



Banff International Research Station

for Mathematical Innovation and Discovery

Hochschild Cohomology of Algebras: Structure and Applications September 2–7, 2007

MEALS

*Breakfast (Buffet): 7:00–9:00 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SUNDAY - MONDAY SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–10:00	Nicole Snashall, <i>The Hochschild cohomology ring modulo nilpotence</i>
10:00–10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45–11:45	Srikanth Iyengar, <i>Gorenstein algebras and Hochschild cohomology</i>
11:45–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–14:15	Group Photo; meet on the front steps of Corbett Hall
14:15–15:15	Andrei Căldăraru, <i>Non-flat base change and the orbifold HKR isomorphism</i>
15:15–16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:00–17:00	Wendy Lowen, <i>Hochschild cohomology of abelian categories</i>
17:15–18:15	Travis Schedler, <i>Calabi-Yau Frobenius algebras, stable Hochschild cohomology, and pre-projective algebras</i>
18:15–19:30	Dinner

Tuesday, Wednesday, Thursday, Friday

Schedule to be announced.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **



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MONDAY ABSTRACTS

Speaker: **Andrei Căldăraru** (University of Wisconsin, Madison)

Title: *Non-flat base change and the orbifold HKR isomorphism*

Abstract: I shall discuss and prove a base change theorem for derived categories of smooth schemes, in the absence of flatness assumptions. As an application I shall present a Hochschild-Kostant-Rosenberg isomorphism for smooth, global quotient orbifolds.

Speaker: **Srikanth Iyengar** (University of Nebraska, Lincoln)

Title: *Gorenstein algebras and Hochschild cohomology*

Abstract: A classical result of Hochschild, Kostant, and Rosenberg characterizes smoothness of commutative algebras essentially of finite type over a field in terms of its Hochschild cohomology. I will discuss a similar characterization of the Gorenstein property. This is joint work with L. L. Avramov.

Speaker: **Wendy Lowen** (Vrije Universiteit Brussel/Université Denis Diderot Paris 7)

Title: *Hochschild cohomology of abelian categories*

Abstract: In contrast to the well known connection between Hochschild cohomology and deformation theory of associative algebras, Hochschild cohomology of schemes is harder to interpret in terms of deformations. We explain how it describes the deformation theory of suitable abelian categories over the scheme, like the categories of (quasi-coherent) sheaves, and we generalize both Hochschild cohomology and deformation theory to arbitrary abelian categories. There is a characteristic morphism from the Hochschild cohomology of an abelian category into the graded centre of its derived category. This morphism encodes the obstructions to deforming single objects of the (abelian or derived) category, and describes which part of the (enhanced) derived category is deformable as a differential graded category.

Speaker: **Travis Schedler** (University of Chicago)

Title: *Calabi-Yau Frobenius algebras, stable Hochschild cohomology, and preprojective algebras*

Abstract: We will define and study the notion of Calabi-Yau Frobenius algebras over arbitrary base commutative rings k (especially the integers). This includes preprojective algebras of ADE quivers and finite group algebras. Such algebras have Calabi-Yau stable module categories and have a duality between “stable” Hochschild cohomology and homology—concepts which we define for any Frobenius algebra and interpret using the (unbounded) derived category of k -modules. We show that the stable Hochschild cohomology of “periodic” CY Frobenius algebras has a BV Frobenius algebra structure, which is closely related to the BV string topology algebra for compact spherical manifolds. We show that this includes the preprojective algebras above and group algebras for finite groups that act freely on a sphere. As a consequence, we also give a new explanation why any symmetric algebra has a BV algebra structure on ordinary Hochschild cohomology. If time permits, we will also compare the Dynkin preprojective algebra results with the non-Dynkin case (which is usual Calabi-Yau, and infinite-dimensional).

Speaker: **Nicole Snashall** (University of Leicester)

Title: *The Hochschild cohomology ring modulo nilpotence*

Abstract: For a finite-dimensional algebra Λ , it was conjectured by Snashall and Solberg that the Hochschild cohomology ring of Λ modulo nilpotence is itself a finitely generated algebra (Proc. LMS 2004). This talk describes the current position of this conjecture and its connection to support varieties of modules. Reference will also be made as to whether or not it is known that the Hochschild cohomology ring itself is finitely generated in certain cases, as this plays an important role in support varieties for self-injective algebras.

