

Innovations in Mathematics Education via the Arts

BIRS Workshop 07w5062 January 22-26, 2007 FINAL REPORT v. 2

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Summary Introduction

Our primary objective was to bring together a diverse body of mathematically trained professionals who individually incorporate the arts in their educational activities. As a group, we brainstormed to identify promising areas and techniques for a wider movement of math education via the arts.

The following paragraphs are from participants' reports of the experience:

This was a very productive week. I liked the flow of the workshop, that we worked together as a large group to decide on our goals then broke into groups to work on developing the goals, which we then reported back to the group. Then, we discussed other goals for new groups, but we were allowed to also participate in the first groups or the second groups. That we had the freedom to move within groups or stay in groups made it easy to focus on activities targeted towards my interests and talents.

It was intellectually energizing to be a part of a diverse group, comprising people in specialized areas of mathematics and the arts within higher education, teacher education and K-12 school contexts. The challenge of bridging from the specialized areas to making a measurable difference in learning in the K-12 classroom is significant. It involves the ongoing cultivation of multiple perspectives through continuous dialog between all parties.

This workshop was for me a unique experience that provided me with connections to elementary and high school teachers of mathematics that would have been difficult to realize otherwise.

I have formed a new collaboration and started a new project. From presentations by others, I have learned new methods for enhancing mathematical education and new ways to incorporate art into mathematics.

This five day intense workshop within these excellent facilities in BIRS has been a very positive and unique experience for me and, no doubt, for the entire group of participants. It has served not only for my personal professional development but also for reinvigorating the teaching of mathematics through the arts. I believe that this can be a very valuable pedagogical tool. I foresee that in the next few years this workshop will be a reference point in the sense that many of the seminal ideas and personal connections of future projects that involve teaching with some kind of artistic activity started here during these five days.

Web Site

Additional documents, including daily notes and presentation materials, are collected on the web site, which we updated daily during the workshop: <http://www.georgehart.com/birs>

Daily Schedules

Monday January 22, 2007

- Start at 9:00. Welcome by Brenda Shakotko
- Introductory remarks
- Five-to-ten minute introductions. Describe yourself, your art/math interests, past or future projects.
- Late afternoon: Discuss goals.
- Breaks:
 - Coffee: 10:15 and 3:15
 - Lunch: 12:00-1:00
 - Banff tour: 1:00-2:00, by Jim Olver, Corbett 2nd fl. lounge
- Evening: CD sculpture activity, here

Tuesday January 23, 2007

- 9:00-9:15 *Traditional Science* —Barb Frazer
- 9:15-10:15 *Discuss objectives*
- 10:15-10:45 *Coffee Break*
- 10:45-12:00 *Discuss objectives & Form groups*
- 12:00-12:15 *Group photo* — Corbett steps
- 12:15-1:15 *Lunch*
- 1:15-3:15 *Group discussions* — (walk to Banff!)
- 3:15-3:45 *Coffee Break*
- 3:45-4:30 *Groups report. Plan for Wednesday.*
- 4:30-5:30 Bridges/ISAMA/MAA/Math Forum
- 5:30... Dinner
- 8:00 Workshops — Carol Bier: *Islamic cutouts*, Mara and Glyn: *L-Systems*

Wednesday January 24, 2007

- 9:00-9:30 Math Forum, Knitting Network, Textile Society, ISIS, *NEXUS*, Katachi
- 9:30-10:15 *Discuss objectives* — *Gerda de Vries*
- 10:15-10:45 *Coffee Break*
- 10:45-11:45 *Twiddler, Etch-a-sketch, Longsword workshop* — *Susan Gerofsky*
- 12:00-1:30 *Concert (Rolston Recital Hall, Music Bld.) and Lunch*
- 1:30-3:15 *Group work* — (walk to Banff!)
- 3:15-3:45 *Coffee Break*
- 3:45-5:30 *Groups report. Plan for Thursday.* •5:30... Dinner
- 7:00... *Traditional Story-telling* — Smith Hall

Thursday January 25, 2007

- 9:00-10:00 *Music workshop* — Godfried, Paco, David, Susan
- 10:00-10:30 *Outline final outcomes*
- 10:30-11:00 *Coffee*
- 11:00-12:00 *Open problem session*
- 12:00-1:30 *Lunch*
- 1:30-3:15 *Group work*
- 3:15-3:45 *Coffee Break*
- 3:45-5:30 *Groups report. Plan final report.*•5:30-6:30 *Dinner*
- 7:00 PM *Hot springs trip*

Friday January 25, 2007

- 9:00-10:00 *Wrap-up and discussion of joint international congress on mathematics and art*
- 10:00-10:15 *Survey*
- 10:15-10:30 *Final Oulipo workshop— Susan Gerofsky*
- 10:30-11:00 *Coffee and depart*

Workshops

We alternated our discussions with hands-on activities that we felt were models for classroom use.

- CD truncated icosahedron --- George Hart
- Birch bark ornament, traditional science --- Barb Frazer
- Islamic cutouts --- Carol Bier
- L-Systems --- Glyn Rimmington and Mara Alagic
- Twiddler, Etch-a-Sketch, and Long-sword --- Susan Gerofsky
- Math and Rhythm — Godfried Touissant, Paco Gomez, David Rappaport, Susan Gerofsky
- Oulipo — Susan Gerofsky

Outcomes

After brainstorming about many possible outcomes, the group converged on the goal of developing pedagogical materials at various levels. There are various groups of participants who have committed to target their energy towards future projects that were incubated here:

Bob Bosch, Pau Atela, Doug Burkholder, and David Richter will be editing a book of long-term, out-of-class projects that can be incorporated into existing sophomore-junior-senior-level courses. Each project will be a module that builds upon material found in one or more courses in the standard curriculum. Each project will be assigned to a group of students. The final piece of each project will be the creation of a piece of art (a piece of sculpture, for example). In each

case, mathematics will be an integral part of the creation process. Carlo Sequin promised to contribute at least two project ideas that he will write up in the next few months.

Nat Friedman, Mara Alagic, Glyn Rimmington, Stewart Craven, and Phil Wagner formed a group focused on K – 12 education. The group is concerned about activities where art is in some meaningful way ought to be connected to mathematics. Whether the inspiration for the mathematics comes from the art or the mathematics in and of itself leads to artistic representations, there is a need for suggestions/activities for elementary and secondary teachers to use in their classrooms. To this end the group will create a framework that provides the critical information required by teachers to embed these lessons in their programs. They will start by writing 4 – 6 lessons, field test them, and refine them to be published in an appropriate form. Stewart will initially write a lesson based on the construction of a “giant” stellated octahedron followed by a series of lessons about students who use their own photographs imported into Geometer’s Sketchpad to explore transformational geometry. He will additionally submit my workshop plans for two Mathematics and Art sessions that he will be doing over the course of the next four months.

Blake Mellor, Gwen Fisher, Kevin Hartshorn, Doris Schattschneider and Carolyn Yackel formed a collaboration to edit a collection of activities/projects for Mathematics for Liberal Arts. They plan to create some sample projects by July 2007, along with detailed guidelines for the projects, and send out a call for proposals by the end of the summer. They hope to collect a range of projects, on different topics and of different lengths, to be a resource for teachers of college Math for Liberal Arts courses, and possibly also for high school teachers.

Godfried Toussaint, David Rappaport, Paco Gomez, Susan Gerofsky, and Reza Sarhangi started a collaboration to explore the potential of teaching a variety of mathematical concepts through music and rhythm. They are working on an initial article for a mathematics education academic journal (For the Learning of Mathematics or Educational Studies in Mathematics). They will be outlining a program to use Toussaint's innovative circular representations of rhythmic patterns in music to teach concepts in a wide range of mathematical areas, ranging from number theory to geometry, abstract algebra, and combinatorics. They hope to use the analysis from this collaborative article as the basis for the development of a book of lesson ideas and materials for mathematics instructors at a variety of different levels, to encourage thoughtful implementation of mathematics teaching via music. A proposed book may also include a call for articles from other mathematics educators who use music as a means to teach math concepts.

Gwen Fisher wrote a proposal for a mathematical art exhibit “Mathematical Expressions: Bead Weaving with Gwen Fisher” at the *San Jose Museum of Quilts and Textiles* in California that Carol Bier will help her submit.

Gene Klotz is writing a proposal to form a wiki for the math/arts community. Workshop participants helped to develop a taxonomy of the types of content to include.

