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Trends in Applied Harmonic Analysis

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Abstracts

A prediction scheme for the adaptive approximation of nonlinear functions of wavelet expansions

Kai Bittner* and Karsten Urban
Ulm University, Germany

A core ingredient of adaptive wavelet methods for nonlinear operator equations is the adaptive evaluation of nonlinear functions. We present an efficient adaptive method for approximately evaluating nonlinear functions of wavelet expansions using semi-orthogonal spline wavelets. Solving the two tasks of predicting an index set for the approximation in terms of wavelets, as well as the approximative computation of the corresponding wavelet coefficients, we are able to achieve the desired accuracy. The computational complexity of the proposed method has the same asymptotic behavior as the best n-term tree approximation.

Fast Minimization of the Vectorial Total Variation Norm and Applications to Color Image Processing

Xavier Bresson
University of California, Los Angeles, USA

We propose a regularization algorithm for color/vectorial images which is fast, easy to implement and mathematically well-posed. More precisely, the regularization model is based on a dual formulation of the vectorial Total Variation (TV) norm and it may be regarded as a vectorial extension of the dual approach defined by Chambolle in [1] for gray-scale/scalar images. We apply the proposed vectorial regularization model to carry out different color image processing tasks, such as color image decomposition, color image inpainting, color image deblurring, and color image denoising defined on manifolds.

[1] A. Chambolle. An Algorithm for Total Variation Minimization and Applications. Journal of Mathematical Imaging and Vision, 20(1-2):89-97, 2004.

A Robust Algorithm for Total Variation Deblurring and Denoising

Qianshun Chang*, Chinese Academy of Sciences, China,
Wei-Cheng Wang and Jing Xu, National Tsing-Hua University, Taiwan

In this talk, we propose a new deblurring and denoising algorithm for the total variation based image restoration. In the inner iteration, an algebraic multigrid (AMG) method is applied to solve the linearized systems of equations. Convergence of the outer iteration is efficiently improved by adding a linear term on both sides of the system of equations together with the Krylov subspace acceleration technique. We also give a convergence analysis to help determine the choice of the linear stabilizing term. Numerical experiments

demonstrate that our algorithm is efficient and robust over a wide range of parameters. From images with large noise-to-signal ratios (SNR) and strong blur, to purely blurred images without noise. In addition, the proposed AMG method is stable enough to invert mild blurring operators directly.

Sibling frames for interpolatory subdivision
Maria Charina* and Joachim Stöckler
University of Dortmund, Germany

We present a local matrix factorization technique, inspired by the results in [1], for construction of non-stationary sibling frames with at least one vanishing moment. The construction method is demonstrated on the examples of interpolatory subdivision such as 4–point and butterfly schemes.

[1] C.K. Chui, W. He, J. Stöckler, *Nonstationary tight wavelet frames, I: Bounded Intervals*, ACHA, **17** (2004), pp. 141 - 197.

Diffusion PDE's derived from bilateral filtering: An overview
Charles Chui
University of Missouri, St. Louis, USA

This is an on-going joint research program with Jianzhong Wang. One of our main objectives is to develop a unified theory based on three important concepts: anisotropic diffusion, diffusion maps, and nonlinear filtering. In this talk and the one to follow, we will report on our recent result concerning the formulation of diffusion partial differential equations (PDE) derived from the bilateral filter of Tomasi and Manduchi. We will first present an overview that leads to the problem statement, with the goal of uncovering certain mystery on the resemblance between bilateral filtering and anisotropic diffusion as well as shedding some light on distinguishing input-data-dependent filtering from the notion of diffusion maps introduced by Coifman and Lafon.

TV Optimization and "Graph-cuts"
Jerome Darbon
University of California, Los Angeles, USA

I'll present some deep theoretical links between a maximum-flow based approach known as "graph-cuts" and the Total Variation minimization problem.

