, their advisors encouraged them to stay; they have also reported that the sense that their advisors cared kept them going (Herzig, in press; Manzo, 1994). Advisors can help students learn to negotiate the politics of their departments and learn the “rules of the game” (Cooper, 2000). In particular, if students have not been socialized to understand the political strategies necessary to survive in graduate school in science, advisors can help them learn these strategies (Etzkowitz et al., 2000).

One common cause of attrition in the sciences is an incompatible relationship with advisors (Bair & Haworth, 1999; Girves & Wemmerus, 1988; Golde, 1996), which also has the effect of eroding students’ reports of self-confidence (Berg & Ferber, 1983; Golde, 1996). In other disciplines, lack of both faculty mentoring and departmental advising has been associated with high attrition rates and long periods to degree completion (Nerad & Cerny, 1993). Students who have left graduate school have reported that if their advisors or other faculty had been more supportive and sensitive, they might have been more inclined to stay (Lovitts, 2001).

Students who are treated as “junior colleagues” are more likely to remain enrolled in graduate school and complete degrees (Berg & Ferber, 1983; Girves & Wemmerus, 1988; Nerad & Cerny, 1993). On the basis of a survey of 459 graduate students who had been enrolled in 32 departments at one university in a 7-year period, Berg and Ferber (1983) reported that those who earned a doctorate (relative to those who enrolled in doctoral programs but did not earn a doctorate) were 3.4 times as likely (based on an odds ratio) to have reported being treated as a junior colleague by at least one male faculty member and 4.8 times as likely to have come to know two or more male faculty members quite well. (Of course, students who left without completing their degree might have left before these relationships with faculty members had the chance to develop.) Male degree recipients were significantly more likely than female degree recipients to perceive that they had been treated as a junior colleague by a male faculty member. (Relationships with female faculty members could not be analyzed because few students in their sample had sufficient interaction with female faculty to allow a sufficient sample size for analysis.) Conversely, students who feel they are treated as “adolescents” are less likely to complete their degrees (Nerad & Cerny, 1993).

Relationships between faculty and doctoral students of color (both men and women) have not been reported on in the research literature, but negative interactions with faculty are pervasive among women in science. Sonnert and Holton (1995) documented forms of discrimination that women faced in finding mentors, ranging from professors who would not take on women students to mentors who did not seem to tap into their professional networks as vigorously for their female students as they did for their male students. Women’s opportunities have also been limited by being excluded from the informal social networks of their laboratories or departments, being treated as “invisible,” or otherwise having their contributions marginalized (Becker, 1990; Committee on the Participation of Women, 2003; Etzkowitz et al., 1992, 2000; Sonnert & Holton, 1995; Stage & Maple, 1996). The Committee on the Participation of Women of the Mathematical Association of America (2003) reported on sexist behavior experienced by women in graduate mathematics, including unwanted sexual advances from faculty, tolerance of public sexist comments, and professors who openly stated that women are not as smart, dedicated, or talented as men.
Female students in the sciences and mathematics receive less mentoring from male faculty than do male students (Berg & Ferber, 1983; Etzkowitz et al., 2000; Hollenshead et al., 1994; Sonnert & Holton, 1995). There has been a tendency reported for faculty to mentor same-sex students (Berg & Ferber, 1983; Reskin, Koretz, & Francis, 1996). However, there are few women faculty in most STEM disciplines. This is true of mathematics in particular: In the fall of 2002, 13% of full-time doctoral faculty were women, and 31% of full-time graduate students were women (Kirkman et al., 2003). Burton (1999a) interviewed 70 practicing mathematicians in the United Kingdom and found that none of them had had a female advisor; yet, many of the 35 women she interviewed were advising graduate students. She concluded that there may be reason to expect that women in mathematics will have increased opportunities to have women as advisors.

When students do not receive adequate guidance and advisement regarding the nature of graduate school, they can have unrealistic expectations, which contribute to negative experiences in graduate school (Golde, 1996; Lovitts, 2001; National Research Council, 1992). Given the reduced opportunities women and students of color may have to develop substantive relationships with faculty, this represents an additional obstacle to their effective participation. These issues might be addressed by more careful advising.

The small numbers of women faculty, the importance of positive mentoring relationships, and the tendency for faculty to mentor same-sex students combine to pose a serious obstacle for women in mathematics. Although these issues have not been studied among women of different races and ethnicities or among men of color, it seems likely that similar forces may come into play for those students as well. Indeed, if faculty have a tendency to mentor students of the same race or ethnic group, then this would pose an even more significant challenge for students who are members of certain ethnic and racial groups as a result of the extremely small numbers of mathematics faculty from those groups. The mathematics department at the University of Maryland, which has both African American faculty and staff, has had disproportionate success in recruiting and graduating Black graduate students (Cooper, 2000).