Pau Atela and Philip Wagner have joined forces to disseminate to the larger public an exhibition about the biological phenomenon of phyllotaxis and current mathematical models for the phenomenon. This exhibit was prepared a few years back with biologists, mathematicians, and

artists as participants. It has been very popular in a few botanical gardens in Europe but has never been exhibited (outside Smith College) in North America.

Carolyn Yackel, Mara Alagic, and Gwen Fisher plan to develop and conduct an assessment study on the effects of introducing mathematical art in the classroom on spatial reasoning skills.

Also, Pau Atela and Bob Bosch plan to work on a portrait of Fibonacci constructed out of images from Pau's phylotaxis research.

In addition, several topics were discussed which we agree the participants should explore further. One is the idea of a joint interernational congress which combines the art and math communities from many countries into one conference. Participants will explore this idea with the organizers of Bridges, ISAMA, ISIS, NEXUS, Katachi, the Math and Design Conference. Another topic discussed was for participants to follow up on the funding opportunities offered by the NFS for the National Science Digital Library.

Detailed Individual Reports

We asked the participants each to write a page on the following topics. Their responses follow.

- Name, affiliation
- Paragraph about experience here
- Description of math education needs you feel are important and whether they were addressed
- Accomplishments, e.g., partnerships formed and projects planned
- What you see as the long range impact of this week's workshop
- Anything else you think should be mentioned in our final report to BIRS

Mara Alagic & Glyn Rimmington
College of Education, Wichita State University, Kansas, USA

BIRS Experience and Mathematics Education Needs

It was intellectually energizing to be a part of a diverse group, comprising people in specialized areas of mathematics and the arts within higher education, teacher education and K-12 school contexts. The challenge of bridging from the specialized areas to making a measurable difference in learning in the K-12 classroom is significant. It involves the ongoing cultivation of multiple perspectives through continuous dialog between all parties.

Partnerships

We joined the K-12 collaborative group along with Nat, Stewart and Phil to develop a framework for using mathematics and arts in the classroom. It will take into account such issues as prior learning and life experiences of students and teachers. An important part of the framework is the cross-indexing of arts with mathematics resources and vice versa. Such a

framework must include information for K-12 teachers on how to integrate the resources into their classes.

Two NSF RFPs (NSDL and CLII) were identified and investigated to support the provision of more resources for mathematics and arts teachers. This is consistent with the framework proposal. The group investigating the grant proposal comprises Gary, Gene, Dirk, David, Glyn and Mara.

Accomplishments

We learned more about L-systems in terms of how they may be integrated into classrooms to improve student learning of a range of concepts, such as 3D and 2D geometry, recursion, iteration, branching and evolving structures. The music/rhythm activity will be introduced into elementary mathematics education classes and to instructional leaders. There are a couple of other interesting ideas to take to our classrooms.

Long Range Impact

The vision of improving learning outcomes in the K-12 mathematics classroom can only be accomplished through an ongoing dialog between those with new ideas in mathematics and the arts and the classroom teachers and instructional leaders. The proposed framework for integration of resources will help with this process.

Research Questions

We believe two important research questions that relate to the observations above are:

- How is teaching a mathematics concept via art changing/influencing understanding of that concept?
- Are these (if yes, how) representations different from traditional/non-art-based representations?

Pau Atela, Smith College

This five day intense workshop within these excellent facilities in BIRS has been a very positive and unique experience for me and, no doubt, for the entire group of participants. It has served not only for my personal professional development but also for reinvigorating the teaching of mathematics through the arts. I believe that this can be a very valuable pedagogical tool. I foresee that in the next few years this workshop will be a reference point in the sense that many of the seminal ideas and personal connections of future projects that involve teaching with some kind of artistic activity started here during these five days.

I am involved in two main partnerships. One, with Philip Wagner, entails the dissemination to the larger public of an exhibition about the biological phenomenon of phyllotaxis and current mathematical models for the phenomenon. This exhibit was

prepared a few years back with biologists, mathematicians, and artists as participants. It has been very popular in a few botanical gardens in Europe but has never been exhibited (outside Smith College) in this side of the Atlantic.

The second main partnership involves writing a book that will be a resource for College level faculty. It will contain art-math projects aimed at upper level students that have passed at least a calculus course. The idea is that these projects would be flexible enough so that the teacher will be able to implement them either within a course for the whole class, or as supplementary activities for a subgroup of students. Some will also be suitable for semester-long courses or for independent studies. I have volunteered to be one of the editors, together with Robert Bosch (Oberlin College), David Richter (Western Michigan University) and Doug Burkholder (Lenoir-Rhyne College).

A smaller scale project that could take place in the near future is a collaboration with Robert Bosch involving carefully chosen images of mathematical models of plant spiral patterns involving Fibonacci numbers and a computer generated portrait of Fibonacci using those images as shades of grey with algorithms created by Robert Bosch.

Carol Bier
Mills College, Oakland CA
Research Associate, The Textile Museum, Washington DC

The BIRS Workshop, “Innovations in Mathematics Education via the Arts,” provided an outstanding opportunity for key players in the fledgling field of intersections among mathematics and art to address educational needs and to develop plans for our future development. We arrived at some very basic and profound understandings of shared goals and diverse perspectives. Banff offered a unique environment in which to brainstorm, focus, group, and regroup, allowing inspiration, creative leaps, and cross-fertilization of ideas. The Max Bell Building and Corbett Hall were ideally suited to our needs, and the BIRS staff provided a very supportive and nurturing environment for our group. The catering in Donald Cameron Hall is outstanding, and the facilities of the Professional Development Centre also contributed to the inspiring ambience of our intense intellectual engagement.

Intellectually, I feel that my experience here was encountered by others as well, that our work and professional activities in art-math intersections were affirmed, and that I am inspired to continue to pursue them, in spite of all too frequent resistance from within the establishment. We have an important agenda, and it is worth pursuing.

My Own Accomplishments at BIRS Workshop

Offered workshop on folding and cutting, playing with the geometry of the circle, to form patterns that are used in Islamic art and architecture

Learned proof from Doris Schattschneider why the folding results in a 30 degree angle

Invited Gwen F, Daina T, Carolyn Y, Susan G, Barbara F, and others to submit proposals for their work on mathematics related to fiber media, textile technologies, and interlacing

(bead-weaving, crocheting, knitting, longsword dancing, birch-bark chomping) for Textile Society of America 2008 Symposium, to be held in Honolulu, Hawai'i
Deadline for proposals October 1, 2007

Renewed contact with Gary Greenfield for *Journal of Mathematics and the Arts*, to consider papers derived from our plenary session, “Textiles – Math = 0/Textiles + Math = ∞ ,” organized w/ Dave Masunaga at Textile Society of America 2006 Symposium in Toronto
Promoted Textile Society of America interests in textiles and math to art-math community
Edited revised draft by Gerda de Vries for publication in TSA 2006 Symposium Proceedings
Met Barbara Frazer; acknowledged shared interests in traditional science, and parallels between traditional science and values in classical Islamic world and First Nations in Canada
Sent her the syllabus for my course, “Sufism, Spirituality, and Science” (Fall 2007, SFSU)
Introduced Dirk Huylebrouck to work of Eric Broug, www.broug.com
Explored Banff Centre as possible symposium venue for TSA 2012; met with Nancy Sande of Conference Sales; sought (and received) proposal