De Rham Transform of Subdivision Schemes
Serge Dubuc* and Jean-Louis Merrien
Université de Montréal and INSA de Rennes, Canada

For any subdivision scheme $\mathcal{S}$, we define its de Rham transform $\tilde{\mathcal{S}}$ which generalizes the Chaikin-de Rham corner cutting. When $\mathcal{S}$ is affine, if T and $\tilde{T}$ are the respective subdivision matrices of the difference subdivision schemes $\Delta\mathcal{S}$ and $\Delta\tilde{\mathcal{S}}$, then the spectral radii $\rho(T)$ and $\rho(\tilde{T})$ are good tools for comparing the smoothness of the limit functions of $\mathcal{S}$ and $\tilde{\mathcal{S}}$.

In an analogous way, for any Hermite subdivision scheme $\mathcal{H}$, we define its de Rham transform $\tilde{\mathcal{H}}$. For a Hermite subdivision scheme $\mathcal{H}$, we introduce a spectral condition. We assume that $\mathcal{H}$ satisfies this condition. Firstly, its de Rham transform fulfils it. Secondly, if $\mathcal{S}_1$ and $\mathcal{S}_2$ are the associated subdivision schemes to $\mathcal{H}$ and $\tilde{\mathcal{H}}$, if T and $\tilde{T}$ are the respective subdivision matrices corresponding to $\Delta\mathcal{S}_1$ and $\Delta\mathcal{S}_2$, then the spectral radii $\rho(T)$ and $\rho(\tilde{T})$ allow the comparison between the indices of smoothness of the limit functions of the schemes $\mathcal{H}$ and $\tilde{\mathcal{H}}$. We apply these results to the Merrien class of Hermite subdivision schemes.

Analysis of a discrete, ill-posed, fourth order PDE from image processing
Catherine Dupuis* and Selim Esedoglu
University of Michigan, USA

We study the discrete version of the ill-posed, fourth order, nonlinear diffusion equation that was proposed by You and Kaveh as an edge preserving image denoising and segmentation model. It is the high order analogue of the Perona-Malik model, and was intended to address some of its well-known drawbacks such as staircasing. Like the Perona-Malik model, the continuum version of the You-Kaveh equation violates parabolicity and lacks a well-posedness theory. In particular, numerical simulations indicate sensitive dependence of its solutions on initial data. We prove a weak upper bound on the coarsening rate of the discrete-in-space version of this equation in one space dimension. Our bounds are obtained by following a recent technique of Kohn and Otto. They constitute a rigorous step towards understanding how important parameters in the model should be chosen.

Stable polynomials, Christoffel-Darboux formula and applications
Jeff Geronimo
Georgia Institute of Technology, USA

A stable polynomial in one variable is one that has all its zeros exterior to the closed unit disk. Such polynomials play an important role in the theory of polynomials orthogonal on the unit circle, the construction of Szego-Bernstein measures, and auto-regressive models. One variable stable polynomials satisfy a Christoffel-Darboux formula. In two variables there is a similar link between stable polynomials and a Christofell-Darboux formula which leads to a surprising number of applications.

Adaptive Wavelet Schemes for a Nonlinear Variational Problem in Image Processing
Bin Han
University of Alberta, Canada

Based on the framework of adaptive wavelet schemes developed by Cohen, Dahmen and Devore for nonlinear variational problems, in this talk we apply the general adaptive wavelet schemes to a particular total-variation-based nonlinear partial differential equation studied by Nashed and Scherzer in image processing. Theoretical convergence rates of the adaptive wavelet scheme for such nonlinear variational problem have been obtained for any dimension. This is joint work with W. Dahmen and V. Pasyuga.

A Sufficient Condition for Existence of Vector-valued Tight Frames
Charles Chui and Wenjie He*
University of Missouri, St. Louis, USA

In this paper, we study the construction of vector-valued tight frames based on the Unitary Extension Principle (UEP). Let $\Phi = [\phi_1, \ldots, \phi_r]^T \in (\mathrm{L}_2(\mathbb{R}^s))^r$ be a compactly supported refinable function vector with dilation matrix A and two-scale symbol $P(\mathbf{z})$. We are interested in developing a condition on $P(\mathbf{z})$, such that the tight frames based on the UEP exist under this condition.

For the univariate case, this problem can be solved by the matrix version of the Riesz Lemma. But for the multivariate case, this problem is much harder due to the fact that the Riesz Lemma cannot be used in general. We develop a sufficient condition on $P(\mathbf{z})$ under which the compactly supported vector-valued tight frames can be constructed based on the UEP.