**Feelings of Isolation**

Collaboration is an important part of the work of research mathematicians (Burton, 1999b; Henrion, 1997). Despite this, research and study in most mathematics doctoral programs are solitary activities (National Research Council, 1992). Stage and Maple (1996) asked former mathematics graduate students who had left mathematics to pursue degrees in other fields what it meant to be a mathematician. The students described patterns of isolation and lack of social interaction, expectations that involved extensive time commitments, and few interests outside of mathematics. Women doctoral students in the sciences and mathematics have reported how isolating graduate study had been, how seldom people worked together, the rare use of group work in classes, that faculty did not bother to learn the names of graduate students in their classes, and that students were not taken seriously until after they had passed their qualifying exams (Etzkowitz et al., 2000; Herzig, 2002; Hollenshead et al., 1994; Stage & Maple, 1996).

Students in several programs have reported the importance of having a "critical mass" of women or students of color (Cooper, 2000; Manzo, 1994). Graduate
women in mathematics, computer science, and physics have reported feeling isolated or alienated in their male-dominated departments, and some have specifically described ways in which they feel they do not fit in (Becker, 1990; Etzkowitz et al., 2000; Herzig, in press; Hollenshead et al., 1994).

**Perceived Competition**

Despite common stereotypes of mathematicians working in isolation, interviews with practicing mathematicians reveal that they do much of their work collaboratively (Burton, 1999b; Henrion, 1997; Herzig, 2002). Henrion (1997) describes the myth of the mathematical loner, which she calls the “Mathematical Marlboro Man” (p. 1), and attempts to discredit this myth through interviews with six successful women mathematicians. While these women viewed themselves as highly autonomous and independent, they also highly valued their interactions with other members of the mathematics community. These connections were important for their political value as well as for their contribution to the “doing” of mathematics. Henrion provides ample evidence of the social nature of mathematical activity and then argues that the image of a mathematician as a loner serves as a filter to keep certain types of people out of mathematics.

Despite the collaborative nature of much of their work, some mathematicians have described the intense competition of their work worlds, including the confrontational atmosphere at conferences (Burton, 1999a, 2000) and relationships with their PhD supervisors that featured “arrogance, bullying, favoritism, and the need for tenacity” (Burton, 2000, p. 4). Some of the mathematicians interviewed by Burton described communication styles commonly used in mathematics that were meant to impress and mystify rather than to explain and illuminate. In an essay about communication styles in mathematics, Keith (1988) described the confrontational language that is a normal part of mathematics discourse. Common terms used in mathematics such as “we claim,” “the proof is trivial,” “it is obvious,” and “it is commonly known that,” which are “a vocabulary of mathematical punctuation” (p. 5), can be intimidating. Burton (2000) echoed these findings, citing mathematicians’ descriptions of published mathematical results using terms such as “important,” “significant,” and “trivial.” This assertive communication style may be a mode of communication that is uncomfortable for some students, particularly women.

The mode in which mathematics is usually written and spoken is one of advocacy, of claims and assertions, one which generally ignores its audience. It is a language which I feel is more easily adopted by men than women, if we can believe what we are told about women using fewer declarative sentences in conversation. And it is a language, which particularly when spoken, is frequently abused with impatience, frustration, and defensiveness. Mathematical “arguments” in the rhetorical sense of the word, move easily into “argumentative” encounters. (Keith, 1988, p. 8)

Positive interactions with peers are important to students’ experiences and persistence in graduate school (Bair & Haworth, 1999; Hollenshead et al., 1994). The perceived competition of graduate study has been cited as a factor in students’ decisions to leave graduate school (Golde, 1996; Lovitts, 2001; Stage & Maple, 1996) and in enrolled students’ dissatisfaction with their experiences in graduate
school (Hollenshead et al., 1994). Working scientists have also complained about the perceived competitiveness of work in STEM (Sonnert & Holton, 1995), a feature to which some students of science object (Golde, 1996). “For some, the reality of competition between faculty for success was contrasted with an idealized model of selfless, cooperative search for scientific truth. The reality dismayed and frustrated them” (Golde, 1996, p. 208).

Doctoral study in mathematics is a competitive enterprise (Cooper, 2000; Hollenshead et al., 1994; Stage & Maple, 1996). In response to concern about a difficult job market for PhD mathematicians, some members of the mathematical community advocate increasing the competition among doctoral students so that smaller numbers succeed.

For people who view the profession as a kind of priesthood, it is appealing to reduce numbers by keeping out all but the most worthy. However, there might be several negative consequences to such an approach. First, there would be the terrible human waste of labeling a large group of our most talented people as failures and choking them out. . . . Second, while Darwinian selection appeals to many mathematicians as a fair way to choose who succeeds, the playing field is often not as level as many would like to believe. In many cases, it is as though someone taught some of the animals how to use weapons and then accepted the outcome of which animals survived as having been dictated by nature. (Douglas, 1997, p. 43)

Since 1987, fewer than 50% of mathematics PhDs awarded in the United States have been awarded to U.S. citizens (Loftsgaarden et al., 2003). Students who receive their undergraduate mathematics training outside the United States often enter graduate school with stronger mathematical backgrounds than their U.S.-educated peers (Madison & Hart, 1990; National Research Council, 1992). As a result, foreign-educated students often have an advantage over domestically educated students, and, when enrolled in the same introductory classes, they often perform better. U.S.-educated students can feel intimidated by competing with foreign-educated students (Herzig, 2002), which may contribute to increases in attrition among U.S.-educated students (National Research Council, 1992).

