Report on the Workshop on “Stochastic Dynamical Systems and Climate Modelling”

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Organizers: Jinqiao Duan (Illinois Institute of Technology), Boualem Khouider (University of Victoria), Richard Kleeman (Courant Institute, New York University), Adam Monahan (University of Victoria).

1 Overview

The workshop on Stochastic Dynamical Systems and Climate Modelling gathered about 11 mathematicians and 15 geoscientists, from around the world, to talk about stochastic dynamical systems and their application to atmosphere and ocean sciences. At least 9 of the geoscientists work for a government lab and thus use stochastic and/or probabilistic methods in real world applications for climate predictions and/or day-to-day weather forecasts. On top of that 14 students and post-docs have attended the workshop. There were 7 women participants.

The speakers have covered a quite rich and diverse scientific program, ranging from pure mathematical issues in the new area of stochastic dynamical systems, such as,

- Lyapunov exponents for random dynamical systems,
- dynamical boundary conditions,
- synchronization in stochastic dynamical systems,
- singular perturbation theory and Navier-Stokes shell models
- stochastic mode reduction in large deterministic systems,

to systematic applied math strategies for modelling and simulating geophysical flow phenomena based on stochastic and statistical techniques:

- mode reduction and hidden Markov techniques for atmospheric low frequency variability and for the predictability of the associated annular modes,

- first passage time for stochastic climate models,
- nonlinear optimal perturbation for prediction and ensemble forecast,
- stochastic models for unresolved processes such as wave breaking, cumulus convection, and the underlying moist dynamics

to application of those techniques in the reality of climate predictions and day to day weather forecast and to explore the variability in the coupled atmosphere-ocean system, including:

- sea-surface temperature and sea surface winds,
- El Nineo-Southern Oscillation,
- low stratospheric dynamics,
- quasi-geostrophic turbulence,
- climate predictability,
- data assimilation,
- model error in weather and climate models,
- regimes and metastability in atmospheric dynamics, etc.

2 Some specific main topics

2.1 Mode reduction and hidden Markov chains

Given a dynamical system with a large number of dynamical variables. Imagine that we only are interested in the large scale/low-frequency variability associated with this system. This is particularly typical for climate predictions. Common sense suggests that in a suitable variable–coordinate system only a few 'slow' variables are capable are enough to capture at least a qualitative behaviour of those large scale/low-frequency features. However, often in practise the 'fast' variables have affect the slow dynamics. How many mode should be considered and what should one with the fast variables are some of the questions the systematic mode reduction techniques try to answer. This was on of the topics addressed by Andrew J. Majda from Courant Institute (NYU). His lecture was about systematic mathematical strategies for low-dimensional mode reduction for large dimensional dynamical system using stochastic methods and their application for low frequency variability. The main idea is to exploit well know teleconnection patterns that dominate the atmospheric low frequency variance in the Northern hemisphere and use them as basis functions to develop cheap climate predictability models with highly reduced degrees of freedom. To explain the basics of the stochastic mode reduction technique, he started his lecture with a simple solvable pedagogical model with three modes to explain the underlying MTV (for Majda-Timofeyev-Vanden Ejden, see Timofeyev's talk) theory. Then, he showed how the feasibility of the mode reduction when applied to an actual atmospheric model with a thousand degrees of freedom effectively reduced to a stochastic model will only ten modes, which captures the essential statistical dynamics. The rest of the lecture was devoted to investigating the existence of metastable states in atmospheric low-frequency variability, despite the nearly Gaussian behaviour in the associated probability distribution, through a technique know as the hidden Markov chains (HMV, see Franke's talk). Apparently, those metastable regimes are not part of the low-frequency dynamics but related to turbulent effect from high frequency mode. However, the mode reduction methods were capable in capturing those metastable low frequency regimes when both a suitable nonlinearity and a multiplicative noise were added into the stochastic differential equation.

Ilya Timofeyev, from U of Houston, explained that the mode reduction techniques, introduced earlier by Majda, consists on reducing a large dimensional dynamical system into a few stochastic differential equations for the slow variables, under the assumptions of ergoticity and a time scale separation, between the slow and fast variables. This is demonstrated by using a truncated Hopf-Burgers (THB) equation system as a test case. The THB obtained basically by using discrete Fourier transforms for the Hopf-Burger equation and non-paying attention to the aliasing errors introduced by the unresolved non-linear interaction, which introduce a huge amount of noise into the discretized system. He then showed how the MTV method is applied to derive a reduced number of differential equations where the noise carried by the high frequency modes is represented by a stochastic noise. He also demonstrated the importance of the scale separation by artificially increasing the timescale gap between the slow and fast variables.

