Banff International Research Station

for Mathematical Innovation and Discovery

Geometric Mechanics: Continuous and discrete, finite and infinite dimensional
August 12 - August 17, 2007

MEALS

*Breakfast (Buffet): 7:00–9:00 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours) Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Donald Cameron Hall
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and small assortment of snacks available on a cash honour-system.

Over...

Monday

7:00–8:45	Breakfast
9:00–9:15	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:15–9:45	Antonio Hernandez-Garduno/*On the averaging of Lagrangian systems*
9:45–10:15	Nawaf Bou-Rabee/*Stochastic variational integrators*
10:15–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:15	Oliver Junge/*Optimal control of formation flying spacecraft*
11:15–12:00	Andrew Lewis/*Some results on energy shaping feedback*
12:00–13:00	Lunch
13:30–14:00	Ivan Struchiner/*Local equivalence and classification of finite type G-structures*
14:00–14:30	Ari Stern/*Computational electromagnetism with variational integrators and discrete differential forms*
14:30–15:00	Katlin Grubits/*Self-assembly of particles using isotropic potentials*
15:00–15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15–16:00	Eva Kanso/*Low-order models of swimming*
16:00–16:45	Rouslan Krechetnikov/*Dissipation-induced instability phenomena*
17:30–19:00	Dinner
19:00–20:00	Jerrold Marsden/*Computational mechanics and dynamical systems*

Tuesday

7:00–8:45	Breakfast
8:45–9:15	Oleg Kirillov/*How to play a disc brake*
9:15–9:45	Melvin Leok/*Lie group and homogeneous variational integrators*
9:45–10:15	Jeroen Lamb/*Homoclinic bifurcations with symmetry*
10:15–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:15	George Patrick/*Discretizations of Lagrangian systems*
11:15–12:00	Rui Loja Fernandes/*Singular reduction of Poisson manifolds and integrability*
12:00–13:00	Lunch
14:00–14:30	Tanya Schmah/*Symplectic tubes for cotangent-lifted symmetries*
14:30–15:00	Stephane Chretien/*Decay of correlation for the Euler approximation of a chaotic delay differential equation*
15:00–15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15–16:00	James Montaldi/*Bifurcations of relative equilibria at zero momentum*
16:00–16:45	Anthony Bloch/*Hill's equation with random forcing terms*
17:30–19:00	Dinner
19:00–20:00	Jedrzej Sniatycki/*Commutativity of quantization and reduction*

Over...

Wednesday

7:00–8:45	Breakfast
8:45–9:15	Bob Rink/*Branching patterns of resonant wave trains in a Hamiltonian lattice*
9:15–9:45	Dan Offin/*Maslov index and some questions of global stability*
9:45–10:15	Claudia Wulff/*Stability Transitions for axisymmetric relative equilibria of Euclidean symmetric Hamiltonian systems*
10:15–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:15	Sujit Nair/*Coordination with instabilities*
11:15–12:00	Manuel de Leon/*Hamilton-Jacobi theory on Lie algebroids. Applications to nonholonomic mechanical systems*

Thursday

7:00–8:45	Breakfast
8:45–9:15	Miguel Rodriguez-Olmos/*Nonlinear stability of Riemann ellipsoids with symmetric configurations*
9:15–9:45	Luciano Buono/*Steady-state bifurcation in reversible-equivariant vector fields*
9:45–10:15	Andreu Lazaro/*Stochastic Hamiltonian systems and reduction*
10:15–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:15	Mark Roberts/*The geometry of satellite attitude control*
11:15–12:00	Cristina Stoica/*Constant inertia trajectories: Saari's conjecture and more*
12:00–13:00	Lunch
14:00	Group Photo; meet on the front steps of Corbett Hall
15:15–16:00	Frank Schilder and Claudia Wulff/*Numerical bifurcation analysis of Hamiltonian relative periodic orbits*
16:00–16:45	Mark Gotay/*Concatenating variational principles and the kinetic stress-energy-momentum tensor*
17:30–19:00	Dinner
19:00–20:00	Tudor Ratiu/*The Hamiltonian strucutre of the Euler-Yang-Mills equations*

Friday	Travel day. Informal Discussions — no lectures
7:00–9:00	Breakfast
10:15	Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Over...