Features of the Communities of Graduate Study

While there is a literature focusing on disciplinary cultures more generally (e.g., Becher, 1989; Gerholm, 1990), few aspects of these cultures have been studied in the particular context of the graduate student experience. These cultures form an important part of the broader context of doctoral education. In this section, I review aspects of doctoral education in mathematics that are more visible on the community plane: department and program structure, financial support, support for students with families, epistemology, and the state of the job market.

Department and Program Structure

Lower attrition rates and shorter average degree completion times have been associated with programs that offer the most structure and supervision (California Postsecondary Education Commission, 1990). Well-structured programs may have a positive impact on degree progress through the effects of enhancing students’ integration into the department.
Research training in the sciences tends to be laboratory- and group-oriented, highly structured, closely supervised, and has shorter time to degree [and lower attrition]. Conversely, research training in the arts, humanities, social sciences, and the professions tends to be individualistic and less structured, with less supervision of day-to-day progress in research activities. Time to degree is longer in those individualistic disciplines with lower levels of structure. (California Postsecondary Education Commission, 1990, p. 31)

Graduate study in mathematics is more similar to study in the humanities than it is to study in the sciences (cf. Becher, 1989), which may contribute to the long average times to degree completion and relatively high attrition rates in mathematics.

The Mathematical Sciences Board of the National Research Council (1992) found that successful doctoral programs in mathematics are those with a “focused, realistic mission.” The standard program of doctoral study in mathematics, which aims to prepare students for careers in academic research, is too broad and lacks this quality. Instead, programs that focus on educating graduate students in specific subdisciplines or in interdisciplinary areas might be more successful in recruiting and graduating students.

Some mathematics departments, under the twin pressures to admit more domestic students and to staff undergraduate courses with teaching assistants, have liberal admissions policies. The more successful programs are those that provide the support necessary for students to make up for deficiencies in their preparation; however, in many programs this is not done (National Research Council, 1992), so students who need something other than the standard program of study might not succeed. Other structural factors that have been associated with high attrition rates or long times to degree completion include requirement of a master’s degree before completion of a PhD (California Postsecondary Education Commission, 1990; Nerad & Cerny, 1993); sporadic evaluations and irregular, infrequent interactions between students and faculty (Nerad & Cerny, 1993); the definition of a dissertation as contributing to existing knowledge as compared with a test of future ability to do research (Nerad & Cerny, 1993); and the size of the student cohort (smaller cohorts exhibit less attrition) (Bowen & Rudenstine, 1992).

Financial Support

In comparison with students who do not receive financial support during their graduate studies, students who receive such support are more likely to complete their PhDs (Bair & Haworth, 1999; Berg & Ferber, 1983; Bowen & Rudenstine, 1992; Girves & Wemmerus, 1988) and complete their degrees in one half to two thirds the time (Bowen & Rudenstine, 1992; California Postsecondary Education Commission, 1990; Nerad & Cerny, 1993). Students employed as teaching or research assistants have more opportunities to interact with faculty and other students, are more likely to receive office space, and are more likely to learn about research or teaching norms within the field. Consequently, students supported as teaching or research assistants are more likely to become involved and integrated in their departments, which has a positive effect on degree progress (Girves & Wemmerus, 1988; Lovitts, 2001; National Science Foundation, 1998; Tinto, 1993).

Not surprisingly, students whose support comes in the form of teaching assistantships take longer to complete their degrees than students whose support comes in the form of fellowships or research assistantships (Bowen & Rudenstine, 1992;
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California Postsecondary Education Commission, 1990), since teaching responsibilities can consume a great deal of time without contributing directly to research. In contrast, in disciplines in which students’ research work supports their dissertations, a research assistantship can facilitate degree completion (Nerad & Cerny, 1993). In comparison with doctoral students in the physical sciences and engineering, mathematics graduate students are much more likely to receive support in the form of teaching assistantships and are much less likely to have research assistantships (Madison & Hart, 1990; National Research Council, 1992).

Support for Students With Families and Other Campus Resources

Certainly, some of the effect of family life on graduate student progress could be ameliorated by appropriate institutional responses such as affordable child care, tolerance of (and indeed support for) part-time study, flexible deadlines and a slower pace for students who are also parents, financial support, and flexibility in scheduling (Etzkowitz et al., 2000; Manzo, 1994). Availability of child care has been associated with lower attrition rates and shorter times to degree completion in doctoral programs (Nerad & Cerny, 1993). Women of color were particularly successful in one doctoral program that had made a commitment to “accommodating the busy professional and personal lives of the women, many of whom are working mothers” (Manzo, 1994, p. 40); students and graduates of that program reported that such flexibility was a critical factor in their persistence.