Christian Franzke from the National Center for Atmospheric Research, used the hidden Markov model (HMM) technique, discussed in Majda talk, to identify meta-stable states in time series (data) taken from different atmospheric models. According to Christian the HMM approach is useful for describing situations where we are given a time series of a zonal flow, for e.g., but without knowledge of other flow fields which are crucial for dynamics of the given variable. Those 'hidden'–unknown variables are represented by a Markov chain, whose probability density function is calculated from the given time series. It turns out that even if the "observed"–given time series has a nearly Gaussian distribution the hidden Markov chain can exhibit meta-stable states. Tests are then carried for some atmospheric models, such as a barotropic flow over topography and multilayer quasi-geostrophic flow, with well know atmospheric flow patterns, and the HMM technique was able to effectively describe those regimes.

2.2 Stochastic models for cumulus convection

The physical processes associated with clouds occur at length and time scales ranging from a few millimeters and a few fractions of seconds for the droplet growth and formation to a few kilometers and a few hours for the updrafts and downdrafts in convective cells including various intermediate radiative and turbulent effects. This particularly makes it impossible in practice to represent all those processes, from first principle, on a single grid. Rather, the grid is truncated at a certain level and all the unresolved feature are represented by often ad hoc or empirical set of equations called a parameterisation. In typical general circulation model (GCM) used for climate and long range weather predictions, the grid resolution varies from 10 km to 200 km. At this level even the convective cell and the associated latent heat release feeding back into the large scale–resolved variables need to be parameterised. This is often referred to as the cumulus or convective parameterisation problem. The latent heat release from deep convective clouds is the main heat source that drives tropical atmospheric circulation and storms.

Two talks at this meeting were concerned with convective parameterisations using stochastic models. The first talk was by George Craig from DLR, Germany. In his group, they propose a statistical mechanic model based on an equilibrium–canonical distribution for the convective upward mass flux of an an ensemble of non-interacting clouds, given the cloud base mass flux which can be estimated from the large scale thermodynamics. He then presented some (direct) numerical simulation using cloud resolving modeling–where the convection is explicitly represented, to demonstrate the theoretical predictions. He particularly put emphasis on the fact the simulated results approach the theory when the external forcing (here radiation) is increased so that the convective clouds are not strongly linked with each, they appear and disappear in almost ad hoc fashion. This is known as the unorganized convection regime, as opposed to the organized convection were clouds clusters and superclusters as seen to form and propagate, especially in the tropical region. Finally, G. C. stressed on the efforts his group are putting together to apply this model in a realistic GCM simulation.

The second talk on this subject was by Boualem Khouider form the University of Victoria, and was on his joint work with A. Majda on birth-death stochastic models for convective inhibition. Convective inhibition refers to a stable layer located above the mixed boundary layer, where convective parcels typically originate. It constitutes an energy barrier for the rising parcel to reach it level of free convection where it becomes positively buoyant through condensational heating. They propose a simple order parameter defined on microscopic sites 1 to 10 km apart and takes values 1 or 0 according to whether convection is inhibited or there is potential for deep convection. The sites interact with each other and with the ambient (external) large-scale variables in manner similar to the Ising model used for magnetization and phase transition, i.e, via a Hamiltonian and an invariant Gibs measure. The resulting microscopic stochastic model evolves in time according to some intuitive spin-flip rules so that convection will preserve its organization character. B.K then derived a coarse grained-stochastic birth-death process by averaging the microscopic model on the "GCM" grid box. The resulting model is then tested on a toy GCM model with a bad convective parametrization and showed how the stochastic model can change the model results and capable to drive the system back and forth to a climate regime know to persist in the tropics. Moreover, B.K. showed some new results exploring some interesting parameter regimes with metastable states elucidating the behavior seen in his simulations.

2.3 Data assimilation and predictability

Cecile Penland (Climate Diagnostics Center, CIRES, University of Colorado and Physical Sciences Division/ESRL/NOAA) presented her research on adaptive stochastic modeling using data assimilation. Most operational centers are at least investigating stochastic parameterizations of unresolved processes in the ensemble forecasts, and ECMWF has already implemented a version for increasing their ensemble spread. The problem is that stochastic modeling is a real pain. To do it right, we need to look into issues of both parameter estimation and numerical integration. She and Jim Hansen and came up with a neat way to approach both problems at once and figured out why it worked.