Immediate Plans

Used proof Doris Schattschneider provided in my class on Thursday, 1/25
Edited paper by Reza Sarhangi on “Geometric Constructions and their Arts” for Bridges 2007 (Friday 1/26)
Met with curator of textiles at the De Young Museum to propose art-math workshops for upcoming exhibition of Turkmen rugs (Fall 2007) (Friday 1/26)
Take Banff Centre proposal for TSA 2012 to TSA Board for consideration at February meeting
Use proof DS provided in future classes and workshops on geometry and Islamic art
Submit proposal to San Jose Quilt and Textile Museum for a series of exhibitions called “Mathematical Expressions,” of which Gwen Fischer’s Beaded Weavings will be the first
Pursue plans to make contacts with venues in the San Francisco Bay Area for art-math activities (De Young Museum; Oakland Museum; Asian Art Museum; SF Craft and Folk Art Museum; Lawrence Hall of Science; Exploratorium; Mathematical Sciences Research Institute; UC-Berkeley’s Center for Middle Eastern Studies; San Jose Quilt and Textile Museum; Santa Rosa Gallery) Will welcome support from Bridges Organization.
Prepare a workshop proposal for Bridges-to-Teachers/Teachers-for-Bridges 2007
Hoping to introduce Doris Schattschneider to a friend who is a digital artist in NJ in February
Seek to apply principles of interlacing used in longsword-dancing for programs with children
Engage children in soap film activities using Nat Friedman’s knots to form minimal surfaces
Establish link on TSA website to Carolyn Y’s Knitting Network
Encourage Bridges’ website to establish links to Ethnomathematics Digital Library, and to my students’ work at the Math Forum
Pursuing several references I learned about from colleagues at BIRS: *Mathematics and Aesthetics*, *Mathematics and Beauty*, books on Ornament, *Experiencing Geometry*, *Homoestheticus*
Suggest to colleagues at University of Hawaii, Hawaii Pacific University, Honolulu Academy of Art, Iolani School, Ethnomathematics Digital Library that they sponsor individual speakers on art/math subjects, so they could then participate in the TSA 2008 symposium in Honolulu

Longer-term ideas

Want to consider more contributions I can offer to The Math Forum at Drexel University
Want to consider ways I can contribute to Gene Klotz' wiki initiative, introduced at BIRS
Hope to plan TSA 2012 at Banff Centre; and consider keynote address on "Traditional Fiber
Technologies among First Nations" (bark-chewing; pattern-making; snowshoe
construction, skin clothing, etc.); encourage participation of U. Alberta, U. Calgary,
Nickle Museum
Seek grants/develop proposals to support proposed activities from the BIRS Workshop
Put together list of art-math museum exhibits for Bridges website

Robert Bosch, Oberlin College

The BIRS Workshop "Innovations in Mathematics Education via the Arts" was an amazing, enlightening, and invigorating gathering. I came to it with great excitement, and it did not disappoint; in fact, it exceeded my very high expectations.

I have seen in my own work as a mathematician/artist/educator how beneficial it is to combine math and art. (I've taken great pleasure in using mathematics to help me create works of visual art, I've found that combining math and art provides many opportunities for incorporating my students in publishable research projects, and I've seen that it can inspire students who---beforehand---felt that math was boring and useless.)

I strongly believe that many mathematics educators---at all levels---would jump at the chance to bring some art into their classrooms. All that they need are some resources. And this BIRS workshop has been the genesis of an entire collection of high quality materials.

For me, perhaps the most significant outcome of this workshop is that I will be editing (with Pau Atela, Doug Burkholder, and David Richter) a book of long-term, out-of-class projects that can be incorporated into existing sophomore-/junior-/senior-level courses. Each project will be a module that builds upon material found in one or more courses in the standard curriculum. Each project will be assigned to a group of students. The final piece of each project will be the creation of a piece of art (a piece of sculpture, for example). In each case, mathematics will be an integral part of the creation process.

I also made many contacts. I think that there's a very good chance that some of these will lead to additional collaborations in the near future. (One example: Pau Atela and I have talked about working on a portrait of Fibonacci constructed out of images from his phylotaxis research.)

Summary Report by Doug Burkholder

BIRS Math and Art Workshop

January, 2007

Experience:

The time spent here in Banff this past week has been a productive week. In addition to building friendships with faculty with common interest, I have formed a new collaboration and started a new project. From presentations by others, I have learned new methods for enhancing mathematical education and new ways to incorporate art into mathematics.

Prior Needs:

As I came to this conference, I was hoping to gain new insights into projects that I could take into my courses. While I am always looking for ideas and projects for all levels of courses, I am specifically interested in projects for the upper-level mathematics courses. Like many faculty at small private liberal arts colleges, I teach a wide range of courses and I find that I do not have the time or expertise to develop projects in all of my courses. Specifically, I want to incorporate more opportunities for visualization of the mathematical material.

Partnerships Formed:

Through brainstorming sessions and breakout group discussion David Richter, Pau Atela, Bob Bosch, and I decided to form a partnership leading to the publication of resource material for upper-level mathematics course. We also identified several others, such as Carlo Sequin, Doris Schattscheider, and Gary Greenfield who are willing to assist us in various aspects of our project such as creation of material and expertise in the publication process.

Project Planned:

Our project is to compile a set of 15 to 20 projects which use art to enhance mathematical instruction in upper-level mathematics courses. These projects will be self-contained and ready to distribute to students in traditional upper-level courses. They should enhance the mathematical experience both through the visual arts and through alternate applications and extensions of the mathematical material being taught within the course. Each project should culminate in artwork produced individually or through group effort. Generally, these projects will be designed to take 2-4 weeks, but they should be flexible and open-ended to allow situations where they may extend over the semester or where they could be used for undergraduate research projects. There is also the possibility of using these projects, perhaps in a condensed version, in math clubs. There is also the opportunity for these projects to be used as the core of an upper-level seminar course devoted to mathematics and art.

Long Range Impact:

In addition to assisting faculty currently excited about the ability for art to enhance mathematical instruction, the publication of our resource material should attract other faculty to math and art. This should also assist young faculty as they begin their teaching career. Likewise,

I anticipate taking advantage of projects begun by other partnerships here are BIRS, such as the liberal arts math and art project.

Stewart Craven, Toronto District School Board

I came into this week wondering if I could make a contribution to the proceedings and wondering about what new things I might learn. I was concerned that my mathematics knowledge would not be sufficient. My concerns were allayed as the week played out. Shortly after I arrived in Banff I encountered Reza and George on the main street here in Banff. We proceeded to a coffee shop and the rich discussions began. The week was orchestrated in such away that the participants grouped and regrouped in a various combinations that of course led to discussing a vast array of topics from numerous perspectives. I discovered that my knowledge of elementary and secondary school teaching and learning was a critical piece of the mosaic particularly given the kinds of products that have been proposed. Nevertheless, I continue to be in awe of those participants whose mathematics understanding and creative abilities in art are seemingly beyond my grasp. It would be remiss of me not to comment on our surroundings. The mountains, the deer, the birds, the Banff Centre, and the village all contribute to an environment where learning and creativity will inevitably flourish. Also, activities such as, the Circus, the Banff Centre tour, the native story telling, the walk up Tunnel Mountain, and the excursion to the hot springs all serve to activate the senses. One last note is to commend the staff at the Banff Centre who have been so friendly and helpful throughout the week.

Accomplishments

First and foremost, I learned about ideas in topology and knots. I learned how to construct a truncated icosahedrons and six-point or eight point stars through paper folding and I learned about how represent rhythm in music as polygons.

Projects

The K – 12 group is concerned about activities where art is in some meaningful way ought to be connected to mathematics. Whether the inspiration for the mathematics comes from the art or the mathematics in and of itself leads to artistic representations, there is a need for suggestions/activities for elementary and secondary teachers to use in their classrooms. To this end our group will create a framework that provides the critical information required by teachers to embed these lessons in their programs. We will start by writing 4 – 6 lessons, field test them, and refine them to be published in an appropriate form. I will initially write a lesson based on the construction of a “giant” stellated octahedron followed by a series of lessons about students who use their own photographs imported into Geometer’s Sketchpad to explore transformational geometry. I will additionally submit my workshop plans for two Mathematics and Art sessions that I will be doing over the course of the next four months. I will work closely with my group (Nat, Mara, Glyn, and Phil) to achieve our goals.

Gerda de Vries
Department of Mathematical and Statistical Sciences

University of Alberta

Paragraph about experience here.

I learned a lot about the math&art community - what types of activities people are involved in, what their educational interests are, etc.

I am in awe of the leeway that educators have at liberal arts colleges, and have come to the realization that most innovations in education will come from colleagues at such institutions.

Description of math education needs you feel are important and whether they were addressed.

The members of the group primarily were academics, and so were most comfortable identifying areas at the undergraduate level where we can have impact.

I think that members of our group have and can develop engaging activities that can have impact on education at the K-12 level when combined with (tested) pedagogical support materials. Unfortunately, due to a lack of critical mass of K-12 education specialists in our group, we were not able to 1) identify areas of the K-12 curriculum that need to be addressed in the first place, nor 2) address the development of pedagogical support materials.

Accomplishments, e.g., partnerships formed and projects planned.

Through discussions with colleagues, I now have ideas about how to improve my outreach activities.

I am inspired to contact my local science museum and find out whether there is interest in math&art there.