Other campus resources, such as availability of affordable housing and convenient transportation, have also been associated with lower attrition rates and shorter times to degree completion (Nerad & Cerny, 1993). These factors allow students increased opportunities to participate in the practices of their disciplines, despite their competing family or other responsibilities.

Epistemology

Some students have described being attracted to mathematics because of its abstract nature (Becker, 1984; Herzig, 2002, in press). Other students have reported disillusionment with the degree to which their studies or research in mathematics or science were removed from meaningful or relevant questions (Golde, 1996; Herzig, 2002; Stage & Maple, 1996). In interviews with seven women who had left graduate programs in mathematics, Stage and Maple (1996) found that

most described a growing frustration with the seeming lack of connection of mathematics with the world surrounding them. Mathematics began to seem an endless series of puzzles that could be solved if enough time or effort was invested. . . . Solutions bore little relationship to others’ learning of mathematics, social issues, or the people in their own lives. (p. 32)

Biology students interviewed by Golde (1996) cited the irrelevance of research to important real-world questions. Also, Sonnert and Holton (1995) found that

a frequent cause [of leaving science] mentioned by both men and women was rooted in the culture of science: they felt their science jobs required too many hours of hard work, the work itself was too fragmented and meaningless, and the results lacked applicability and relevance. (p. 170)

Doctoral study in mathematics is largely focused on traditional, theoretical mathematics (National Research Council, 1992). As a result, “many researchers...
lack the broad knowledge needed to address real-world problems . . . and the sys-

tem of education is more or less self-contained, with graduates teaching what they

have been taught in the same manner they have been taught” (p. 15).

Mathematics is often taught in highly abstracted ways, with little or no explicit

connection to other mathematical ideas, ideas outside of mathematics, or the math-

eatical “big picture” (Herzig, 2002). Women may tend to be more interested in

relationships and interaction among ideas than men (Belenky, Clinchy, Goldberger,

& Tarule, 1986), and in science women are more eager to learn how scientific ideas

and facts fit together than are men, who may be more content to examine informa-

tion out of context (Rosser, 1995). One project of some feminist writers has been to

challenge the predominance of abstraction in mathematics; for example, Johnston

(1995) argues that abstraction in mathematics is a consequence of modern indus-

trial society, based on the idea of separating things into manageable pieces, apart

from their context.

Much research on attrition, and on a host of other educational issues, includes

mathematics in categories along with the physical or natural sciences. Indeed,

mathematics shares features with the sciences such as high consensus on research

paradigms, including both methods and problems for future research and their loca-


qualities of “hard” and “soft” knowledge disciplines and found that mathematics

bears many similarities to the sciences (“hard knowledge”) in areas such as crite-

ria for and meanings of new knowledge, research questions and paradigms, the

nature of argument, and epistemology.

These analyses imply that the nature of mathematical knowledge is much like

that of the “hard” sciences. However, the nature of graduate research in mathe-

matics has strong similarities to research in the humanities. Science students gen-

erally begin research early in graduate school and work in organized research

teams, which is rarely the case in the humanities (Golde, 1996; Lovitts, 1996;

Tinto, 1993) or in mathematics, where students often do not begin research until

they have completed their graduate coursework (National Research Coun-

cil, 1992). Research in mathematics and the humanities is more likely to be indi-

vidual and isolated (National Research Council, 1992), as opposed to the high

degree of collaboration that is more typical in the sciences (Becher, 1989; Califor-

nia Postsecondary Education Commission, 1990; Golde, 1996; Nerad & Cerny,

1993; Tinto, 1993). As do graduate studies in mathematics, graduate studies in

the humanities emphasize absorbing canonical knowledge and teaching, activi-

ties that are downplayed in the sciences in favor of actively conducting research

(Golde, 1996).

Attrition rates in mathematics are generally estimated to be closer to those in the

physical sciences than to those in the humanities (Bowen & Rudenstine, 1992;

National Science Foundation, 2002; Sells, 1973; Zwick, 1991). This observation

implies that the similarities between mathematics and the sciences may be more

important in understanding attrition from mathematics than the similarities between

mathematics and the humanities. In other words, since mathematics and the physi-

cal sciences share some epistemological foundations, and since attrition in these dis-

ciplines seems to be similar, mathematics epistemology, at least as it is represented

in and constructed through graduate study, might be important in understanding

attrition and might warrant further investigation.
State of the Job Market

Doctoral completion rates have been shown to deteriorate during periods in which the economy results in a poor job market for academics (Bowen & Rudenstine, 1992). Students in disciplines with poor academic job markets (few available positions or low salaries) are more likely to leave graduate school or to take long times to complete their degree (Nerad & Cerny, 1993). In contrast, Golde (1996) reported a higher attrition rate from geology, a science discipline in which students perceived that they would be able to find high-status, well-paying jobs with an MS, than from biology, in which students perceived that an MS would earn them primarily low-status jobs with limited room for advancement.7

The current job market in mathematics is improving. People with master’s degrees in mathematics have had very low unemployment rates (Madison & Hart, 1990), which may contribute to the relatively high attrition rate from mathematics doctoral programs. The unemployment rate for new doctoral recipients has been below 5% since 1997, down from a high of 11% in 1994 and 1995 (Kirkman et al., 2003).

