Energy Markets III: Carbon Emission Trading

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Emission Trading

- **SOx and NOx Trading**
  - Have existed in the US for a long time
  - Does it create Pollution Hot Spots?
  - Not enough liquidity
- **Cap & Trade for Green House Gases** (**Kyoto**)
  - The Lessons from the EU Experience
  - Carbon Markets **soon to exist in the US**
- **Equilibrium Models**
  - For emission credits (**Fehr-Hinz**)
  - Joint for Electricity and Emission credits (**RC-Porchet**)
  - ibidem simultaneously (**Fehr-Hinz**)

Market Model

- **Borrowing** from future allowances allowed
- **Banking and No Borrowing** Agents must offset their emissions at every time step (EU ETS)
- **Combined Model**
  - Production & Trading allowed at each time step
  - Emission offsetting at the end of each period

Regulator Input

- **TAX** π for each ton of carbon **not** offset by credit certificate
- For each agent $i \in \mathcal{I}$ and each time $t \geq 0$
  - **Credit allocation** $E_t^i \geq 0$ given at the beginning of each time period

Individual Agent Equilibrium Problem

Assume two **stochastic processes** $\{P_t\}_{t\geq 0}$ and $\{Q_t\}_{t\geq 0}$ are **GIVEN**

At each time period t

- Agent i holds a position
  - θ_t^i in emission credits
  - $\theta_t^{0,i}$ in cash in a self-financing portfolio
- $(\theta_0^i, \theta_0^{0,i}) = (x^i, 0)$
- **Emission credits** used for offsetting purposes α_t^i
- **Self-financing condition**

$$\theta_{t+1}^i - \theta_t^i = -\frac{\theta_{t+1}^{0,i} - \theta^{0,i} - t}{Q_t} + E_t^i - \alpha_t^i$$

- Change in the number of credits comes from
  - sale/purchase on the market
  - allocation
  - emission offset

Model cont.

Portfolio at time T

$$X_T^{\theta^i,\alpha^i,Q} = x^i + \sum_{t=0}^{T-1} \theta_{t+1}^i (Q_{t+1} - Q_t) + \sum_{t=0}^{T-1} (E_t^i - \alpha_t^i) Q_t$$

Cash Market Clearing

$$\sum_{i \in \mathcal{I}} \theta_t^{0,i} = \sum_{i \in \mathcal{I}} x^i, \qquad 0 \leq t \leq T$$

Credit Market Clearing

$$\sum_{i \in \mathcal{I}} \theta_t^i = \sum_{i \in \mathcal{I}} \sum_{s=0}^{t-1} (E_s^i - \alpha_s^i), \qquad 0 \leq t \leq T$$

We also need

$$\theta_t^i \geq 0, \qquad 0 \leq t \leq T, \; i \in \mathcal{I}$$

Optimization Problem for Agent i

$$\sup_{S^i,\theta^i,\alpha^i} \mathbb{E}\left\{ \sum_{t=0}^{T-1} \left(P_t \sum_{j\in\mathcal{J}} S_t^{i,j} - \sum_{j\in\mathcal{J}} c_{i,j} S_t^{i,j} - \pi \left(\sum_{j\in\mathcal{J}} e_{i,j} S_t^{i,j} - \alpha_t^i \right)^+ + X_T^{\theta^i,\alpha^i,Q} \right) \right\}$$

Remarks:

- $Q_t \in [0, \pi]$
- $\{Q_t\}_t$ is a super-martingale

Existence

Social Cost Minimization Problem

$$\sup_{S,\alpha} \mathbb{E}\left\{\sum_{t=0}^{T-1}\left(-\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} c_{i,j} S_t^{i,j} - \pi\left(\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} e_{i,j} S_t^{i,j} - \alpha_t^i\right)^+\right)\right\}$$

under the constraints

$$\begin{aligned} \sum_{0\le s\le t}\sum_{i\in\mathcal{I}} E_s^i &\ge \sum_{0\le s\le t}\alpha_s \qquad 0\le t\le T \\ \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} S_t^{i,j} &= D_t, \qquad 0\le t\le T \end{aligned}$$

Existence

- Include constraints in a Lagrangian
- Look for a saddle point

Sol. to Social Cost Minimization Problem

- Linearize positive part
- First order conditions give *Lagrange Multipliers*
- Determine two processes $\{P_t^*\}_t$ and $\{Q_t^*\}_t$
- Solve Optimization Problem of Agent i, get $(S^{i*}, \theta^{i*}, \alpha^{i*})$
- Check that the *sums over i* give a saddle point

VOILA !

Next

- Compare prices with prices obtained with borrowing
- Compare agent profits with and without credit trading
- Does a utility benefit via electricity price increase
- Include **investment in (cleaner) technologies**

INVERSE PROBLEM

*How should we adjust the **credit allocation schedule** and the **tax** to meet expected pollution reduction target at horizon T?*

Dynamic Programming Equation

Value Function

$$V(t, d, a)$$

Dynamic Programming Equation

$$V(t,d,a) = \sup_{(S_t,\alpha_t)} \mathbb{E}\left\{ \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} c_{i,j} S_t^{i,j} - \pi \left(\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} e_{i,j} S_t^{i,j} - \alpha_t \right)^+ \right.$$
$$\left. + V(t+1, D_{t+1}, a + \sum_{i\in\mathcal{I}} E_t^i - \alpha_t) \middle| D_t = d \right\}$$