In particular, I will give an overview of what is known for finite-dimensional self-injective algebras, before discussing the cases where Λ is a monomial algebra (Green, Snashall, Solberg, J. Alg. and Appl. 2006) or is in a class of special biserial algebras which arise from the representation theory of $U_q(\mathfrak{sl}_2)$ (Erdmann, Snashall, Taillefer).



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TUESDAY - FRIDAY SCHEDULE

Tuesday

7:00-9:00	Breakfast
9:00-10:00	Thorsten Holm, <i>Bilinear forms, Hochschild (co)homology and invariants of derived module categories</i>
10:00-10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45-11:45	Maria Julia Redondo, <i>Hochschild cohomology via incidence algebras</i>
12:00-12:30	Marco Farinati, <i>The cohomology of monogenic extensions in the non-commutative setting</i>
12:30-14:30	Lunch break
14:30-15:30	Amnon Neeman, <i>Brown representability via Rosicky</i>
15:30-16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:00-16:50	Emily Burgunder, <i>Leibniz homology and Kontsevich's graph complexes</i>
17:00-17:30	Petter Andreas Bergh, <i>Hochschild (co)homology of quantum complete intersections</i>
17:30-19:30	Dinner

Wednesday

7:00-8:30	Breakfast
8:30-9:30	Hubert Flenner, <i>Hochschild cohomology of singular spaces</i>
9:45-10:45	Joseph Lipman, <i>Hochschild homology, the fundamental class of a scheme-morphism, and residues</i>
10:45-11:15	Coffee Break, 2nd floor lounge, Corbett Hall
11:15-12:15	Amnon Yekutieli, <i>Twisted Deformation Quantization of Algebraic Varieties</i>
12:15-1:30	Lunch
Afternoon	No lectures
17:30-19:30	Dinner

Thursday

7:00-9:00	Breakfast
9:00-10:00	Henning Krause, <i>Localising subcategories of the stable module category of a finite group</i>
10:00-10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45-11:30	Andrea Solotar, <i>Representations of Yang-Mills algebras</i>
11:45-12:30	Mariano Suarez-Alvarez, <i>Applications of the change-of-rings spectral sequence to the computation of Hochschild cohomology</i>
12:30-14:30	Lunch break
14:30-15:30	Claude Cibils, <i>The Intrinsic Fundamental Group of a Linear Category</i>
15:30-16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:00-17:00	Silvia Montarini, <i>Finite dimensional representations of symplectic reflection algebras associated with wreath products</i>
17:15-17:45	James Zhang, <i>Twisted Hochschild (co)homology for Hopf algebras</i>
18:00-18:30	Ching-Hwa Eu, <i>Hochschild cohomology of preprojective algebras of ADE quivers</i>
18:30-19:30	Dinner



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ABSTRACTS

Speaker: **Petter Andreas Bergh** (NTNU)

Title: *Hochschild (co)homology of quantum complete intersections*

Abstract: This is joint work with Karin Erdmann. We construct a minimal projective bimodule resolution for finite dimensional quantum complete intersections of codimension 2. Then we use this resolution to compute the Hochschild homology and cohomology for such an algebra. In particular, we show that when the commutator element is not a root of unity, then the cohomology vanishes in high degrees, while the homology is always nonzero. Thus these algebras provide further counterexamples to “Happel’s question”, a question for which the first counterexample was given by Buchweitz, Green, Madsen and Solberg. On the other hand, the homology of the quantum complete intersections behave in accordance with Han’s conjecture, i.e. the homology version of Happel’s question.