I am inspired to contact colleagues in education to develop pedagogical support materials for at least one of my outreach activities (support materials that can be used by classroom teachers to follow up on concepts explored during my classroom visits, for example).

I look forward to receiving updates on the book projects initiated at this workshop - at the moment, I have very vague ideas about how I might contribute, but these ideas may become more concrete after some incubation time.

What you see as the long range impact of this week's workshop.

Connection to a new group of people, with possibilities to collaborate on projects in the future.

Anything else you think should be mentioned in our final report to BIRS.

This is the most productive workshop I have attended at BIRS. It truly was a workshop, with participants working together to articulate goals and develop plans towards achieving those goals.

Name: Gwen Fisher

Affiliation: Mathematics Department, California Polytechnic State University, San Luis Obispo

Paragraph about experience:

This was a very productive week. I liked the flow of the workshop, that we worked together as a large group to decide on our goals then broke into groups to work on developing the goals, which we then reported back to the group. Then, we discussed other goals for new groups, but we were allowed to also participate in the first groups or the second groups. That we had the freedom to move within groups or stay in groups made it easy to focus on activities targeted towards my interests and talents.

Description of math education needs you feel are important and whether they were addressed:

I believe that assessment of math/art programs is an imperative educational need that was not addressed sufficiently here. I proposed a study to assess the effects of our project, and while I received moral support for such work, and suggestions that others might be interested in working on this in the future, nobody had sufficient interest to work with me to develop this project during this week. Most of the final products appear to be the creation and collection of mathematical art resource materials. While I believe that this work is also very important, at some point, the math/art/education group of scholars will likely need to justify that our work is actually teaching mathematical skills and concepts to get more widespread support of our work from our education colleagues and government agencies.

Partnerships formed:

Carolyn Yackel, Mercer University, yackel_ca@mercer.edu
Kevin Hartshorn, Moravian College, hartshorn@moravian.edu
Doris Schattschneider, Moravian College, schattdo@moravian.edu
Blake Mellor, Loyola Marymount University, bmellor@lmu.edu
Carol Bier, Mills College / The Textile Museum, carol.bier@gmail.com
Mara Alagic, Wichita State University mara.alagic@wichita.edu

Accomplishments:

We wrote a proposal for a book of activities joining mathematics and art in a liberal arts environment.

I wrote a proposal for a mathematical art exhibit “Mathematical Expressions: Bead Weaving with Gwen Fisher” at the *San Jose Museum of Quilts and Textiles* in California that Carol will help me submit, and hopefully get accepted.

Projects planned:

Carolyn, Kevin, Blake, Doris and I plan to complete the book of activities described above: I will be providing at least one of the activities, and I will be the illustrator for the book. The other four will be the co-editors.

Carolyn, Mara, and I plan to develop and conduct a study on the effects of studying mathematical art on spatial reasoning skills, at least I hope so.

What you see as the long range impact of this week's workshop:

I believe that the impact will be the creation and dissemination of more mathematical art projects/lessons/information to the general public, especially students.

Anything else you think should be mentioned in our final report to BIRS:

The facilities are wonderful, and the staff has been very kind and helpful.

Nathaniel Friedman, Dept. of Mathematics, Univ. at Albany-SUNY,
Albany, NY 12222

(1) I had a very important experience at the BIRS workshop on Innovations in Mathematics Education via the Arts. The workshop sessions went very well and all facilities were first-rate.

(2) There were many innovative projects introduced that reflected my basic concern with mathematical understanding through visualization having an arts component. The level of discussion was very high and a variety of significant ideas were introduced. These ideas were definitely developed in a practical manner during the workshop.

(3) I formed a partnership on the development of projects for K-12 education with Mara Alagic and Glyn Rimmington of Wichita State University, Kansas, Stewart Craven of the Toronto School Board, Canada, and Philip Wagner of the Fusion Project, San Francisco. We have very complementary skills and I look forward to our collaboration.

I also plan to collaborate with Doris Schattschneider of Moravian College, Pennsylvania, on her project for developing a collection of iconic images relating mathematics and art for a CD.

Thirdly, I plan to collaborate with Gene Klotz of Swarthmore College, Pennsylvania, on his Web-Based Math Forum project.

(4) I organized the first Arts/ Mathematics Conference at the University at Albany in June, 1992. There have been annual conferences organized by myself as well as Reza Sarhangi and others every year since 1992. I consider this first BIRS Workshop on Mathematics Education via the Arts of historical significance. I envision this Workshop as strongly accelerating the movement in education relating mathematics and art at all levels from K-college. I am totally grateful to BIRS and the Banff Center for making this possible.

Susan Gerofsky, Curriculum Studies
Faculty of Education, University of British Columbia
Vancouver, BC, Canada.
Jan. 26, 2007

I am very pleased to be a participant in the BIRS workshop, Innovations in Mathematics Education Via the Arts. It has been a wonderful opportunity to connect with an exciting and creative group of like-minded colleagues in a place where we could concentrate on our

collaborative work without distractions. To quote Banff Centre Service Director Jim Olver, "there are no excuses" at BIRS to delay the work that you must do. BIRS provides all the necessities and trimmings for a highly successful academic workshop: comfortable accommodations, excellent meeting facilities, delicious meals, computers, internet access, scanning and copying facilities, lounges, a reading room, and the most helpful staff imaginable, all in this spectacularly beautiful setting. With the support of this infrastructure in place, we were able to work very productively and accomplish a great deal during our five-day residency.

Working to establish connections between mathematics and the arts in a faculty of education, I often feel isolated from both mathematicians and artists at my own university. I think that many of us do occupy the position of the lone "math/arts" advocate in our own institutions. It is both a necessity and a delight to gather together for an intensive working session like this one.

Our workshop addressed the need for connections between mathematics and the arts at all levels of education: K-12, college liberal arts courses, university undergraduate mathematics courses, and in terms of lifelong learning through museums, television programs, books, CDs, websites, traveling math/ art shows and other media. My own professional interest centers on secondary school mathematics education, and this was certainly addressed in all our sessions and in the outcomes of the workshop. I was also happy to expand my own view of mathematics education to take in ages "zero to infinity".

One of the most exciting outcomes of this BIRS workshop was a partnership several of us formed around the potential of teaching a very wide variety of mathematical concepts through music and rhythm. I am now collaborating with colleagues Godfried Toussaint (McGill), David Rappaport (Queens), Paco Gomez (Univ. Politecnico de Madrid, visiting at McGill), and Reza Sarhangi (Towson University, Baltimore) on an initial article for a mathematics education academic journal (For the Learning of Mathematics or Educational Studies in Mathematics). We will be outlining a program to use Toussaint's innovative circular representations of rhythmic patterns in music to teach concepts in a wide range of mathematical areas, ranging from number theory to geometry, abstract algebra, and combinatorics. We hope to use the analysis from this collaborative article as the basis for the development of a book of lesson ideas and materials for mathematics instructors at a variety of different levels, to encourage thoughtful implementation of mathematics teaching via music. A proposed book may also include a call for articles from other mathematics educators who use music as a means to teach math concepts.

Workshop leaders and participants are planning to pitch the idea of a book series offering math and art lesson plans and connections aimed at different levels of schooling (say, high school, elementary school, or undergraduate math courses) and different artistic media (math and music, or math and sculpture for example). This is a very exciting prospect, and may result in a coordinated series of resource materials which will promote a practical implementation of an enriched mathematics teaching via the arts, with the potential to reach many more students through an embodied, humanistic grasp of abstract concepts.

Thanks to BIRS for providing us the opportunity to come together to accomplish this important work, in this beautiful place.

Paco Gómez, Department of Applied Mathematics
School of Computer Science, Polytechnique University of Madrid, Madrid, Spain.

Participating in the BIRS workshop, Innovations in Mathematics Education via the Arts has been a great experience for many reasons. For one, it has been a unique opportunity to meet very creative people with my own interests. Sometimes, it is hard for those who like mathematics and art. Also, the settings and facilities were perfect, with no obstacle to attention, allowing all us to concentrate on our work.

I really identify myself with what Susan Gerofsky says in her report. I also feel very isolated from both mathematicians and artists. They both seem to have prejudices on each other. In workshops like this, those prejudices are broken down. I have been in touch with mathematicians who love art and try to apply to their work. They do not do that for the fun of it, but because there are many connections. A good idea to extend this workshop would be to bring over artists so that it becomes a truly interdisciplinary workshop.