In times when the job market for new mathematics PhDs is tight, the response of the mathematical community has been to reduce the number of students who complete their PhDs, either by “raising the bar” (Douglas, 1997) or by encouraging students to choose other disciplines (National Research Council, 1992). “Our over-reliance on academia for jobs for new Ph.D.s is often not considered to be a problem, nor is the matching of doctoral education with the positions that graduates take considered to be a priority” (National Research Council, 1992, p. 5). Doctoral education in mathematics has a strong tradition of training students for positions in academic research; training students for other job possibilities, such as teaching or work in industry, has not been emphasized, leaving doctoral students ill prepared for the jobs many of them take (Douglas, 1997; National Research Council, 1992). Some former graduate students have cited lack of a clear understanding of career options for researchers in science (Golde, 1996; Lovitts, 2001) or mathematics (Becker, 1984; Stage & Maple, 1996) as a factor in their decisions to leave graduate school.

A Leaky Pipeline?

The disproportionate loss of various groups of students from progressive stages of postsecondary STEM is often described via the metaphor of a “leaky pipeline.” This metaphor is flawed in several important ways, not least of which is the implication that students are passive players in their education; those who “leak” out of the pipeline are waste products with the poor fortune of encountering a crack in the pipe. Considering the educational system to be a pipeline poses students as an unwitting and homogeneous natural resource affected only by the global forces of fluid dynamics (and perhaps other molecular-level forces). Furthermore, a “leaky pipeline” does not adequately address why members of some groups stay in STEM while others leave in greater proportions, and it fails to model important features of postsecondary education in STEM that may contribute to attrition. By combining all students into one undifferentiated volume of “fluid,” this metaphor allows researchers and policymakers to overlook the very human implications of the inequitable postsecondary educational environment in STEM.
Herzig

Adelman (1998) proposes a “path” as a more appropriate metaphor, arguing that it incorporates students’ decisions and growth and describes a situation in which there are many possible ways to traverse the educational terrain to many possible destinations. While a “path,” in which students are sentient human beings making decisions about their interests and destinations, is an improvement over the notion of a pipeline, it implies that all groups of students have equal opportunities to traverse all possible paths and overlooks the obstacles that are placed in students’ way and how those obstacles differ for different groups of students. While this metaphor may represent STEM education as we wish it were, the pathway metaphor also ignores important aspects of postsecondary STEM education, such as its culture of fierce competition and harsh weed-out policies.

I prefer the metaphor of a cross-country track event. Like a cross-country race, an education in STEM is individualistic, competitive, and intense. There is a given starting point, a predetermined trail to follow, and a clear finish line. While students can choose alternative routes, which may provide them with equally interesting experiences, these alternatives do not “count” as part of the race and can often disqualify the runner. (This is not to say that the alternative trail does not lead to a worthwhile destination, as long as the runner is willing to accept no longer being eligible to cross the official finish line.) Little support is provided for runners along the way; they are assumed to arrive at the starting line already prepared to complete the event largely on their own.

Athletes competing in such a race come differently equipped. Some runners have the resources to have acquired expensive equipment, such as the latest, lightest running shoes, or to have participated in sophisticated training programs that enhance their chances to succeed over other runners who are not so privileged. Runners also differ in their entering skills—for example, one may be better on uneven terrain, while another can better handle hills. In such a race, the event will go to the runner whose skills best match the particular terrain of the race, even though neither runner might be considered to be superior overall (even though the victor will earn that label, at least temporarily).

Finally, many runners can complete a cross-country race, even if they do not do it very quickly. Given enough time and sufficient resources (better running shoes, water stops along the way, time to train to bring their skills in line with those required by the race, and trail markers when they get lost), entrants with a broad range of equipment, skills, abilities, and talents can enjoy the satisfaction of completing the race, even if they have to walk (or crawl) for parts of the trail. Unfortunately, the race is usually structured in such a way that once the fastest and best-equipped athletes have crossed the finish line, the race is considered over and everyone goes home.

Similarly to the case of runners in a cross-country race, not all participants in the communities of practice of graduate study come equally equipped or with equivalent starting skills, and not all are equally familiar with the rules of the game. Students are members of other communities as well, such as those of family and work. To some extent, persistence depends on the degree to which they successfully negotiate the competing demands of these different communities or, alternatively, the ways in which these other communities interfere with or enhance their participation and integration in their programs (Tinto, 1993). As a result, students do not have equitable opportunities to participate, which leads to unequal chances that they will persist.
Of course, doctoral study is a challenging undertaking for all students, and they all face obstacles to participation and integration. In reviewing the previous research literature on doctoral student persistence, I identified several obstacles that are either unique to women and students of color or are more challenging for those students than for their peers. Reducing attrition and building a more diverse population of mathematicians requires constructing a more equitable race through elimination of the particular obstacles to participation faced by women and students of color. Perhaps an even better goal would be to restructure the education of graduate students so that it is no longer a race at all.