Geometric Mechanics: Continuous and discrete, finite and infinite dimensional
August 12 - August 17, 2007

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Anthony Bloch** (University of Michigan)
Title: *Hill's equation with random forcing terms*
Motivated by a class of orbit problems in astrophysics, we discuss solutions to Hill's equation with forcing strength parameters that vary from cycle to cycle. The results are generalized to include period variations from cycle to cycle. The development of the solutions to the differential equation is governed by a discrete map. For the general case of Hill's equation in the unstable limit, we then find exact expressions, bounds, and working estimates for the growth rates. We also find exact expressions, estimates, and bounds for the infinite products of several 2 by 2 matrices with random variables in the matrix elements. In the limit of sharply spiked forcing terms (the delta function limit), we find analytic solutions for each cycle and for the discrete map that matches solutions from cycle to cycle.

Speaker: **Nawaf Bou-Rabee** (California Institute of Technology)
Title: *Stochastic variational integrators*

Speaker: **Luciano Buono** (University of Ontario Institute of Technology)
Title: *Steady-state Bifurcation in Reversible-Equivariant Vector Fields* Abstract: I will discuss steady-state bifurcation results for vector fields which are both time-reversible and equivariant. I will focus on a general type of bifurcation called "separable bifurcations". The main result is that the analysis can be reduced to the study of bifurcations of an equivalent equivariant vector field (with no time-reversibility), sometimes also having parameter symmetry and for which a bifurcation theory already exists. This is joint work with J. Lamb and M. Roberts.

Speaker: **Stephane Chretien** (Universite de Franche Comte)
Title: *Decay of correlation for the Euler approximation of a chaotic delay differential equation*
Abstract: The goal of this work is to study differential delay equations of the type

$$x'(t) + x(t) = \kappa\Phi(x(t-\tau))$$

coming from optics and used by practicioners to generate chaotic behavior and produce a sequence of random numbers. The problem considered here is twofold. Firstly, we would like to know the invariant measure of this system or a least a good approximation. Secondly, we would like to estimate the decay of correlation of this system in order sample the sequence of iterates so that to obtain an efficient random number generator. In order to simplify the analysis, we first propose to study an equivariant approximation of the Euler scheme obtained by discretizing the equation. More precisely, we consider the recursion

$$x_{n+1} = x_n + h\kappa\phi(x_{nd})$$

which is easily seen to be equivariant under the action of adding any vector in Z^{d+1} to both state space vector $[x_{n+1}, ..., x_{nd+1}]$ and $[x_n, ..., x_{nd}]$ in the case where ϕ is periodic with integer period which is the case in the original equation proposed by the physicists. We study this equivariant equation first and give an estimate of the decay of correlation using the approach of Liverani [Decay of correlations. Ann. of Math. (2) 142 (1995), no. 2, 239301]. We then turn back to the study of the plain Euler scheme that we consider as a perturbation of the equivariant case.

Speaker: **Manuel de Leon** (Instituto de Matematicas y Fisica Fundamental)
Title: *Hamilton-Jacobi theory on Lie algebroids. Applications to nonholonomic mechanical systems*

Speaker: **Rui Loja Fernandes** (nstituto Superior Tecnico)
Title: *Singular reduction of Poisson manifolds and integrability*

Speaker: **Mark Gotay** (University of Hawaii)
Title: *Concatenating variational principles and the kinetic stress-energy-momentum tensor*

Speaker: **Katlin Grubits** (University of Hawaii)
Title: *Self-assembly of particles using isotropic potentials*

Speaker: **Antonio Hernandez-Garduno** (Universidad Nacional Autonoma de Mexico)
Title: *On the averaging of Lagrangian systems*
Abstract: We describe a method for averaging the variational principle in Lagrangian mechanics, illustrating it with the example of the forced inverted pendulum

Speaker: **Oliver Junge** (Munich University of Technology)
Title: *Optimal control of formation flying spacecraft*
Abstract: Future space missions like Terrestrial Planet Finder (NASA) and Darwin (ESA) will make use of a network of formation flying spacecraft. In these missions, the requirements on the accuracy on the relative positioning of the craft are extremely high. In addition, reconfigurations of the formation have to be performed at regular intervals with minimal energetical effort. In this talk, we show how the recently developed variational method DMOC (Discrete Mechanics and Optimal Control) for the numerical computation of optimal open-loop controls for mechanical control systems can be applied to this problem.