In particular, at this workshop I worked on connections between mathematics and the arts at all levels of mathematical education, ranging from K-12 to college liberal arts courses and university undergraduate mathematics courses. Since I have mostly worked with university students (and a little bit with babies), one of the outcomes I was expecting from this workshop was to learn about other levels of education.

Excitingly enough, I am now collaborating with Susan Gerofsky, Godfried Toussaint, David Rappaport, and Reza Sarhangi. We are writing a paper on teaching mathematics via musical rhythm. Our main idea is using geometry, in particular circular representations, in order to teach mathematics concepts. We will concentrate our efforts on both the content and the methodology.

Name, affiliation

Gary Greenfield, University of Richmond, Richmond, Virginia, USA

Paragraph about experience here

This is my third trip to Banff. The previous two visits were for conferences, but this one was for a _workshop_. Banff is arguably one of the finest venues in the world, and being involved with a small focused group is infinitely more rewarding than being associated with the "sprawl" of a conference. My only regret is that our schedule precluded taking better advantage of the excellent facilities and recreational diversions.

Description of math education needs you feel are important and whether they were addressed

It is seminal that a group of educators was organized to consider the broader picture of the role, impact, integration, and future of mathematics and art in education. The fact that so many different levels (K through ∞ !) were considered attests to the effort to address the

perceived need and its value at all levels.

Accomplishments, e.g., partnerships formed and projects planned

Group Projects Planned:

- Contributions to "activities projects" materials for both K-12 and post calculus math classes;

Individual Projects Planned:

- Further investigation of NSF funding opportunities
- Math and art research into L-systems art making and optimization-based art making

Partnerships Formed: In my role as editor of the JMA, solicitations for

- individual contributions
- report to be published about the activities of the workshop.

What you see as the long range impact of this week's workshop

A concerted effort to promote mathematics and art education initiatives.

Anything else you think should be mentioned in our final report to BIRS

The unique inspirational, informational, hands-on activities to energize and engage participants led by George Hart (CD polytope sculpture), Barb Fraser (birch bark ornament), Carol Bier (Islamic art constructions), Glyn Rimmington and Mara Alagic (L-parser), Susan Gerofsky (long sword patterns), and Godfried Toussaint (mathematics of rhythm).

George W. Hart, Stony Brook University

This was an outstanding week for us. A wonderful mix of creative people participated, with varying backgrounds, diverse areas of expertise, and experience working with different levels of students. But we all feel a passion for mathematics education via art. Many of us have had great teaching experiences with art-based math education activities that we have developed. After much sharing and brainstorming, we decided that the greatest need that we as a group could fill is to provide resources for educators at all levels. Other teachers will use these ideas in their classes if we write them up in a useful manner, which conveys some of our excitement and contains clear procedures to follow.

There are three main projects formed, plus smaller groups of participants formed a number of additional partnerships. The main projects are edited collections of activities, one aimed at the K-12 level, one aimed at college-level liberal arts students, and one aimed at post-calculus students. We expect that each collection will be published in the form of a book, probably with an electronic supplement on a CD or a web site. We discussed representative activities for each collection and developed a format that contributors can follow when proposing activities to the editors. The groups of editors for the three collections will format some initial activities and with them as models, announce a call for additional solicitations. Many of the workshop participants will be contributors.

I am certain that everyone left feeling creatively invigorated, with a sense that our work will have a very positive long-term impact on mathematics education.

Kevin Hartshorn (Moravian College)

This was a tremendously rewarding experience. I've made contacts with many people connected to math/art activities, rejuvenated my enthusiasm for teaching my "Math for Design" course (a quantitative literacy course at Moravian), gained many excellent ideas for using art to bring mathematics to a broader community, and established a new and promising book project.

Coming to the conference, my main focus was developing projects and ideas for my Math for Design course. I wanted projects to energize the students (mostly art majors) while teaching them some genuine mathematical concepts. In a broader scope, I wanted projects with a public face - projects that would bring mathematical ideas to the broader campus community.

We (Blake Mellor, Carolyn Yackel, Gwen Fisher, Doris Schattschneider and myself) formed a very enthusiastic coalition. We have all been hungry for activities for a liberal arts mathematics course. Together, we have made some solid plans to develop a book of activities and projects. Blake, Carolyn, and I will offer our energy to the project, Doris has been a font of knowledge and insight from her previous experience, and Gwen has had excellent ideas to create a very strong product.

That book is certainly going to be the long-range output for this project. We hope to have some significant progress, including a proposal for publishers, available by the summer. At that point, we will be soliciting for contributions from the math/art community.

This is a perfect facility for collaboration, creativity and really solid mathematical work. We are removed from the distractions of the day-to-day work. There is a truly inspiring environment in the Banff community. And the facilities are top notch - amazing meeting rooms, internet and computing capabilities, staff, etc. I really look forward to a chance to attend another workshop here.

William Higginson
Queen's University at Kingston
Banff, 2007 01 26

I have been interested in the academic area defined by the overlap of mathematics and art for more than a decade. During that time I have attended several national and international conferences and have written on both theoretical and practical aspects of certain issues in the field. From that perspective my sense is that this workshop may well come to be seen in the future as a distinct developmental landmark in the evolution of an interesting subfield of mathematics with important implications for education. The breadth and intensity of the interests and experience of workshop participants was positively exploited by the ongoing presentation of a rich set of examples to the group. These same characteristics presented some challenges as the workshop progressed in the construction a common vision/sense of purpose. The workshop was

well planned and effectively led. The BIRS facilities were outstanding.

I am concerned about the public image of mathematics and the effect it has in limiting possibilities for learners. I think that the interests of members of this group have considerable potential as one direction from which to address this problem. My sense is that good progress was made in several sub-areas toward making some of this potential more accessible to teachers and learners.

I much enjoyed individual interactions with a number of participants and found several of the group activities to be enlightening and informative for future teaching and research. With respect to future possibilities I was particularly pleased to meet and be able to interact with Gene Klotz whose contributions to mathematics education I have long admired. I was surprised and pleased to learn in the pre-conference exchange of proposed projects by how many commonalities our ideas appeared to have. Following discussions at the workshop I look forward to collaborating with Dr. Klotz and others in the group on the development of a wiki for people interested in Mathematics and Art and the generation of an accessible and well-supported gallery of mathematical images.

I am pleased that the BIRS executive committee saw fit to support this workshop (a wise choice I would contend) and am grateful to have been able to be a participant. The workshop organizers have done a fine job in moving a spirited, knowledgeable and energetic (but not particularly 'herdable' - if that is a word) group of individuals several steps further in the direction of the generation of some significant resources for the field. Thank you.

Dirk Huylebrouck, Hogeschool Wetenschap en Kunst

It was an enriching experience: great people, great site. It is the second time I come here, and if possible, I'll return. This is one of the best environments for a mathematics and art conference.

The promise of working on real proposals was not entirely met. Most people thought more about their own forthcoming publication, their own project, than about the general promotion of the "Math & Art" idea. A few more workshops could help to encourage a climate of collaboration, so that individual concerns would prevail less on immediate individual aspirations. I am not finger pointing the organizing scientist here, who gave a lesson in democratic approach and encouragement of collaboration, nor the participants (I was one myself, so I was co-responsible!), no, it is a human feature, and through more workshops like the present one, something can be done about it.

I was glad to conclude, that, in the end, it was agreed upon some strategies for involving more closely other mathematics and art organizations, though in the beginning the exchanges about this idea were hesitating (to put it mildly). It will need more diplomatic efforts, and hopefully more participants from South-America, Europe, ... will participate in the 2009 meeting in Banff, so that the 2010 conference may turn out to become a conference where Bridges, Isama, ISIS, Katachi, Nexus, Math& Design and whatever other math and art groups may meet. An international gathering, say, every four years, of these different organizations could even attach as a "math and art" movement to the ICM meetings, organized every four years, and, thus, give it more scientific recognition. Maybe the good Banff center dinners and lunches contributed to this idea, as the dinner table served for the talks with the always open-minded Reza Sarhangi, (co)-organizer of the Bridges meeting.