Speaker: **Emily Burgunder** (Université de Montpellier II)

Title: *Leibniz homology and Kontsevich’s graph complexes*

Abstract: The homology of the Lie algebra of matrices $gl(A)$ over an associative algebra A can be computed thanks to the cyclic homology of A :

$$H(gl(A)) = S(HC(A))$$

This theorem is known as the Loday-Quillen-Tsygan theorem. Another well-known theorem in homology is due to Kontsevich which says that the homology of the symplectic Lie algebra $K[p_1, \dots, p_n, q_1, \dots, q_n]$ can be explicated thanks to the homology of a certain “graph complex” G :

$$H(K[p_1, \dots, p_n, q_1, \dots, q_n]) = S(H(G))$$

These two theorems are, in fact, examples of a more general theorem in the operadic setting, that we will present.

If we replace the Lie homology by the Leibniz homology, then the cyclic homology has to be replaced by the Hochschild homology (Cuvier-Loday theorem). We show that, in Kontsevich case, there exists a “nonsymmetric graph complex” which computes the Leibniz homology of the symplectic Lie algebra.

Speaker: **Claude Cibils** (Université de Montpellier II)

Title: *The Intrinsic Fundamental Group of a Linear Category*

Abstract: Joint work with Maria Julia Redondo and Andrea Solotar.

The main purpose is to provide a positive answer to the question of the existence of an intrinsic and canonical fundamental group associated to a linear category. The fundamental group we introduce takes into account the linear structure of the category, it differs from the fundamental group of the underlying category obtained as the classifying space of its nerve (see for instance G. Segal [1968] or D. Quillen [1973]).

We provide an intrinsic definition of the fundamental group as the automorphism group of the fibre functor on Galois coverings. We prove that this group is isomorphic to the inverse limit of the Galois groups associated to Galois coverings. Moreover, the graduation deduced from a Galois covering enables

us to describe in a conceptual way the canonical monomorphism from its automorphism group to the first Hochschild-Mitchell cohomology.

The fundamental group that we define is intrinsic in the sense that it does not depend on the presentation of the linear category by generators and relations. In case a universal covering exists, we obtain that the fundamental groups constructed by R. Martínez-Villa and J.A. de la Peña depending on a presentation of the category by a quiver and relations are in fact quotients of the intrinsic fundamental group that we introduce.

If a universal covering exists, the fundamental group that we define is isomorphic to its automorphism group. Otherwise we show that the fundamental group is isomorphic to the inverse limit of the automorphisms groups of the Galois coverings of B . In case each connected component of the category of the Galois coverings admits an initial object – in other words if “locally” universal coverings exist – the intrinsic group that we define is isomorphic to the direct product of the corresponding automorphism groups.

The methods we use are inspired from the topological case as presented for instance in R. Douady and A. Douady’s book. They are closely related to the way the fundamental group is considered in algebraic geometry after A. Grothendieck and C. Chevalley.

This work is very much indebted to the pioneer work of P. Le Meur [2006].

Speaker: Ching-Hwa Eu (MIT)

Title: *Hochschild cohomology of preprojective algebras of ADE quivers*

Abstract: Preprojective algebras of ADE quivers are Calabi-Yau and Frobenius. We use these properties to compute the structure of the Hochschild cohomology and its product.

Speaker: Marco Farinati (Universidad de Buenos Aires)

Title: *The cohomology of monogenic extensions in the noncommutative setting*

Abstract: We extend the notion of monogenic extension to the noncommutative setting, and we study the Hochschild cohomology ring of such an extension. As an application we complete the computation of the cohomology ring of the rank one Hopf algebras begun in [S. M. Burciu and S. J. Witherspoon, Hochschild cohomology of smash products and rank one Hopf algebras].

Speaker: Hubert Flenner (University of Bochum)

Title: *Hochschild cohomology of singular spaces*

Abstract: This is a report on joint work with R.O.Buchweitz.

In analogy with the cotangent complex we introduce the so called (derived) Hochschild complex of a morphism of analytic spaces or schemes; the Hochschild cohomology and homology groups are then the Ext and Tor groups of that complex. We prove that these objects are well defined, extend the known cases, and have the expected functorial and homological properties such as graded commutativity of Hochschild cohomology and existence of the characteristic homomorphism from Hochschild cohomology to the (graded) centre of the derived category.