Some Applications of Multiscale Tools in Medical Research
Don Hong
Middle Tennessee State University, USA

Challenges concerning medical data processing involve dimension reduction, efficient representation and approximation of empirical functions, and biological feature extraction. We will discuss some applications of multiscale tools in medical research including wavelet-based method for proteomic data analysis and multiscale models in cancer study for tumor growth.

The Projection Method in the Development of Wavelet Bases
Rong-Qing Jia
University of Alberta, Canada

Most of the research work on wavelet analysis so far has been concentrated on wavelets on uniform meshes in Euclidean spaces. We are interested in wavelet bases on bounded domains with possibly nonuniform or irregular meshes. For this purpose, we introduce the projection method for construction of wavelet bases.

Let $(V_n)_{n=0,1,2,\ldots}$ be a family of closed subspaces in a Hilbert space H. Suppose that $V_0 = \{0\}$ and $V_{n-1} \subset V_n$ for $n = 1, 2, \ldots$. Let P_n be a linear projection from V_n onto V_{n-1}. If W_{n-1} denotes the kernel space of P_n, then V_n is the direct sum of V_{n-1} and W_{n-1}. We give necessary and sufficient conditions on the projections P_n $(n = 1, 2, \ldots)$ such that the combination of Riesz bases of W_n $(n = 0, 1, \ldots)$ forms a Riesz basis of H.

Under the guidance of the general theory, we introduce discrete wavelets on intervals and discuss their applications to image processing. We also investigate wavelet bases of splines for Sobolev spaces on bounded domains and their applications to numerical solutions of partial differential equations.

Shearlet Multiresolution Analysis and
Adaptive Directional Subdivision Schemes
Gitta Kutyniok

Princeton University, USA

In data analysis, one main focus of current research is on the development of directional representation systems which precisely detect orientations of singularities like edges in a 2-D image while providing optimally sparse representations. The shearlet systems are the first directional representation systems, which not only possess those properties, but are moreover equipped with a rich mathematical structure similar to wavelets.

In this talk we will first give an introduction to the theory of shearlets. The main properties of the shearlet transform will then be presented, thereby in particular illustrating the detection of orientations and emphasizing the usefulness of the structural similarity to wavelets. Secondly, we will discuss some very recent results on deriving a Multiresolution Analysis associated with discrete shearlets aiming at a fast shearlet decomposition using FIR-filters. To achieve this goal we introduce a new class of bivariate directional subdivision schemes which have the capability of adaptively changing the orientation of the data during the subdivision process.

This is joint work with T. Sauer (U. Gießen).

Approximation of functional regression models by bivariate splines

Ming-Jun Lai
University of Georgia, USA

We consider a functional linear regression model where the explanatory variable is a random surface and the response is a real randowm variable with noise. Bivariate splines will be used to represent the random surfaces. Under the assumptions that the regressors in the sample are bounded and span a large enough space of functions, bivariate splines approximation properties yield the consistency of the estimators. Simulations illustrate the quality of the asymptotic properties in various situations.

Mean Value Representation and Curvatures of Compact Convex Sets

S. L. Lee
National University of Singapore

Representation of points and functionals on a set by its extreme points or boundary is an important problem in mathematics and its applications. Barycentric coordinates, the Krein-Millman Theorem and Choquet's Theorem are examples of such a representation. Recently, in conjunction with the construction of one-one transformation and parametrization of meshes in $\mathbf{R}^3$, Floater has constructed new coordinates, called the *mean value coordinates*, for the representation of points in the kernel of star-shaped polygons in $\mathbb{R}^2$ and polyhedrons in $\mathbf{R}^3$ in terms of extreme points of the star-shaped regions. Floater's construction was motivated by the mean value property of harmonic functions, but it was shown recently that the mean value coordinates can be derived from a *mean value representation,* which is more naturally associated with conservative vector fields in $\mathbb{R}^2$ and divergence free vector fields in $\mathbf{R}^3$.