I learned about the existence of two great books, and met (one of) their authors:

- “Experiences in geometry: Euclidean and non-Euclidean with History”, David W. Henderson and Daina Taimina (ISBN 0131437488)

- “Mathematics and the Aesthetics: New Approaches to an Ancient Affinity (CMS Books in Mathematics) by Nathalie Sinclair, David Pimm, William Higginson, Editors (ISBN-10: 0387305262, ISBN-13: 978-0387305264)

There are many books with similar titles and with enchanting reviews, so that meeting the authors, and going through some pages, is actually more convincing. I think I’ll soon (=next academic year) teach a course based on the first, with the second as an indispensable “reader”. This is a down-to-earth simple consequence of this meeting, but a very useful one to my students and me.

Finally, here is the summary along the idea of Carlo Séquin. If I were to meet Bill Gates in Banff and had one minute to talk to him, I would say: “Hello, ho, you’re Bill Gates! I am here in Banff for a meeting about mathematics and art. You know, I am often surprised architects very proudly assert everyone knows the Egyptian pyramids are the first great human realization, while in fact, mister Gates, it is YOUR field, the field of reasoning, logic and mathematics which started the odyssey of humanity. Yes, yes, in black Africa, the continent you want to help in particular, there, the first record of logically grouped was found, 22000 years ago. No joke, mister Gates, it has been confirmed in many scientific publications and in the media, and here at the meeting all find it an evident statement many civilizations contributed to mathematics. However, we need help to overcome colonial and neo-colonial unwillingness to reach the general public: some findings were only revealed on the dying bed of a discoverer. And in Africa, Western publications do not reach the audience. So, 50 years after its discovery in Congo, many Africans still think it was YOU who invented information recording, ha! (- to be followed by a demand for money).”

Craig S. Kaplan, University of Waterloo

I arrived in Banff without a specific agenda, simply hoping to find some way to contribute my knowledge of mathematics and software development to the group. I wanted to suggest ways that other projects might be able to benefit from a computer-based component in general, and possibly my own computer graphics research in particular. The ideal situation would be to find a small group of collaborators with whom I could develop educational materials that simultaneously have a CS research component.

The experience has been enjoyable and inspiring. Through productive discussion with other participants, I came away with ideas for math education and for my own undergraduate computer science teaching. Naturally, some of the discussion also led to new ideas for research (which, being related to art-math, might always filter down to the educational level).

Everyone at this workshop believes that there is a role for art to play in the teaching of mathematics. Art provides a context for the math, motivates it by demonstrating an application, and makes it visible and tactile. Student projects based on art-math can engage them across multiple modalities, in a way that pencil-and-paper work never could.

I believe the most important aspect of using art to teach math is that the presenter not simply treat the math as being self-evident. It isn't sufficient to show an example and proclaim

"behold!". To be educational, a piece of mathematical art must be the launching point for a deep presentation of the underlying concepts, and never an end in itself. I believe that this attitude is shared with many of the other participants, which makes me confident that the educational materials that result from this workshop will have real value.

My impression is that during the week, the group arrived at a concrete goal: a series of books containing lesson plans and project ideas that math educators could incorporate into their curricula. This is a worthwhile goal, and one to which every participant is able to contribute. These books could definitely help bring the beauty of art-math to a wider audience in an educational setting.

Gene Klotz, Swarthmore / Math Forum at Drexel

Paragraph about experience here

I had a very interesting and generally worthwhile experience, even as a newcomer to this community. My interests and focus are different from most people but I found numerous persons to talk with, and some interest in my ideas. I wish I'd had more time to present my ideas at first and more time to interact with people in a focused way at the end.

Description of math education needs you feel are important and whether they were addressed

My interests are in web-based projects and this was addressed but not in the depth I would have liked. However, my Math Image project ideas were strengthened and I plan to submit a proposal to various funding agencies.

Accomplishments, e.g., partnerships formed and projects planned

I've formed potentially useful contacts and expect to continue interacting with a couple people at least. I'll give the community the opportunity to comment on my proposal before submitting it to funding agencies and will encourage people to come up with wiki ideas for the Math & Art community as an associated project.

What you see as the long range impact of this week's workshop

There are likely to be several projects that come out of the workshop, it would appear. Several groups seem to have coalesced with common interests and clear ideas. I also would not be surprised if this workshop were to have a lasting impact on the Math & Art community: spending a week together looking for common goals and projects was a good thing.

Anything else you think should be mentioned in our final report to BIRS

Very nice facilities and support. BIRS provided a fine context in which to grow ideas and to interact in productive ways.

Blake Mellor, Loyola Marymount University

Paragraph about Experience at BIRS

This was a very valuable experience for me personally. I had the chance to meet with many other people interested in connections between mathematics and art, and in particular to meet some of the more established people in the field. I also was able to work on projects with my collaborator, Gwen Fisher, and to become involved in a larger collaboration (which I describe more below) which I believe will fit very naturally with my ongoing projects.

The facilities were excellent – I look forward to returning for Bridges 2009!

Description of math education needs you feel are important and whether addressed

The needs I feel are most important are two fold. The first is the need for easily accessible and ready-to-use materials and modules which integrate mathematics and art in a substantial way that teaches real mathematics and relates to real art. The second is the need to assess whether the integration of mathematics and art is successful in (1) improving the learning of the mathematical content and/or problem-solving skills and/or (2) improving the students' attitudes towards mathematics and their ability to do mathematics.

I think the first need has been addressed at several levels, with proposals to create collections of materials for K-12, college courses for liberal arts students, and college courses for more mathematically advanced students. They are also supported by proposals to collect images and other materials on CD or a website.

The second need was raised, but was not addressed as centrally, though a few of the participants (specifically Gwen Fisher) are planning to do some research on this question.

Partnerships formed/Accomplishments/Projects Planned

I formed a collaboration with Gwen Fisher, Kevin Hartshorn, Doris Schattschneider and Carolyn Yackel to edit a collection of activities/projects for Mathematics for Liberal Arts. We plan to create some sample projects by July 2007, along with detailed guidelines for the projects, and send out a call for proposals by the end of the summer. We hope to collect a range of projects, on different topics and of different lengths, to be a resource for teachers of college Math for Liberal Arts courses, and possibly also for high school teachers.

Long range impact

Collecting resources in a form easier for novice teachers to use will greatly increase the use of art in mathematics education, and that will lead naturally to greater interest in the more research-level questions at the intersection of mathematics and art.

David Rappaport, School of Computing, Queen's University.

Paragraph about experience here

My mathematical teaching experience is constrained to university lecturing to computer science students. I have an ongoing interest in teaching mathematics and think that some of my ideas of using music to teach concepts in mathematics would be useful at other educational levels. The interaction this week has been a tremendous learning experience, where some of my impressions were confirmed and others reformed.

Description of math education needs you feel are important and whether they were addressed

I think that it is important to engage mathematics students so that they can gain a deeper understanding. The hands-on workshops this week, for example George Hart's workshop on building a truncated icosahedron using recycled compact discs, addressed this aspect of learning. In fact this aspect of deep learning seems to be a common trait of all of the presentations and ideas that were proposed this week.

Accomplishments, e.g., partnerships formed and projects planned

I have had a very productive week. My focus on using music to teach mathematical concepts match well with several participants. We are in the process of organizing a paper to be written to articulate and expand some of the ideas that we demonstrated here at BIRS on Thursday morning. I will be continuing a long collaboration with Paco Gomez and Godfried Toussaint. I am very pleased that Susan Gerofsky and Reza Sarhangi will be a part of our collaboration. I am also very excited to have made a connection with Bill Higginson, and Stewart Craven. Bill is at Queen's my home institution, and Stewart is down the road at the Toronto Board of Education. They have both shown interest in the ideas we presented. I feel that their knowledge and experience in the field of education will be an enormous help for us, and may lead to fruitful collaborations.

What you see as the long range impact of this week's workshop

I would like to think that the work we are preparing will have a positive practical impact on how mathematics is taught at a variety of levels. At the very least I will embark on a new and stimulating area of study and research.

Anything else you think should be mentioned in our final report to BIRS

The facilities here are very conducive to group interaction and I have nothing but good things to say about my experience here. My only regret is that I could not interact with more of the people at our workshop. Furthermore it would have been interesting to be able to interact with other groups at the Banff Centre. However, I guess there is only so much that can be done in 5 days.

David A. Richter, Ph.D.
BIRS Workshop 07w5062
Innovations in Mathematics Education via the Arts
Summary of Outcomes
January 26, 2007

I wrote these notes about my personal outcomes. I don't think they answer all the questions you asked above, and certainly you won't find the answers in the order you requested. Nevertheless, the substance is here:

For me, there were four identifiable outcomes. The first is that I got more people to think about holding a student mathematical art contest. The second is that I am more determined, perhaps inspired, to complete an individual project in mathematical art. The third is that I have joined a

collaborative team to work on a project in mathematics instruction via the arts. Finally, I have expressed some interest in contributing to a wiki on mathematical art. Here follow some details about these outcomes.