We further generalize the HKR-decomposition theorem to Hochschild (co-)homology of arbitrary morphisms between complex spaces or schemes over a field of characteristic zero. To be precise, we show that for each such morphism $X \rightarrow Y$, the Hochschild complex decomposes naturally in the derived category $D(X)$ into $\bigoplus_{p \geq 0} \mathbb{S}^p(\mathbb{L}_{X/Y}[1])$, the direct sum of the derived symmetric powers of the shifted cotangent complex, a result due to Quillen in the affine case. The proof shows that the decomposition is given explicitly and naturally by the *universal Atiyah-Chern character*, the exponential of the universal Atiyah class. We also give further applications, in particular to the semiregularity map.

Speaker: Thorsten Holm (Universität Magdeburg)

Title: *Bilinear forms, Hochschild (co-)homology and invariants of derived module categories*

Abstract: Let A be a symmetric algebra over a perfect field k of positive characteristic p . For such algebras, B. Külshammer introduced, for any n , spaces $T_n(A)$ as those elements of A whose p^n -th power lies in the

commutator subspace $K(A)$. He then considered the orthogonal spaces with respect to the symmetrizing bilinear form on the symmetric algebra A . These $T_n(A)^\perp$ are ideals of the center of A , i.e. of the degree 0 Hochschild cohomology of A .

It has been shown by A. Zimmermann that the sequence of these ideals is invariant under derived equivalences of the symmetric k -algebras.

In the talk we first briefly discuss Zimmermann's results and then explain how to extend this theory to arbitrary finite-dimensional, not necessarily symmetric, k -algebras (joint work with C. Bessenrodt and A. Zimmermann). The way to achieve this is by passing to the trivial extension algebras. In this way we obtain new invariants of the derived module categories of finite-dimensional k -algebras. This can be seen as an extension of the well-known fact that the degree 0 Hochschild homology $A/K(A)$ is invariant under derived equivalence.

We also present recent applications of the above results, e.g. to blocks of group algebras (joint work with A. Zimmermann).

Speaker: **Henning Krause** (Universität Paderborn)

Title: *Localising subcategories of the stable module category of a finite group*

Abstract: This is a report on recent joint work with Srikanth Iyengar and Dave Benson. We classify the localising subcategories of the stable module category for a finite group. This enables us to prove the telescope conjecture in this context, as well as give a new proof of the tensor product theorem for support varieties.

In my talk I explain the history of this classification problem as well as the strategy of the proof. The challenge is basically to make proper use of the group cohomology ring which acts as a ring of cohomological operators. Thus we reduce this classification to a problem from commutative algebra. The main tools are support varieties and local cohomology. Then we use a similar classification of localising subcategories for the derived category of a commutative noetherian ring, which Neeman obtained some 15 years ago.

Speaker: **Joseph Lipman** (Purdue University)

Title: *Hochschild homology, the fundamental class of a scheme-morphism, and residues*

Abstract: Let S be a noetherian scheme, $g : X \rightarrow Y$ a flat finite-type separated S -morphism, $\delta : X \rightarrow X \times_S X$ and $\gamma : Y \rightarrow Y \times_S Y$ the diagonal maps. We define a natural functorial map $\mathbf{L}\delta^*\delta_*g^* \rightarrow g^!\mathbf{L}\gamma^*\gamma_*$, the “relative fundamental class of g ” (where $g^!$ is the twisted inverse image functor from Grothendieck duality). This can be interpreted as an orientation in a bivariant theory involving Hochschild complexes. It globalizes interesting commutative-algebra maps, like residues (which can also be described via Hochschild homology), and traces of differential forms. The talk will be about this theory, which, though at least 25 years old, still needs to be properly exposed.

Speaker: **Silvia Montarani** (Massachusetts Institute of Technology)

Title: *Finite dimensional representations of symplectic reflection algebras associated with wreath products*

Abstract: Symplectic reflection algebras were introduced by Etingof and Ginzburg. They arise from the action of a finite group G of automorphisms on a symplectic vector space V , and are a multi-parameter deformation of the algebra $S(V) \rtimes G$, smash product of G with the symmetric algebra of V . A series of interesting examples is provided by the wreath product symplectic reflection algebras, when G is the semidirect product of the symmetric group S_n of rank n with G'^n , where G' is a finite subgroup of $SL(2, C)$.