Interestingly, in this divergence free framework, the mean value representation is intimately connected with Minkowski problem that relates positive functions on the unit $(n-1)$-dimensional sphere in $\mathbb{R}^n$ that are orthogonal to the first harmonics, to the curvature of compact strictly convex hypersurfaces through its Gauss map. The vector fields

are also intimately related to homogeneous functions, which provide a general method for the construction of a large class of coordinates based on the curvature of compact strictly convex hypersurfaces, which extend the barycentric coordinates. In the divergence free framework, the vector fields, $\mathbf{F}(\mathbf{r}) = \mathbf{r}/\|\mathbf{r}\|^{n+1}$, $\mathbf{r} \in \mathbf{R}^n$, that produce Floater's mean value coordinates for $n = 2, 3$ are associated the curvature of the unit $(n-1)$-dimensional sphere. This shows that the mean value coordinates are indeed a simple case of mean value representation based on the geometry of compact strictly convex hypersurfaces.

In this talk we give the relationship between homogeneous functions and the corresponding vector fields for mean value representation and use them, in conjunction with Minkowski problem, to construct mean value representation based on the curvatures of compact strictly convex hypersurfaces in $\mathbf{R}^n$.

Riesz Sequences and Bases of Multiwavelets

Song Li
Zhejiang University, China

In this talk, we investigate compactly supported Riesz sequences and bases of multiwavelets for $L_2(\mathbf{R})$. Suppose $\psi = (\psi^1, \dots, \psi^r)^T$ and $\tilde{\psi} = (\tilde{\psi}^1, \dots, \tilde{\psi}^r)^T$ are two compactly supported vectors of functions in the Sobolev space $(H^\mu(\mathbf{R}))^r$ for some $\mu > 0$. We provide a characterization for the sequences $\{\psi^\ell_{jk} : \ell = 1, \dots, r,\ j, k \in \mathbf{Z}\}$ and $\{\tilde{\psi}^\ell_{jk} : \ell = 1, \dots, r,\ j, k \in \mathbf{Z}\}$ to form two Riesz sequences for $L_2(\mathbf{R})$, where $\psi^\ell_{jk} = 2^{j/2}\psi^\ell(2^j \cdot -k)$ and $\tilde{\psi}^\ell_{jk} = 2^{j/2}\tilde{\psi}^\ell(2^j \cdot -k)$. Furthermore, let $\varphi = (\varphi^1, \dots, \varphi^r)^T$ and $\tilde{\varphi} = (\tilde{\varphi}^1, \dots, \tilde{\varphi}^r)^T$ be a pair of compactly supported biorthogonal refinable vectors of functions associated with the refinement masks a and $\tilde{a}$, where a and $\tilde{a}$ are finitely supported sequences of $r \times r$ matrices. We obtain a general principle for constructing vectors of functions $\psi = (\psi^1, \dots, \psi^r)^T$ and $\tilde{\psi} = (\tilde{\psi}^1, \dots, \tilde{\psi}^r)^T$ such that two sequences $\{\psi^\ell_{jk} : \ell = 1, \dots, r,\ j, k \in \mathbf{Z}\}$ and $\{\tilde{\psi}^\ell_{jk} : \ell = 1, \dots, r,\ j, k \in \mathbf{Z}\}$ form two Riesz multiwavelet bases for $L_2(\mathbf{R})$. The bracket product $[f, g]$ of two vectors of functions f, g in $(L_2(\mathbf{R}))^r$ is an indispensable tool for our characterization.

Families of Bivariate Refinable Scaling Functions

Jian-ao Lian
Prairie View A&M University, USA

By reformulating the conditions for polynomial reproduction order with arbitrary dilation matrix, we will establish families of bivariate refinable scaling functions of any polynomial reproduction order and with certain symmetry.

Diffusion Maps and Wasserstein Distance for Discrimination of High Dimensional Data Sets

Linh Lieu* and Naoki Saito
University of California-Davis, USA

We propose a new method for discrimination of data classes or data sets embedded in a high-dimensional space. Our approach combines two important relatively new concepts in high-dimensional data analysis, i.e., Diffusion Maps and a linear-programming

formulation of the Wasserstein Distance known as the Earth Mover's Distance, in a novel manner so that it is more tolerant to noise and honors the characteristic geometry of the data. Our application of Diffusion Maps achieves non-linear dimensionality reduction and at the same time reorganizes the data to yield natural clustering patterns. We follow these patterns to cluster each data set to form a *signature* (i.e., a high-dimensional histogram) for the data set. Then we measure the discrepancy between any two signatures to determine the dissimilarity between the two corresponding data sets. We will illustrate that this method can be used for a variety of applications in high dimensional data analysis and pattern classification, such as quantifying shape deformations and discrimination of acoustic waveforms.