Enhancing the Participation of Women and Students of Color in Mathematics

Previous research on the doctoral student experience has focused on identifying which features of individuals, relationships, and programs are correlated with doctoral student persistence and attrition. This plethora of causes presents a complex picture for faculty or policymakers because it is not clear what changes in the structure of graduate education might effect real change. Furthermore, while some of these factors have a stronger impact on particular groups of students, many of them affect all students; from this picture alone, it is not clear why some groups of students leave mathematics at higher rates than others. In order to further understand the issues that lead students from some demographic groups into and out of mathematics at different rates than others, we need to know more about the processes underlying these diverse “causes” of doctoral student persistence and attrition. Through the persistence framework, I attempted to identify these processes.

In developing the persistence framework, I argued that the persistence of a doctoral student in mathematics, and the quality of her experience in graduate school, is a function of how well integrated she is in the academic communities of her department and discipline. This argument has been made elsewhere (Lovitts, 2001; Tinto, 1993). The new contribution here is the questioning of the process by which students become integrated. Theories of situated learning posit that learning occurs through participation in social practices and, indeed, that learning is inseparable from that participation (Boaler, 2000). For doctoral students, learning occurs as they participate in the communities of practice (Lave & Wenger, 1991; Rogoff, 1995; Rogoff et al., 1995; Wenger, 1998) found in their programs and departments. The extent and nature of students’ participation, viewed on the individual, interpersonal, and community planes of focus, determine and limit their opportunities to learn to become mathematicians (Rogoff, 1995; Rogoff et al., 1995). I turned an eye toward aspects of individuals, relationships, and communities of doctoral study that inhibit or enhance students’ opportunities to participate in the activities and practices of their department, program, and discipline.

Throughout this description of features of individual graduate students, interrelationships among community members, and aspects of the community itself, as well as their effects on persistence, we see the interactions among the three planes of analysis: individual, interpersonal, and community. Many of these factors affect the participation of students on other planes of focus. Perceptions of a student’s ability, confidence, or autonomy have a profound effect on the relationships she develops with other members of the academic community, both in and out of class.
These relationships are also bound by the participatory structures of the department and program; the common practice in many mathematics departments of isolating students from authentic mathematics practice limits the types of relationships they are likely to develop with faculty. In particular, the effects of family responsibilities on student outcomes are minimized in programs and institutions that provide a range of supports for students who are parents; moral support from significant others should be less critical when students are enrolled in supportive programs. Together, this web of individual, interpersonal, and community forces acts to limit students’ opportunities to appropriate the knowledge, practices, and identities of mathematicians.

There are several ways in which women and students of color have distinctly different opportunities to participate in the communities of practice of their departments and programs. First, if a student has commitments to an ethnic, cultural, or family community (as is the case with students who are parents), it may be difficult for her to participate in the activities of the academic community. These competing communities of practice in which students participate can isolate them from the communities of their departments and programs, particularly in the case of programs that are inflexible or are built on narrow models of how students can or should be available to participate in departmental communities. Second, a student who is not accepted by the other community members or who is perceived to have a particular set of skills, abilities, and dispositions—such as is the case of women who are constructed to be lacking in confidence or autonomy—will have fewer opportunities to develop effective relationships with mentors and others. By constraining her participation on the interpersonal and community planes, these perceptions of her as a learner will indeed make it difficult for her to appropriate the knowledge, practices, and identity of a mathematician.

Empirical research needs to investigate (a) the nature of effective and authentic participation in academic communities, attending to the particular features of specific disciplinary and institutional communities; (b) the processes by which doctoral students come to be full participants in those communities; (c) the extent and manner in which personal characteristics of students (e.g., their gender, race, social class, ability, and family responsibilities) affect their participation and integration in those communities; and (d) educational models that facilitate the participation of a more diverse range of students. Research is currently in progress to investigate several doctoral programs in which women or students of color have had far more success than is typical in the mathematical sciences, as a way of exploring these issues further.

The persistence framework shifts the research lens away from questions such as “What features of students, universities, programs, and departments are associated with the success of doctoral students?” to questions such as “What happens in the interactions among students, faculty, programs, and departments that leads to the success of doctoral students?” and, ultimately, “What can universities and their faculties do to enhance and facilitate the participation and integration of a broad range of students in doctoral programs?” The persistence framework implies that the key to retaining diverse students in graduate mathematics is to develop ways to help students participate fully and effectively in authentic mathematics practice.