In this talk we will explain how to produce finite dimensional representations of these algebras. Our method is deformation theoretic and uses some properties of the Hochschild cohomology for this kind of algebras.

Time permitting, we will illustrate how Wee Liang Gan was able to recover the same representations by defining “reflection functors” between the categories of modules over wreath product symplectic reflection algebras corresponding to different values of the deformation parameters.

Speaker: **Amnon Neeman** (Australian National University)

Title: *Brown representability via Rosicky*

Abstract: For some years we have known that well generated triangulated categories satisfy Brown representability; there are three proofs in the literature. The corresponding statement for the dual has stumped us all; we had no idea how to proceed. Then came a remarkable result of Rosicky's.

Speaker: **María Julia Redondo** (Universidad Nacional del Sur)

Title: *Hochschild cohomology via incidence algebras*

Abstract: Let A be an associative, finite dimensional algebra over an algebraically closed field k . It is well known that if A is basic then there exists a unique finite quiver Q and a surjective morphism of k -algebras $\nu : kQ \rightarrow A$, which is not unique in general, with $I_\nu = \text{Ker } \nu$ admissible. The pair (Q, I_ν) is called a *presentation* of A . Given a presentation (Q, I) of A we associate an incidence algebra $A(\Sigma_\nu)$ and study the connection between their Hochschild cohomology groups. If the chosen presentation is homotopy coherent we define a morphism between the complexes computing these cohomology groups, which induces morphisms $\text{HH}(\Phi^n) : \text{HH}^n(A(\Sigma_\nu)) \rightarrow \text{HH}^n(A)$. Finally we find conditions for these morphisms to be injective.

Speaker: **Andrea Soltar** (Universidad de Buenos Aires)

Title: *Representations of Yang-Mills algebras*

Abstract: Joint work with Estanislao Herscovich.

Given $n \in \mathbb{N}$ and a field k , let $\mathfrak{f}(n)$ be the k -free Lie algebra with n generators x_1, \dots, x_n . Consider the k -Lie algebra

$$\mathfrak{ym}(n) := \mathfrak{f}(n)/\langle \sum_{i=1}^n [x_i, [x_i, x_j]] : 1 \leq j \leq n \rangle,$$

which has been called the **Yang-Mills algebra with n generators** (cf. [CD1] and [CD2]). It is a \mathbb{N} -graded Lie algebra, locally finite dimensional.

We denote $\text{YM}(n)$ its enveloping algebra $\mathcal{U}(\mathfrak{ym}(n))$. Also,

$$\text{YM}(n) \simeq TV(n)/\langle \sum_{i=1}^n [x_i, [x_i, x_j]] : 1 \leq j \leq n \rangle,$$

where $V(n)$ is the k -vector space with basis $\{x_1, \dots, x_n\}$. As a consequence $\text{YM}(n)$ is an homogeneous cubic algebra. As it has been noticed by Connes and Dubois-Violette, $\text{YM}(n)$ is 3-Koszul. They also have computed some of the Hochschild cohomology k -vector spaces of this algebra.

However, $\text{YM}(n)$ is not easy to handle: while for $n = 2$, $\mathfrak{ym}(2)$ is isomorphic to the Heisenberg algebra with generators x, y, z subject to relations $[x, y] = z, [x, z] = [y, z] = 0$, and so $\text{YM}(n)$ is isomorphic to the down-up algebra $A(2, -1, 0)$ (see [BR1], [BR2] and [CM]), for $n > 2$ the associative algebra $\text{YM}(n)$ is non-noetherian. So, the representation theory of $\text{YM}(n)$ for $n > 2$ is highly non-trivial.