Initially I presented the idea of an international student mathematical art contest. I will continue to think about realizing this project. However, I do not have the resources to take on a project on the scale that I envisioned. Therefore I will continue to contemplate trying it on a local level, maybe at my university or my town. Throughout the conference, several participants expressed interest in this project and offered their advice.

This workshop has provided incentive for me to continue on an individual project. My wife and I are working on a quilt which illustrates the conjugacy classes of the “quilt group”, the group of all rigid motions preserving the integer lattice in the plane. We are nearly finished with the entire quilt, but we have barely started the write-up. I plan to take some notes on the project which a quilter with a modest background in geometry can understand, and present the work either the Bridges or ISAMA conference. I call this an “individual” project because my wife did not attend this workshop and obviously, being married, we have the capacity to work very closely as single unit. Thus, I do not consider this to be a new collaboration.

I have joined an editorial board to assemble mathematical art projects into a resource book. The other members who have expressed the most interest are Doug Burkholder, Pau Atela, and Bob Bosch, and it seems likely that we will include others. These projects will be designed to supplement standard core courses for students majoring in mathematics and/or computer science. The students will work on each project outside of class, and each project should require about 2-4 weeks to obtain a final product. We plan to assemble about 10-20 projects into a single book after some field experience. Given that we need at least one or two more semesters to field-test these projects, and given that we wish to invite others to contribute, I anticipate that at least a couple years will elapse before we have a finished product.

Gene Klotz has shown a lot of interest in warehousing mathematical art images using some of the infrastructure of the Math Forum and also he likes the idea of a wiki on mathematical art. I have expressed interest to him about contributing to either of these projects. Due to the scale and complicity of this project, however, I cannot lead this project at this time.

That's it. Thank-you for a very productive and enjoyable workshop!

Name: Reza Sarhangi

Affiliation: Department of Mathematics, Towson University, Towson, Maryland, USA

Paragraph about experience at Banff: My experience at Banff included the following important mathematics education: (a) I became familiar with several mathematics education internet resource that have connections with art and I was not aware of them, (b) meet with some individuals who are key people in developing art related mathematics activity materials and manipulative sets for K-12 teachers, (c) I also learned about new college level mathematics

education materials that were related to the arts, (d) participated in some mathematics and music proposals for writing joint papers and producing workshop activities

Description of math education needs you feel are important and whether they were addressed: We need to reach public and present mathematics in an appropriate way. I believe one appropriate and important approach for this action is to use art as a media for presentation or as a field for integration. Poincaré says: “The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful.” By proper use of art, which is not the only approach but one to be indicated during our workshop, we hope that the new generations become introduced to the beauty of mathematics. In a traditional synthetic geometry course we are introduced to rigorous treatment of axiomatic systems. During which process we also learn historical and philosophical implications of various discoveries in Euclidean and non-Euclidean geometries. In addition, as a part of reasoning or as a mathematical challenge, we also learn how to make geometric constructions using compass and straightedge. Geometric constructions and the logic behind of the taken steps bring excitements to us while exercising our intelligence to justify the steps to reach to a conclusion. Geometric constructions have formed a substantial part of mathematics trainings of mathematicians throughout human life. Nevertheless, we are witnessing a lack of attention to the impertinence of geometry in shaping our understanding of mathematics in colleges and universities. One way of addressing this problem is to produce textbooks and workshop activities in connections of mathematics and the arts.

Accomplishments, e.g., partnerships formed and projects planned: During the workshop we were introduced to some innovative and integrative techniques that promote interdisciplinary work in the fields of mathematics and art education. I found the following areas that I can contribute: (a) I will participate in writing chapters for three “mathematics education via art” textbooks that will be produced in near future. The textbooks will be produced for college students majoring in elementary, middle school, and secondary education, (b) I also will participate in writing chapters for liberal art college level textbooks in mathematics and art connections, (c) Another area that I will participate will be in creating mathematics workshop activity packages for teachers in different levels that use visual art or music.

What you see as the long range impact of this week's workshop: I see that this workshop will impact our education system in producing interdisciplinary resource materials in mathematics education in areas such as textbooks, workshop activity books, and internet recourses.

Doris Schattschneider, Moravian College

My experience at this workshop has been an enriching one. I have met many new people and gotten to know many others better and have substantive discussions with them (I was at Bridges conferences with several of the participants in the past). At times, I had some frustration at the lack of pre-planned programming, but letting the connections and conversations happen and drive the schedule was a good idea. The presentations/workshops were very interesting and there were just enough of them, with very different flavors.

In a short period of time, many plans for materials on math/art have been discussed, and some detailed outlines for some projects have been formulated. I felt that I was able to contribute to many of the conversations, in formal groups and in informal settings. One unexpected connection and collaboration was with Philip Wagner, who came looking for input from conference participants on his proposed "Fusion" project in San Francisco. Since I'm a consultant for a curriculum project and have curated some art/math exhibitions at art museums in the past, I was able to give several suggestions to him. I hope that this collaboration will continue.

The greatest need for integrating art(s) in the teaching of mathematics, in my opinion, are good resources, readily available, and mentoring of teachers in how to use these. At the elementary and secondary level, this is especially important, since these teachers have enormous constraints under which they teach and they often feel that there is no way they can go beyond the text, or integrate other material with the required material. The BIRS workshop addressed many aspects of these needs, and if the proposed projects are culminated, at least some of this need will be met.

I joined discussions with two groups-- those preparing materials to integrate art into the teaching of math at the college level--both the liberal arts level and the math major level. I expect to be actively involved with both these projects. Also, mentioned above, I collaborated with Philip Wagner. My own proposed project, of preparing a CD with art images to be used in teaching math/art courses, was not pursued by any group, but several participants who are teaching such courses in liberal arts colleges told me they really would like to have such a resource. I hope to pursue this further.

I learned several new and interesting mathematical ideas/facts-- how to fold a perfect 6-pointed star, how to dance a sword-star, how to clap a polygon. Most of all, I felt part of a group of educators who are very interested in promoting the use of art in math education. I'm sure that these partnerships will continue well beyond the workshop at Banff.

There is a real possibility that what started at this workshop will have long-range impact. No one will leave here without having learned many new ideas and everyone seems to have the commitment to carrying out the various proposed projects.

Carlo H. Séquin

EECS Computer Science Division, University of California, Berkeley, CA 94720-1776

Overall Experience – General Impressions

The Banff Center presents a wonderful environment for a productive workshop – to make new contacts, to exchange ideas, and to plan new activities. The facilities (A/V, computers, conference rooms) are first-rate. The surroundings are spectacular and inspiring. Good ideas came from walks in this nice scenery.

Positive Results, Benefits

A primary result of this workshop was to make new, lasting contacts and to become aware of various existing resources related to the connections between art and math – in particular, web

sites and publications. I also gathered useful ideas for new student projects, classroom activities, and works of geometrical art. Examples are:

Class room activities: Rhythmic clapping exercises related to n -gons and periodic motions on a circle.

Geometrical sculptures: Make models of manifold surfaces from CDs connected with cable ties.

Student projects: Define sets of points or line segments in 3-space that yield different semantically meaningful shadows when projected in different directions.

Accomplishments

I had a secondary personal goal for these five days in Banff. I hoped to find time to write the bulk of a paper for Bridges 2007 for which I was still missing a crucial results related to the challenge of mapping the Hurwitz group of order 504 onto a genus-7 surface in a nice and symmetrical way. I had hoped that in Banff I would meet some knowledgeable topologists or group theoreticians who could me help with this problem. Indeed, I did make several crucial contacts (e.g., David Richter) and got myself educated on some fundamental issues that had been puzzling me for quite some time. While at this point I do not yet have the final solution for the genus-7 embedding, I am definitely a couple of steps closer. I have been able to tackle a very similar, but less complex problem: Mapping a manifold of 24 pentagons onto a genus-4 surface. Insight gained in solving this simpler task will help me in my quest for the final solution.