Speaker: **Eva Kanso** (University of Southern California)
Title: *Low-order models of swimming*

Speaker: **Oleg Kirillov** (Moscow M.V. Lomonosov State University)
Title: *How to play a disc brake*
Abstract: We consider a gyroscopic system with two degrees of freedom under the action of small dissipative and non-conservative positional forces, which has its origin in the models of rotating bodies of revolution being in frictional contact. The spectrum of the unperturbed gyroscopic system forms a "spectral mesh" in the plane "frequency–gyroscopic parameter" with double semi-simple purely imaginary eigenvalues at zero value of the gyroscopic parameter. It is shown that dissipative forces lead to the splitting of the semi-simple eigenvalue with the creation of the so-called "bubble of instability" — a ring in the three-dimensional space of the gyroscopic parameter and real and imaginary parts of eigenvalues, which corresponds to complex eigenvalues. In case of full dissipation with the positive-definite damping matrix the eigenvalues of the ring have negative real parts making the bubble a latent source of instability because it can "emerge" to the region of eigenvalues with positive real parts due to action of both indefinite damping and non-conservative positional forces. In the paper, the instability mechanism is analytically described with the use of the perturbation theory of multiple eigenvalues. Explicit conditions are established for the origination of the bubble of instability and its transition from the latent to active phase, clarifying the key role of indefinite damping and non-conservative positional forces in the development and localization of the subcritical

flutter instability. As an example stability of a rotating circular string constrained by a stationary load system is studied in detail. The theory developed seems to give a first clear explanation of the mechanism of self-excited vibrations in the rotating structures being in frictional contact that is responsible for such well-known phenomena of acoustics of friction as the squealing disc brake and the singing wine glass.

Speaker: **Rouslan Krechetnikov** (Carleton University)
Title: *Dissipation-induced instability phenomena*
Abstract: In this talk we present a coherent theory of the counter-intuitive phenomena of dynamical destabilization under the action of dissipation. While the existence of one class of dissipation-induced instabilities in finite-dimensional mechanical systems was known to Sir Thomson (Lord Kelvin), until recently it has not been realized that there is another major type of these phenomena hinted by one of Merkin's theorems; in fact, these two cases exhaust all the generic possibilities in finite dimensions. We put the main theoretical achievements in a general context of geometric mechanics, thus unifying the current knowledge in this area and the multitude of relevant physical problems scattered over a vast literature.

Next we develop a rigorous notion of dissipation-induced instability in the infinite-dimensional case, whose inherent differences from classical finite degree of freedom mechanical systems makes uncovering this concept more intricate. In building this concept of dissipation-induced instability we found Arnold's and Yudovich's nonlinear stability methods, for conservative and dissipative systems respectively, along with some new existence theory for solutions to be the essential ingredients. As a paradigm and the first infinite-dimensional example to be carefully analyzed, we use a two-layer quasi-geostrophic beta-plane model, which describes the fundamental baroclinic instability in atmospheric and ocean dynamics.

Speaker: **Jeroen Lamb** (Imperial College London)
Title: *Homoclinic bifurcations with symmetry*

Speaker: **Andreu Lazaro** (University of Zaragoza)
Title: *Stochastic Hamiltonian systems and reduction*

Speaker: **Melvin Leok** (Purdue University)
Title: *Lie group and homogeneous variational integrators*
Abstract: I will survey recent work on the synthesis of Lie group techniques and variational integrators to construct symplectic-momentum methods which automatically stay on Lie groups and homogeneous spaces without the need for constraints, local coordinates, or reprojection.

In contrast to traditional Lie group integrators, issues of equivariance and order-of-accuracy are independent of the choice of retraction in the variational formulation. Homogeneous variational integrators are derived using constrained variations restricted to a complementary section of the isotropy subgroup. However, as is the case of the choice of a connection in reduction theory, while the form of the resulting equations depend on the choice of the complementary section, the flow on the homogeneous space is independent of that choice.

The importance of simultaneously preserving the symplectic and Lie group properties is also demonstrated.

Speaker: **Andrew Lewis** (Queens University)
Title: *Some results on energy shaping feedback*
Abstract: Energy shaping is a control strategy wherein one converts a given mechanical system (called the open-loop system) to another mechanical system (called the closed-loop system) with desired properties. For example, if the open-loop system possesses q_0 as an unstable equilibrium point, one may wish to design a feedback which makes q_0 stable for the closed-loop system. While the strategy is attractive in many respects, it is difficult to implement since it leads to a system of quasi-linear partial differential equations that are generally over-determined.