Our main result is then, given $n > 2$, to find families of representations of $\text{YM}(n)$ big enough to separate points of the algebra. We manage to do so by showing that some well-known algebras, such as all the Weyl algebras and enveloping algebras of nilpotent finite-dimensional Lie algebras are quotients of $\text{YM}(n)$. A mix of both methods allows us to describe several families of representations of $\text{YM}(n)$, including infinite dimensional ones—via the Weyl algebras—and also finite dimensional representations of $\text{YM}(n)$. The tools we use include A_∞ -algebras and results of Bavula and Bekkert [BB] on representations of generalized Weyl algebras.

The interest on the representations of this family of algebras is mainly motivated by its physical applications, related to classical field theory and also to the study of D -branes [Ne].

References:

[BB] Bavula, V.; Bekkert, V. Indecomposable representations of generalized Weyl algebras. *Comm. Algebra* 28, (2000), no. 11, pp. 5067–5100.

[BR1] Benkart, G., Roby, T. Down-up algebras. *J. Algebra* 209, (1998), no. 1, pp. 305–344.

[BR2] Benkart, G., Roby, T. Addendum: “Down-up algebras”. *J. Algebra* 213, (1999), no. 1, p. 378.

[CM] Carvalho, P.; Musson, I. Down-up algebras and their representation theory. *J. Algebra* 228, (2000), no. 1, pp. 286–310.

[CD1] Connes, A.; Dubois-Violette, M. Yang-Mills Algebra. *Lett. Math. Phys.* 61, (2002), no. 2, pp. 149–158.

[CD2] Connes, A.; Dubois-Violette, M. Yang-Mills and some related algebras. To appear in *Rigorous Quantum Field Theory*. math-ph/0411062v2.

[Ne] Nekrasov, N. Lectures on open strings and noncommutative gauge fields. Unity from duality: gravity, gauge theory and strings (Les Houches, 2001), pp. 477–495, NATO Adv. Study Inst., EDP Sci., Les Ulis, 2003.

Speaker: **Mariano Suárez-Alvarez** (Universidad de Buenos Aires)

Title: *Applications of the change-of-rings spectral sequence to the computation of Hochschild cohomology*

Abstract: We consider the change-of-rings spectral sequence as it applies to Hochschild cohomology, obtaining a description of the differentials on the first page which relates it to the multiplicative structure on cohomology. Using this information, we are able to completely describe the cohomology structure of monogenic algebras as well as some information on the structure of the cohomology in more general situations.

We also show how to use the spectral sequence to reprove and generalize results of M. Auslander et al. about homological epimorphisms. We derive from this a rather general version of the long exact sequence due to D. Happel for a one-point (co)-extension of a finite dimensional algebra and show how it can be put to use in concrete examples.

Speaker: **Amnon Yekutieli** (Ben Gurion University)

Title: *Twisted Deformation Quantization of Algebraic Varieties*

Abstract: Let X be a smooth algebraic variety over a field of characteristic 0, endowed with a Poisson bracket. A quantization of this Poisson bracket is a formal associative deformation of the structure sheaf \mathcal{O}_X , which realizes the Poisson bracket as its first order commutator. More generally one can consider Poisson deformations of \mathcal{O}_X and their quantizations.

I will explain what these deformations are. Then I'll state a theorem which says that under certain cohomological conditions on X , there is a canonical quantization map (up to gauge equivalence). This is an algebro-geometric analogue of the celebrated result of Kontsevich (which talks about differentiable manifolds).

It appears that in general, without these cohomological conditions, the quantization will not be a sheaf of algebras, but rather a stack of algebroids, otherwise called a twisted associative deformation of \mathcal{O}_X .

In the second half of the talk I'll talk about twisted deformations and twisted quantization, finishing with a conjecture.

The work is joint with F. Leitner (BGU).

Speaker: **James Zhang** (University of Washington)

Title: *Twisted Hochschild (co)homology for Hopf algebras*

Abstract: The Hochschild homology and cohomology groups with coefficients in a suitably twisted free bimodule are shown to be non-zero in the top dimension d , when A is an Artin-Schelter regular noetherian Hopf algebra of global dimension d . (Twisted) Poincaré duality holds in this setting, as is deduced from a theorem of Van den Bergh.

This is a joint work with K. Brown.