Planned Follow-up Activities

In three small group discussions and various lunch and breakfast conversations, we were able to outline some promising and potentially very exciting student activities and projects. Different group members will write up those ideas in more detail and put them in a shared place on the web. After the activities have been tested at least once with some actual students, the result from those trials will also be described and shared in that same place. Ultimately, the fleshed-out material will be carefully edited, so that the write-ups can become chapters in a book on college-level art-math projects. Here are two outlines of projects that I am committed to writing up in detail:

Graph Embeddings: Study knot and/or graph embeddings in 2-manifolds embedded in 3-space. What is the lowest-genus surface in which a given graph or knot can be embedded without crossings? What is the maximal symmetry that can be achieved? Make a large, sculpture-like model for long-term display in a public place.

Knot-spanning Surfaces: Learn about simple knots and about the 2-manifolds that use the path of the knot as their boundaries. Study different such knot-spanning surfaces and determine their genus and whether they are one-sided or two-sided. Deform some promising looking surfaces into artistically shaped forms and then build a real, tangible 3D artifact to represent this configuration. Possible realization techniques could be masking-tape surfaces attached to a knot made of copper tubing, or shapes formed from thick wires and from wire meshing.

Daina Taimina, Cornell University

BIRS is a wonderful place for work -everything that could possibly distract you or take your time away is taken care of and no excuses left not to be productive. I was finding people all around campus here very welcoming and helpful. It was a key to successful workshop. Even if we had sessions in certain times, there were so many other opportunities to get together informally and share our ideas and do brainstorming. I was finding it very useful to do some of brainstorming sessions in small groups and walking.

I liked very much beginning of the conference with introductions by all participants - it really gave a good overview what people are doing and what they are interested in, I think it helped later to make connections. There is a lot of need for informal math education. During our workshop issues in K-12 and college level math education were addressed but I think another direction to think is life long learning. It has been my experience that people are getting interested in math behind objects they see as art objects, and then they want to have an explanation and understanding. I am doing quite a lot internet correspondence with people later in their lives who now want to learn more mathematics. Talking about mathematics for general audience is different than teaching it in class. General audience is more interested to grasp concepts and do not need details. And that is really good opportunity for arts to come in. In our workshop there were many activities showed and developed that can be used for such audiences also but I think that can be continued.

At this point I can not give a strong partnership formed for particular project but I feel that I got an excellent opportunity to get to know many people I did not know before and learn about what they are doing. Concrete project is to connect with Carol Bier and Textile Society of America and write a proposal for a workshop. Also I will be writing a paper for Mathematical Intelligencer.

Thank you very much for the opportunity to be a part of this group and learn a lot!

Professor Godfried T. Toussaint, School of Computer Science and
Centre for Interdisciplinary Research in Music Media and Technology
Schulich School of Music
McGill University, Montreal, Quebec, Canada

Paragraph about experience:

This workshop was for me a unique experience that provided me with connections to elementary and high school teachers of mathematics that would have been difficult to realize otherwise. The group and the surroundings at BIRS provided a most stimulating environment in which to do research. In short this was one of the most positive workshop experiences of my life.

Math education needs that are important and how they were addressed:

Many young students today are turned off from mathematics because it is presented to them in a non-exploratory, non-creative dry manner. with little applications or connections to other exciting subjects such as art and music. The need to remedy this situation in education was considered in depth at this workshop through many discussions in both large and small energetic study groups.

Accomplishments, partnerships formed, and projects planned:

My proposal for this workshop was to explore and outline which concepts in mathematics would be suitable for teaching students at all levels by using musical rhythm to act as an accessible spring board and motivating factor. Several participants were excited about my proposal and joined me to form a group. These participants were: Susan Gerofsky, Francisco (Paco) Gomez, David Rappaport, and Reza Sarhangi. As a group the five of us met every afternoon to explore our ideas. We also met in pairs and threes in the mornings and evenings. We presented progress reports to the entire group every day, and received valuable suggestions from participants not in our group. In particular we benefitted from suggestions and useful references given to us by William Higginson and Dirk Huylebrouk. By the end of the workshop we had put together the skeleton of a paper we will write on this topic addressed to high school teachers. We also presented an example one-hour lesson to the entire group.

Long range impact of this week's workshop:

In general I think this workshop will in the long range increase awareness among teachers at all levels, that mathematics can be taught in a more interesting and fun manner for both students and teachers. On a personal level I have started a new collaboration with Susan Gerofsky who is in an education department. This collaboration will result in the publication of materials that will improve mathematics education in the future.

Additional comments for the final report to BIRS:

The workshop lasted four full days. Perhaps for this type of exploratory workshop seven days is more appropriate?

(1) **Philip Wagner**, Fusion Project, Integrating Mathematics and Art, San Francisco, CA.

(2) Excellent workshop with terrific people. I learned a great deal and everyone was anxious to share.

(3) I will collaborate with Stewart Craven and Nat Friedman and work with teaching facilitators to instruct teachers at 3-5 pilot schools in San Francisco. I will also work with art museum docents on these interdisciplinary projects that relate the museum collections with public mathematical education. In addition, I will coordinate and evaluate these programs.

(4) The long term goal is a mutual benefit for public education and art museums relating art and mathematics.

(5) Our short term goal is to improve understanding of MATH among middle school teachers and their students using ART. Working with the District we will identify those subject areas on standardized tests most likely to be improved using this approach. We must improve test scores. That is the reality.

Carolyn Yackel

Mercer University

My experience at BIRS was both intellectually invigorating and physically exhausting. I am proud of the work I accomplished with my team--Gwen Fisher, Kevin Hartshorn, Blake Mellor, and Doris Schattschneider--this week, but I am also apprehensive about the large amount of work to which we have committed ourselves over the next one to two years by agreeing to edit and write the liberal arts math and art book. In one sense, I am very happy about having partners in this endeavor that I felt sure I was destined to undertake during the next few years. In another sense, this conference has moved up the timeline of my individual plans for a book by several years. Again, having collaborators means we'll have more ideas and a much broader set of skills between us. However, it will also mean that I will have to compromise my goals and aspirations for the final project to fit with those of the group.

Aside from the identifiable project in which I have agreed to participate, I believe that this conference has also been a success for me in that I have made the acquaintance of and begun friendships with many colleagues who I greatly admire. I often find that these relationships blossom over time into collaborations. For example, I could imagine that in the future I would work with some of my new acquaintances to plan a conference, to serve on a committee, or even to write a paper in addition to the book we are already planning to edit.

My math education needs are for projects for my liberal arts math course taught through fiber arts. I suppose I gained some ideas for this course. Through discussions with my team, I certainly gained some ideas about the importance of not ignoring the artistic aspect of the course. In addition, I will be happy to have access to the book we will put out. Another big issue that faces education is the problem that in US culture not knowing math is perfectly acceptable. I think that we may be able to begin to make some inroads into this problem by using math and art to change attitudes towards mathematics. However, I think that this process will neither cause attitudes to change quickly, nor will it have an immediate impact on student scores. Furthermore, I believe that the math and art projects that experts choose to do with students during school time must be carefully chosen to fit with the curriculum students are already learning. Notice that this is not the case for special projects that take place outside of the classroom environment. Finally, I think that the mathematics contained in any activity or project is best if it is implicit in the project or activity rather than being superimposed. Implicit mathematics calls out to be discovered and investigated, whereas superimposed mathematics is artificial and tends not to engage curiosity.

As stated above, I plan to work on the Liberal Arts math and art book as an editor together with Kevin Hartshorn, Blake Mellor, and Doris Schattschneider as coeditors and Gwen Fisher as illustrator until she allows us to promote her to coeditor. My current obligation is to write a sample chapter about temari balls. It is due to the group by May 10th. We intend to have a proposal to a publisher by the end of June, and a call for chapters to potential authors by MathFest 2007. Other planned projects include digging back out a temari ball project/problem I was working on a year ago. Carlo Sequin re-inspired me with some new information which may enable me to either solve the problem or remember the solution I thought I had, but never had time to write up. Having Gwen Fisher teach me to bead led me to ask a beading question, which I may follow up on, if I have time. Gwen also wanted to work on a math ed project with me, which we may do, if she is still interested, and if I have time.

I think the long range outcome of this workshop will be at least two or three books in the next couple of years. I also think that a much more subtle result will be the collaboration of a younger group of mathematical artists or the incorporation of that younger group into the collaboration of the slightly older generation. The results of those collaborations may not be known for years. However, having had this time to meet, talk, and really try to work together has allowed us to know each other in a way that would usually take the time of several conferences.