Adam Monahan from the University of Victoria presented some new results regarding the probability density function of sea-surface momentum fluxes. These surface momentum exchanges, which exert a drag on surface winds and play an essential role in driving the ocean circulation, are largely determined by the surface wind field itself. Starting from previous work in which the pdf of surface vector winds was characterised, Monahan presented empirical and mechanistic models of the pdf of surface momentum fluxes. He demonstrated that the simulation of the first four moments (mean, standard deviation, skewness, and kurtosis, the last two of which may be parameterised in terms of the first) of the surface vector wind field is sufficient to characterise these moments of the momentum fluxes. Furthermore, an idealised stochastic boundary layer model was shown to provide a qualitatively accurate characterisation of the relationships between momentum flux moments seen in observations.

Mu Mu (Institute of Atmospheric Physics, Chinese Academy of Sciences) reported his recent work on conditional nonlinear optimal perturbation and its applications in predictability study, ensemble forecast, and adaptive observation.

Peter C. Chu (Department of Oceanography, Naval Postgraduate School, USA) reported his work on the first passage time for climate index and model prediction. Climate variability is simply represented by teleconnection patterns such as the Arctic Oscillation (AO), Antarctic Oscillation (AAO), North Atlantic Oscillation (NAO), Pacific/North American Pattern (PNA), and Southern Oscillation (SO) with associated indices. Two approaches can be used to predict the indices: forward and backward methods. The forward method is commonly used to predict the index fluctuation at time t with a given temporal increment . Using this method, it was found that the index (such as for NAO) has the Brownian fluctuations. On the base of the first passage time (FPT) concept, the backward method is introduced in this study to predict the typical time span ( ) needed to generate a fluctuation in the index of a given increment . After the five monthly indices (AO, AAO, NAO, PNA, SO) running through the past history, the FPT density functions (inverse Gaussian distribution) are obtained. FPT presents a new way to detect the temporal variability of the climate indices.

FPT can also be used as metrics to evaluate climate model predictability. FPT is defined as the time period when the prediction error first exceeds a pre-determined criterion (i.e., the tolerance level). It depends not only on the instantaneous error growth, but also on the noise level, the initial error, and tolerance level. The model predictability skill is then represented by a single scalar, FPT. The longer the FPT, the higher the model predictability skill is. A theoretical framework on the base of the backward Fokker-Planck equation is developed to determine FPT.

Youmin Tang (University of Northern BC) and Richard Kleeman (Courant Institute, NYU) presented their work on comparison of information-based measures of predictability in ensemble ENSO prediction. Ensemble predictions of the El Nino Southern Oscillation (ENSO) were conducted for the period from 1981-1998 using two hybrid coupled models. Several recently proposed information-based measures of predictability, including relative entropy (R), predictive information (PI), predictive power (PP) and mutual information (MI), were explored in terms of their ability of estimating a *priori* the predictive skill of the ENSO ensemble predictions. The address was put on examining the relationship between these measures of predictability that do not use observations and the model prediction skills of correlation and root mean square error (RMSE) that make use of observations. The relationship identified here offers a practical means of estimating the potential predictability and the confidence level of an individual prediction.

It was found that the MI is a good indicator of overall skill. When it is large, the prediction system has high prediction skill whereas small MI often corresponds to a low prediction skill. In a perfect model scenario, this suggests the MI to be a good indicator of the actual skill of the models. The R and PI have a nearly identical average (over all predictions) as should be the case in theory. Comparing the different information-based measures reveals that R is a better predictor of prediction skill than PI and PP, especially when correlation-based metrics are used to evaluate model skill. A “triangular relationship” emerges between R and the model skill, namely that when R is large, the prediction is likely to be reliable whereas when R is small, the prediction skill is much variable. A small R is often accompanied by a relatively weak ENSO variability. The possible reasons why R is superior to PI and PP as a measure of ENSO predictability will also be discussed.

2.4 Stochastic modeling and parameterizations

Geoff Vallis (Princeton University) discussed about deterministic and stochastic variability in the coupled atmosphere-ocean-climate system. Geoff also presented a poster on the dynamics of the NAO and annular modes with insight from stochastic models.

Prashant Sardeshmukh (Climate Diagnostics Center, CIRES, University of Colorado and Physical Sciences Division/ESRL/NOAA) discussed how to reconcile non-Gaussian climate statistics with linear dynamics.

Juan Restrepo (University of Arizona) presented his research on stochastic parametrization of wave breaking.

Balasubramanya Nadiga (Los Alamos National Laboratory) discussed stochastic parameterizations of unresolved scales in geophysical large scale flows, in the context of large eddy simulations.