Some Numerical methods for the ROF model for BV image smoothing
Bradley J. Lucier
Purdue University University, USA

We consider numerical methods for the Rudin-Osher-Fatemi minimization problem for bounded variation image smoothing: If $I = [0,1]^2$, given $g \in L_2(I)$ and $\lambda > 0$, find u that minimizes over all $v \in L_2(I)$

$$\frac{1}{2}\|v-g\|^2_{L_2(I)} + \lambda |v|_{\mathrm{BV}(I)}.$$

We prove the convergence of methods based on Chambolle's projection technique for both anisotropic and "isotropic" upwind discrete numerical methods. We also introduce multigrid/multiscale algorithms that speed the computations significantly. Some qualitative properties of solutions of the numerical schemes will be given through examples.

This is joint work with Stacey Levine and Antonin Chambolle.

Harmonic Analysis on data sets
Mauro Maggioni
Duke University, USA

Analysis of and on data sets is of importance in a wide variety of applications where one is confronted with large amount of data, and tasks such as finding features, compressing, denoising, and building predictors are of importance. It appears that in many disparate contexts data sets have a certain geometry that can be analyzed quantitatively, and exploited in order to achieve superior performance in the tasks above. It also allows to build useful dictionaries of functions on the data, similar to generalized Fourier and wavelet base, thereby allowing to lift signal processing tools to the analysis and manipulations of functions on data sets.

Diffusion polynomial frames on metric measure spaces
H. N. Mhaskar
California State University, Los Angeles, USA

We construct a multiscale tight frame based on an arbitrary orthonormal basis for the L^2 space of an arbitrary sigma finite measure space. The approximation properties of

the resulting multiscale are studied in the context of Besov approximation spaces, which are characterized both in terms of suitable K–functionals and the frame transforms. The only major condition required is the uniform boundedness of a summabilility operator. We give sufficient conditions for this to hold in the context of a very general class of metric measure spaces, where the assumption of finite speed of wave propagation might not hold. The theory is illustrated using the approximation of characteristic functions of caps on a dumbell manifold, and applied to the problem of recognition of hand–written digits. Our methods outperforms comparable methods for semi–supervised learning. This is joint work with Professor Mauro Maggioni, Duke University.

A digital diffusion-reaction type filter for nonlinear denoising

Gerlind Plonka
University of Duisburg-Essen, Germany

Image denoising is a field where one is typically interested in removing noise, which may be introduced by the image information process, image recording, image transmission, etc. In a wide variety of applications, the images are discontinuous, and the challenge is to smooth them while preserving their edges and important structures.

Different nonlinear methods have been proposed to tackle this problem including global stochastic methods, adaptive smoothing, wavelet techniques, anisotropic diffusion and variational methods.

In this talk, we propose a new nonlinear data-dependent denoising filter, called digital diffusion-reaction filter. It can be seen as an adaption of the digital total variation filter introduced by Chan, Osher and Shen [COS]. The TV-filter in [COS] can be interpreted as a precise translation of the classical analog TV restoration model invented by Rudin, Osher and Fatemi [ROF] to the digital case.

Our model can be understood as a digital variant of a diffusion-reaction type equation suggested by Nordström [No]. The Euler equations associated with the energy functional of Nordström are equivalent with the steady state of a certain nonlinear diffusion-reaction equation. Our digitized formulation of the variational and PDE method in [No] leads to nonlinear algebraic equations instead of PDE's and the analysis and application of the digital method needs no knowledge on numerical approximation of PDE's.

We will compare the denoising properties of our filter with different image denoising methods in the literature by numerical examples.

[COS] Chan, T. F., S. Osher, and J. Shen, The digital TV filter and nonlinear denoising, IEEE Trans. Image Process. **10** (2001), 231–241.