In a recent interview, the former chair of the Department of Mathematics at the University of Nebraska, when asked about the relatively large proportion of PhDs
earned by women in that department, said, "We just remove obstacles" (J. Lewis, personal communication, September 28, 2003). In order to improve the experiences and numbers of women and students of color in doctoral mathematics, faculty and policymakers need to consider things they can do to minimize or eliminate the obstacles these students face to participation in all aspects of the academic and social communities of their departments and programs. This framework should direct the attention of mathematics faculty and policymakers to those features of institutions, disciplines, and programs that interfere with some students' participation in the practices of the discipline; in addition, it should direct their attention to designing programs that will make it possible for more students to become integrated into their departments and programs.

Notes

I would like to thank the editors and anonymous reviewers for their thoughtful and truly helpful feedback on earlier versions of this article. Thanks also to Leone Burton, Thomas Carpenter, Eric Knuth, Julia Koza, Walter Secada, and Robert Wilson for their support and guidance in the writing of an early draft.

Although these statistics give an idea of rates of degree completion by women, they do not directly measure attrition, since students enrolled or graduating in a given year may have entered in different cohorts with unknown enrollment and completion/attrition patterns.

Six years earlier, of the U.S. citizens and permanent residents receiving a bachelor's degree in mathematics, 7% were Black, 4% were Latino, and 0.4% were Native American (National Science Foundation, 2000b). These figures provide an indication of the demographic characteristics of those students who were still enrolled in mathematics at the stage just before graduate school.

However, as Noddings (1992) argues, "giving all [students] access to privileged knowledge probably won't work. . . . Power structures do not crumble easily" (p. 32).

Many mathematics graduate students also participate in a third community of practice: that of teaching or working as teaching assistants. Thanks to Jennifer Szydlik for this observation.

Interestingly, this metaphor comes from physics, one of the most male dominated of the academic disciplines.

Sonnert and Holton (1995) offer women's marginalization within the social system of science as a possible explanation for women scientists' lower tendency to collaborate, given that such collaboration posed a risk of being dominated or not receiving credit for their work.

There were other differences between these two departments that may have contributed to attrition as well, such as policies and their related obstacles; students in geology perceived more obstacles (Golde, 1996).

References


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Herzig, A. H. (2002). Where have all the students gone? Participation of doctoral students in authentic mathematical activity as a necessary condition for persistence toward the Ph.D. Educational Studies in Mathematics, 50, 177–212.


Herzig, A. H. (in press). “Slaughtering this beautiful math”: Graduate women choosing and leaving mathematics. Gender and Education.


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Wiles, P. (1999). *Graduate students as legitimate peripheral participants in a mathematical community*. Unpublished manuscript.