In this talk we will (1) give an affine differential geometric formulation of the energy shaping problem and (2) give a complete description of part of the problem using techniques from the formal theory of partial differential equations.

Speaker: **Jerrold Marsden** (California Institute of Technology)
Title: *Computational mechanics and dynamical systems*

Speaker: **James Montaldi** (University of Manchester)
Title: *Bifurcations of relative equilibria at zero momentum*

Speaker: **Sujit Nair** (California Institute if Technology)
Title: *Coordination with instabilities*

Speaker: **Dan Offin** (Queen's University)
Title: *Maslov index and some questions of global stability*
Abstract: I will review some translations of the Maslov index, and explain how these abstract indices may be used to predict stability and instability for global periodic solutions determined by variational principles. I apply this to some open questions in celestial mechanics.

Speaker: **George Patrick** (University of Saskatchewan)
Title: *Discretizations of Lagrangian systems*
Abstract: We develop variational discretizations of mechanics, to the generality of discretizing nonholonomic mechanical systems with nonlinear constraints. This development is based on discrete analogues of tangent bundles obtained by systematically extending tangent vectors to finite curve segments. We show existence and uniqueness of the discrete evolutions by blowing up the variational principles at zero time-step. The blown-up variational principles have a past-future symmetry, which we use to prove the order of consistency between the the continuous and discrete evolutions, a problem which has been oversimplified in existing work. Our discretzation methods can automatically convert any one-step numerical method to a variational method of the same order.

Speaker: **Tudor Ratiu** (Ecole Polytechnique Federale de Lausanne)
Title: *The Hamiltonian strucutre of the Euler-Yang-Mills equations*
Abstract: The Lagrangian and Hamiltonian structures for an ideal gauge-charged fluid is determined. Using a Kaluza-Klein point of view, the equations of motion are obtained by Lagrangian and Poisson reductions associated to the automorphism group of a principal bundle. As a consequence of the Lagrangian approach, a Kelvin-Noether theorem is obtained. The Hamiltonian formulation determines the non-canonical Poisson bracket associated to these equations.

Speaker: **Bob Rink** (Vrije Universiteit Amsterdam)
Title: *Branching patterns of resonant wave trains in a Hamiltonian lattice*

Speaker: **Mark Roberts** (University of Surrey)
Title: *The geometry of satellite attitude control*

Speaker: **Miguel Rodriguez-Olmos** (Affiliation)
Title: *Nonlinear stability of Riemann ellipsoids with symmetric configurations*
Abstract: The problem of Riemann Ellipsoids has an long and important history, going back to Newton, MacLaurin, Dirichlet, Riemann and Poincare. It has its origins in the attempt to provide an explanation to the rotating figure of the Earth. In this talk, we will review this problem from the point of view of Differential Geometry and discuss how the geometric perspective can give some insight into the nonlinear stability of some of its classical solutions.

Speaker: **Frank Schilder and Claudia Wulff** (University of Surrey)
Title: *Numerical bifurcation analysis of Hamiltonian relative periodic orbits*

Speaker: **Tanya Schmah** (Macquarie University)
Title: *Symplectic tubes for cotangent-lifted symmetries*

Speaker: **Jedrzej Sniatycki** (University of Calgary)
Title: *Commutativity of quantization and reduction*

Speaker: **Ari Stern** (California Institute of Technology)
Title: *Computational electromagnetism with variational integrators and discrete differential forms*

Speaker: **Cristina Stoica** (Wilfrid Laurier)
Title: *Constant inertia trajectories: Saari's conjecture and more*

Speaker: **Ivan Struchiner** (UniCamp - Campinas - Brasil)
Title: *Local equivalence and classification of finite type G-structure*

Speaker: **Claudia Wulff** (University of Surrey)
Title: *Stability Transitions for axisymmetric relative equilibria of Euclidean symmetric Hamiltonian systems*
Abstract: In the presence of noncompact symmetry, the stability of relative equilibria under momentum preserving perturbations does not generally imply robust stability under momentum changing perturbations. For axisymmetric relative equilibria of Hamiltonian systems with Euclidean symmetry, we investigate different mechanisms of stability: stability by energy-momentum confinement, KAM, and Nekhoroshev stability, and we explain the transitions between these. We apply our results to the Kirchhoff model for the motion of an axisymmetric underwater vehicle, and we numerically study dissipation induced instability of KAM stable relative equilibria for this system.