[No] Nordström, N., Biased anisotropic diffusion - a unified regularization and diffusion approach to edge detection, Image Vision Comput. **8** (1990), 318–327.

[ROF] Rudin, L., S. Osher, and E. Fatemi, Nonlinear total variation based noise removal algorithms, Physica D **60** (1992), 259–268.

Exponentially localized polynomial bases and frames

Jürgen Prestin
University of Lübeck, Germany

In this joint work with F. Filbir and H. N. Mhaskar we present exponentially localized polynomial frames on the interval $[-1, 1]$, and on the unit sphere of a Euclidean space. In case of periodic functions we give also an example of an exponenetially localized basis of trigonometric polynomials.

Even though the frame or basis coefficients may be computed using the coefficients of a function in an orthogonal polynomial expansion, the behavior of these coefficients near a point on the interval characterizes the possibility of an analytic continuation of the function in a complex neighborhood of the point in question.

Particularly, we construct a sequence of polynomials which converges uniformly in the order of best approximation to the given function f and geometrically fast at each point where f is analytic. Our main interest is in the characterization of local smoothness of a function f in terms of the sequence of Fourier coefficients $\{\hat{f}(k)\}$. But from the point of view of computations, we will also describe our results when samples of the functions in question are available, instead of the coefficients.

Our construction allows one to construct exponentially localized kernels based only on some summability estimates. In turn, the localization enables us to obtain a characterization of local Besov spaces on the interval also in the case of some more general systems of orthogonal polynomials.

Some aspects of time frequency representations

Sherman D. Riemenschneider
West Virginia University, USA

This talk will be selected from topics touching on the Empirical Mode Decomposition and possibly windowed Fourier Transform techniques; perhaps with preliminary applications to detection of pulsars in radio astronomy.

A Rudin Osher Fatemi Approach to Optical Flow

Martin Rumpf
University of Bonn, Germany

The estimation of motion in an image sequence is a fundamental task in image processing. Frequently, the image sequence is corrupted by noise and one simultaneously asks for a restored sequence and the underlying motion field. In smoothly shaded regions of the restored image sequence the brightness constancy assumption along motion paths leads to a pointwise differential condition on the motion field. At object boundaries which are edge discontinuities both for the image intensity and the motion field this condition is no longer well-defined. In this paper a total variation type functional is discussed for the joint image restoration and motion estimation.

This functionals appears to be not lower semicontinuous. By the general theory on vector valued BV functionals its relaxation comes along with a singular part of the energy density which is given by the solution of a local minimization problem at edges. In the case of one space dimension this energy density can be identified explicitly. This relaxation leads to a generalized variational formulation of the brightness constancy assumption.

Furthermore, it enables to resolve an ambiguity problem in the selection of foreground and background motion In case of fast motion a minimization of the functional favors

microstructures at object boundaries, which can be avoided by a suitable scaling of the model.

This is joint work with Sergio Conti from Duisburg-Essen University.

Restoration of Chopped and Nodded Images by Framelets

Jian-Feng Cai, Raymond Chan, Lixin Shen*, and Zuowei Shen
Syracuse University, USA

In infrared astronomy, an observed image from a chop and nod process can be considered as the result of passing the original image through a highpass filter. Here we propose a restoration algorithm which builds up a tight framelet system that has the high pass filter as one of the framelet filters. Our approach reduces the solution of restoration problem to that of recovering the missing coefficients of the original image in the tight framelet decomposition. The framelet approach provides a natural setting to apply various sophisticated framelet denoising schemes to remove the noise without reducing the intensity of major stars in the image. A proof of the convergence of the algorithm based on convex analysis is also provided. Simulated and real images are tested to illustrate the efficiency of our method over the projected Landweber method.

A Unified Tight Frame Approach for Missing Data Recovery in Images

Zuowei Shen
National University of Singapore

In many practical problems in image processing, the observed data sets are often in complete in the sense that features of interest in the image are missing partially or corrupted by noise. The recovery of missing data from incomplete data is an essential part of any image processing procedures whether the final image is utilized for visual interpretation or for automatic analysis. In this talk, we will discuss our new iterative algorithm for image recovery for missing data which is based on spline tight framelets. We consider in particular few main applications in image processing, inpainting, impulse noise removal and super-resolution image reconstruction.