The immense number of degrees of freedom in large scale turbulent flows as encountered in the world oceans and atmosphere makes it impossible to simulate these flows in all their detail in the foreseeable future. On the other hand, it is essential to represent these flows reasonably accurately in Ocean and Atmospheric General Circulation Models (OGCMs and AGCMs) so as to improve the confidence in these model components of the earth system in ongoing effort to study climate and its variability. Furthermore, it is very often the case that in highly resolved computations, a rather disproportionately large fraction of the computational effort is expended on the small scales) whereas a large fraction of the energy resides in the large scales. It is for these reasons that the ideas of Large Eddy Simulation (LES)—wherein the large scale unsteady motions driven by specifics of the flow are explicitly computed, but the small (and presumably more universal) scales are modelled—are natural in this context.

Given the great interest in large scale geophysical flows with its small vertical to horizontal aspect ratio, we restrict ourselves to two-dimensional or quasi two-dimensional flows. Previous models of the small scales in how they affect the large scales in the momentum equations or equivalently the vorticity equation in incompressible settings have mostly been confined to an enhanced eddy viscosity or non-linear eddy viscosity like that of Smagorinsky or biharmonic viscosity. Given the non-unique nature of the small scales with respect to the large scales, the aforementioned use of deterministic and dissipative closures seem rather highly restrictive. On the other hand, it would seem desirable to actually represent a population of eddies that satisfy overall constraints of the flow rather than make flow specific parametric assumptions. This has led to recent investigations of the possibility of using stochastic processes to model the effects of unresolved scales in geophysical flows. More recently, subgrid scale (SGS) stresses have been analysed in simple but resolved flows as a possible way to suggest stochastic parameterizations. These efforts have been preceded, of course, by various attempts to model anomalies in geophysical flow systems as linear Langevin equations and the analysis of stochastic models in isotropic and homogeneous three dimensional turbulence.

Nadiga (with Jinqiao Duan) analyzed the stochastic approach to parameterization in the barotropic vorticity equation and show that (i) if the stochastic parameterization approximates the SGS stresses, then the stochastic large eddy solution approximates the "true" solution at appropriate scale sizes; and that (ii) when the filter scale size approaches zero, then the solution of the stochastic LES approaches the true solution.

Leslie Smith (University of Wisconsin) reported research on reduced models for wave and vortical interactions in stochastically forced dispersive systems.

Timothy DelSole (George Mason University) presented his work on stochastic models of quasigeostrophic turbulence.

Judith Berner (European Centre for Medium-Range Weather Forecasts, UK) talked about stochastic parametrizations for representing model error in weather and climate models.

Jinqiao Duan (Illinois Institute of Technology, Chicago, USA) discussed about a stochastic approach for parameterizing unresolved scales in a simple system with a time-integral memory term. When applying to more complicated systems relevant to climate dynamics, it is noted that more physical mechanisms should be incorporated.

Philip Sura (Climate Diagnostics Center, CIRES, University of Colorado and Physical Sciences Division/ESRL/NOAA) talked about non-Gaussian SST variability.

Paul Williams (University of Reading, UK) presented work on noise-induced phenomena in low-order stratospheric dynamics.

2.5 Recent advances in random dynamical systems

Kening Lu (Brigham Young University, USA) presented a new exciting result on multiplicative ergodic theorem for random dynamical systems in a Banach space. This sets the foundation for further study of random dynamical systems in infinite dimensions.

Bjorn Schmalfuss (University of Paderborn, Germany) talked about stochastic partial differential equations models with dynamical boundary conditions. The results include random attractors and asymptotic dynamics.

Tomas Caraballo (Universidad de Sevilla, Spain) discussed synchronization of a stochastic reaction-diffusion system on a thin two-layer domain.

Hakima Bessaih (University of Wyoming, USA) discussed about a stochastic shell model, i.e., the stochastic Gledzer-Ohkitani-Yamada model. It is a simplified Fourier system. the topics include existence and uniqueness of invariant measure, non-viscous limit and asymptotic exponents.

Barbara Gentz (University of Bielefeld, Germany) discussed a geometric singular perturbation theory with application to simple stochastic climate models. This is a constructive approach to the quantitative description of the effect of noise on multiscale dynamical systems. This method, developed in collaboration with Nils Berglund (CPT-CNRS Marseille, France), consists in the construction of small sets in which the sample paths of the corresponding coupled system of stochastic differential equations are typically concentrated, and provides precise bounds on the exponentially small probability to observe atypical behaviour.

A variant of Stommel's box model for the North-Atlantic thermohaline circulation served as our key example. We first showed how to estimate the effect of random fluctuations on the fast variable which models the difference in temperature between boxes. This allows to study the reduced system for the slow variable, modelling the difference in salinity between the boxes. Depending on the freshwater flux, the salinity difference may approach a bifurcation point. In such a situation, due to noise, a transition to a different stable regime may occur even before the deterministic bifurcation point is reached. The presented approach yields qualitative estimates on such transition times and probabilities.