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### APPENDIX

**Studies included in the review**

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<th>Focus</th>
<th>Methods</th>
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<td>Becker (1984, 1990)</td>
<td>Mathematics and computer science</td>
<td>Factors influencing women to pursue advanced study in mathematics and computer science</td>
<td>Interviews with 9 male and 6 female master's students in mathematics, and 8 female and 8 male master's students in computer science, at 2 universities in 1983 and 1985</td>
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<tr>
<td>Carlson (1999)</td>
<td>Mathematics</td>
<td>Factors influencing students' persistence in the study of mathematics; problem-solving behaviors; beliefs about mathematics</td>
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<td>Committee on the</td>
<td>Mathematics</td>
<td>Experiences of women in graduate school in the mathematical sciences</td>
<td>In-person or on-line interviews conducted in 2002 with 12 enrolled, attrited, or graduated female doctoral students</td>
</tr>
<tr>
<td>Participation of Women (2003)</td>
<td>Mathematics</td>
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<td>Cooper (2000)</td>
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<tr>
<td>Herzig (2002)</td>
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<td>Hollenshead, Younce, and Wenzel (1994)</td>
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<td>Manzo (1994)</td>
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<td>Mathematical Association of America and National Association of Mathematicians (1997)</td>
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<td>National Research Council (1992)</td>
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<td>Stage and Maple (1996)</td>
<td>Mathematics</td>
<td>Women’s experiences and participation in mathematics; perceptions of mathematics and themselves within it; their relationship to the subject</td>
<td>Interviews with 7 women doctoral students in education at a midwestern research university; participants had completed bachelor’s degrees in mathematics, and 5 had previously begun graduate study in mathematics</td>
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<tr>
<td>Authors, Years</td>
<td>Field(s)</td>
<td>Overview</td>
<td>Methodology</td>
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<tr>
<td>Etzkowitz, Kemelgor, Neuschatz, and Uzzi (1992)</td>
<td>Physics, chemistry, electrical engineering, and computer science</td>
<td>Understanding why women leave academic science and what happens to them when they leave</td>
<td>Data from 4 departments at a Research I university; interviews with 25 enrolled and recently graduated female graduate students, and 21 faculty and administrators (including all 5 women faculty and 2 former women faculty); quantitative data from departmental records on graduate students, recent PhDs, and students who left programs before earning their doctorates</td>
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<tr>
<td>Etzkowitz, Kemelgor, and Uzzi (2000)</td>
<td>Sciences</td>
<td>&quot;Life-course analysis&quot; of women in science from early childhood through academic work, to explore why there are so few women scientists</td>
<td>Synthesis of research literature, integrated with extensive interviews with female and male graduate students, faculty, postdoctoral fellows, and children, and a quantitative survey of women graduate students and faculty detailing women's experiences in science</td>
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<tr>
<td>Grant, Kennelly, and Ward (2000)</td>
<td>Natural, physical, and social sciences</td>
<td>Marriage, motherhood, and scientific productivity; impact of science careers on personal lives</td>
<td>Semistructured interviews with men of color or women ($n = 41$) and White men ($n = 14$) who teach in doctoral-granting institutions, conducted in the early 1990s; part of a larger study based on a national sample of female and male scientists</td>
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<td>Sonnert and Holton (1995)</td>
<td>Sciences, engineering, and mathematics</td>
<td>Detailing the career paths of female and male scientists, with particular attention to the effect of gender on careers in science</td>
<td>Survey responses from 508 men and 191 women and interviews with 108 women and 92 men (including 91 fellows in the physical sciences, mathematics, and engineering), conducted in 1989–1990; participants had received postdoctoral fellowships from the National Science Foundation, the National Research Council, or the Bunting Institute (and Bunting finalists) between 1952 and 1986</td>
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<tr>
<td>Bair and Haworth (1999)</td>
<td>Across disciplines</td>
<td>Across disciplines more broadly</td>
<td>Meta-synthesis of findings from 118 research studies on doctoral student attrition and persistence published between 1970 and 1988</td>
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<tr>
<td>Berg and Ferber (1983)</td>
<td>32 academic departments</td>
<td>Measures of success of women and men in and after graduate school; differences across departments</td>
<td>Quantitative analysis of responses to a 1979 survey of people enrolled as graduate students in 32 academic departments during the years 1968–1975 at the University of Illinois at Urbana-Champaign</td>
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<td>Bowen and Rudenstine (1992)</td>
<td>English, history, political science, economics, mathematics, and physics</td>
<td>Efficiency and effectiveness of graduate programs; attrition and time to degree</td>
<td>Quantitative analyses of institutional data from 10 universities on graduate students in 6 disciplines; data from the Doctorate Records File maintained by the National Research Council; and data on fellowship recipients</td>
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<tr>
<td>California Postsecondary Education Commission (1990)</td>
<td>Across disciplines</td>
<td>Recommendations for shortening time to PhD and for increasing the numbers of women and minorities receiving PhDs from the University of California</td>
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<td>Girves and Wemmerus (1988)</td>
<td>Across disciplines</td>
<td>Development of a model to predict progress toward master's and doctoral degrees</td>
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<td>Golde (1996)</td>
<td>Biology, geology, English, and history</td>
<td>Effects of departmental policies, practices, and relationships on the doctoral attrition process</td>
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<td>Quantitative analysis of institutional data describing demographic characteristics and degree progress of doctoral students in the University of California system between 1968 and 1988; interviews with 300 graduate students across the system's 9 campuses</td>
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<td>Quantitative analysis of institutional data on 948 students who entered graduate school in a large midwestern university in 1977, representing 42 departments; 486 responses to a 1985 survey of that sample</td>
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<td>Cases studies of 4 departments at a large university, based on interviews with 58 attrited doctoral students and observations, artifacts, and interviews with enrolled and graduated students, departmental staff, and faculty; quantitative analysis of enrollment data for all students who began a PhD granting program at the university between 1984 and 1993</td>
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<tr>
<td>Lovitts (2001)</td>
<td>Mathematics, biology, chemistry, sociology, economics, psychology, English, history, and music</td>
<td>High rates of attrition and causes of attrition; aspects of organizational structure of graduate school, process of graduate education, and how they affect attrition; opportunities for integration and cognitive map development</td>
<td>1994–1995 survey of 816 degree completers and noncompleters (62 in mathematics) who entered PhD-granting programs between 1982 and 1984 in 9 departments at 2 universities; telephone interviews with 30 noncompleters and the director of graduate studies in each department and in-person interviews with 33 faculty who produced either high or low numbers of PhD students</td>
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<td>Nerad and Cerny (1993)</td>
<td>History, English, French, sociology, psychology, and biochemistry</td>
<td>Student retention and time to degree; policy recommendations</td>
<td>Completion rates among students entering the University of California at Berkeley in 1978–1979 and time to degree for all doctoral degree recipients between 1980 and 1987; interviews with students from 4 departments with long average times to degree and low completion rates (history, English, French, and sociology) and 2 departments with shorter average times to degree and high completion rates (psychology and biochemistry)</td>
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<tr>
<td>Zwick (1991)</td>
<td>11 departments, including mathematics</td>
<td>Differences in degree progress and completion across academic programs and by race, ethnicity, and gender</td>
<td>Quantitative analyses of institutional data from 3 large research universities for PhD-seeking students entering between 1978 and 1985</td>
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