Hilbert transform pairs of tight spline frames

Joachim Stöckler
University of Dortmund, Germany

The characterization of tight wavelet frames on bounded intervals in [1] supplies a constructive method by matrix factorization. In particular, symmetric and anti-symmetric tight frame generators with small support can be achieved by this method. Moreover, the Hilbert transform of an MRA tight frame can be seen to be an MRA tight frame as well.

Selesnick [2] proposed the use of Thiran allpass filters for the construction of Hilbert transform pairs of tight frames. His construction is performed by finding the scaling symbols of the underlying MRA. In our work, we consider generators of tight frames which are linear combinations of B-splines of order $m \geq 2$. We find explicit forms for their approximate Hilbert transforms which are linear combinations of B-splines of order $m+1$. An application of the approximate Hilbert transform to filtered backprojection is sketched.

Most results of this talk were obtained by Kyoung-Yong Lee in his doctoral thesis.

[1] C. K. Chui, W. He, J. Stöckler, Nonstationary tight wavelet frames I: Bounded intervals, Appl. Comput. Harmonic Anal. 17(2004), 141–197.

[2] I. W. Selesnick, The design of approximate Hilbert transform pairs of wavelet bases, IEEE Trans. Signal Proc. 50(2002), 1144–1152.

Sampling and reconstruction of signals with finite rate of innovation

Qiyu Sun
University of Central Florida, USA

Signals with finite rate of innovation are those signals having finite freedom in each unit time. In thistalk, we discuss locally-behaved sampling and reconstruction problems for signals with finite rate of innovation.

Using TV-Stokes equations for digital image denoising and restoration

Xue-Cheng Tai
University of Bergen, Norway

In this talk, we shall try to summarize the work we have done recently about using partial differential equations for image analysis and processing, we shall concentrate on noise removal and restoration. We shall start by the well-known total variation denoising techniques. From numerical experiments and some analysis, we show that better properties can be kept if we use some modified higher oder partial differential equations. In the end, we show that TV-Stokes equations is a naturally choice for noise removal and image inpainting. However, the equations we derive here is not coming from fluid mechanics, but from some geometrical considerations for digital images.

This talk is based on joint works with: S. Osher, R. Holm and T. Rahman.

Diffusion PDE derived from Bilateral Filtering: Proof of Results

Jianzhong Wang
Sam Houston State University, USA

This is a long-term and on-going joint research project with Charles Chui. The present presentation is the continuation of the previous talk given by Charles. The goal is to formulate and derive the class of diffusion PDE's corresponding to Tomasi and Manduchi's bilateral filter which has significant positive impact to the current research areas of image processing and image-based computer graphics. We will show that the iterative bilateral filtered outputs provide the solutions of this class of diffusion PDE's in the sense of infinitestimal. We will also give fairly sharp error estimates. The central idea of our proof and some details will be discussed in this talk.

Multiscale Wavelet Total Variation Method for Image Denoising

Yang Wang
The National Science Foundation, USA

Total Variation (TV) method, pioneered by Rudin, Osher and Fatemi, is a proven effective method for removing noise in an image. It is well known, however, that TV can lead to cartoon like images if left running for too long. In this talk we present a total variation scheme for denoising based on wavelet decomposition, the Multiscale Wavelet TV (MWTV) method. We discuss the mathematics involved in this scheme. We also compare it to the traditional TV method and demonstrate some of the advantages of MWTV.

Kernels for Mathematical Learning Theory
Yuesheng Xu
Syracuse University, USA

We will report recent development in kernels for mathematical learning theory. New development in univasal kernels, refinable kernels and kernels for certain Sobolev spaces will be presented in the talk. Several interesting special examples will be discussed.

The golden ratio encoder – A new robust quantization algorithm for analog-to-digital conversion
Ozgur Yilmaz
University of British Columbia, Canada

The widepspread use of digital technology has placed a high demand on providing accurate conversion between the analog and digital worlds. The technology used in the analog-to-digital conversion involves analog devices which have physical limitations that, at first sight, conflict with accuracy demands. Some state-of-the-art analog-to-digital (A/D) conversion techniques, e.g., sigma-delta modulation and beta encoding, provide methods that are robust with respect to some of these limitations. In this talk, we shall introduce the "golden ratio encoder", a novel A/D conversion scheme that can be implemented robustly on analog hardware and enjoys an exponentially precise conversion performance. In particular, the new scheme exploits certain algebraic properties of the golden mean and produces robust quantized representations without requiring any "precise multiplication" and "precise comparison" to be performed in the analog hardware.

This is joint work with Ingrid Daubechies, Sinan Gunturk, and Yang Wang.

Discrete Wavelets on Intervals and their Applications to Image Compression
Hanqing Zhao
University of Alberta, Canada

In this talk, we shall introduce discrete wavelets and discuss their applications to image processing. We propose a new approach to constructing a family of wavelets which are sequences. Namely, these wavelets form a Riesz basis of ℓ_2 instead of L_2 in which the traditional wavelets are constructed.

Compared to the traditional wavelets, the discrete wavelets have simple expressions and short supports. Being discrete, those wavelets are naturally designed for discrete mathematical models. They can be easily adapted to bounded intervals and boundary wavelets can be constructed without much difficulty.

The fast wavelet transform based on our discrete wavelets will be illustrated. In comparison with the well-known biorthogonal 9/7 wavelets, the performance of the discrete

wavelets is comparable, but the computational cost is about 1/2 of the time consumption of 9/7 wavelets. Moreover, since the discrete wavelets are defined on intervals, no extension is required for the data on the boundary. Therefore, they usually behave much better on the boundary of images than traditional wavelets.

In light of their simplicity and flexibility, we expect discrete wavelets have the potential of wide usage in many applications.

This is joint work with Rong-Qing Jia.

A Wavelet Method for Numerical Solutions of Biharmonic Equations

Wei Zhao
University of Alberta, Canada

Many practical problems in elasticity and fluid dynamics are modeled by biharmonic equations. In order to solve the biharmonic equation on a rectangular domain, we construct inner wavelets and boundary wavelets on the basis of cubic B-splines. These wavelets are then adapted to the corresponding domain.

The wavelet method has many advantages over the traditional methods including the multigrid method. First, the condition number of the stiffness matrix is quite small and uniformly bounded. Second, the idea of using the solution on the coarser grid as a part of the initial guess on the finer grid reduces the computation time significantly. Third, a high order convergence rate is achieved. Fourth, the transition and subdivision matrices are applied so that the algorithm becomes optimal. Fifth, the wavelet bases can be used to solve general elliptic equations of fourth order with variable coefficients. Numerical results are provided to demonstrate these advantages.

This is joint work with Rong-Qing Jia.

Learnability of Gaussians with Flexible Variances

Ding-Xuan Zhou
City University of Hong Kong

Gaussian kernels with flexible variances provide a rich family of Mercer kernels for learning algorithms. We show that the union of the unit balls of reproducing kernel Hilbert spaces generated by Gaussian kernels with flexible variances is a uniform Glivenko-Cantelli class. This result confirms a conjecture concerning learnability of Gaussian kernels and verifies the uniform convergence of many learning algorithms involving Gaussians with changing variances. Rademacher averages and empirical covering numbers are used to estimate sample errors of multi-kernel regularization schemes associated with general loss functions. It is then shown that the regularization error associated with the least square loss and the Gaussian kernels can be greatly improved when flexible variances are allowed. Finally for regularization schemes generated by Gaussian kernels with flexible variances we present explicit learning rates of the regression with the least square loss and the classification with the hinge loss. Extensions to manifold and Markov sampling settings will also be discussed.

Variational Models and PDE Techniques in Wavelet Inpainting

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We proposed variational models for image inpainting in wavelet domain, which aims to filling in missing or damaged wavelet coefficients in image reconstruction. The new variational models, especially total variation minimization in conjunction with wavelets lead to PDE's, in the wavelet domain and can be solved numerically. The proposed models can have effective and automatic control over geometric features of the inpainted images including sharp edges, even in the presence of substantial loss of wavelet coefficients, including in the low frequencies. This work is jointly with Tony Chan (UCLA) and Jackie Shen (Minnesota).