



Banff International Research Station

for Mathematical Innovation and Discovery

Topological and Geometric Rigidity

July 30 - August 3, 2007

MEALS

*Breakfast (Buffet): 7:00–9:00 am, Donald Cameron Hall, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Donald Cameron Hall, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Donald Cameron Hall, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations.

SCHEDULE

Sunday

4:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

5:30–7:30 Buffet Dinner, Donald Cameron Hall

8:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)

Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–9:00 Breakfast

9:00–9:10 Introduction and welcome to BIRS by BIRS Station Manager, Max Bell 159

9:10–9:45 Shmuel Weinberger, *Opening remarks*

10:00–10:45 Lior Silberman, *Random groups, expander graphs and fixed points*

10:45–11:15 Coffee Break, 2nd floor lounge, Corbett Hall

11:15–12:00 Peter Linnell, *L^2 -Betti numbers and the Atiyah conjecture*

12:00–1:30 Lunch

1:30–2:15 Nicolas Monod, *Superrigidity and $Cat(0)$ spaces*

2:30–3:15 Jim Davis, *Remarks on the Farrell-Jones Conjecture*

3:15–3:45 Coffee Break, 2nd floor lounge, Corbett Hall

3:45–4:30 Sylvain Cappell, *Characteristic classes, monodromy and Hodge theory*

4:45–5:30 Andrew Ranicki, *Codimension 1 splitting and the algebraic K -theory of generalized free products*

5:30–7:30 Dinner

Tuesday

- 7:00–8:45 Breakfast
8:45–9:30 Nigel Higson, *The C^* -algebra assembly map for groups that act on, or embed in, Hilbert space*
9:45–10:30 David Rosenthal, *On the K -theory of groups with finite asymptotic dimension*
10:30–11:00 Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45 Mladen Bestvina, *Compactifications of Teichmüller space and outer space*
11:45–1:30 Lunch
1:30–2:15 Lizhen Ji, *Novikov conjectures for linear and related groups*
2:30–3:15 Igor Mineyev, *The symmetric join functor and metric structures on hyperbolic groups*
3:15–3:45 Photo - steps outside Corbett Hall – followed by Coffee Break, 2nd floor lounge
3:45–4:30 Holger Reich, *Algebraic K -theory and hyperbolic groups*
4:45–5:30 Arthur Bartels, *Topological rigidity for aspherical manifolds with word-hyperbolic fundamental groups*
5:30–7:30 Dinner

Wednesday

- 7:00–8:45 Breakfast
8:45–9:30 John Roe, *Analytic structure sets*
9:40–10:25 Jonathan Block, *Searching for Novikov*
10:25–10:45 Coffee Break, 2nd floor lounge, Corbett Hall
10:45–11:30 David Fisher, *Quasi-isometric embeddings of symmetric spaces*
11:40–12:25 Roman Sauer, *A new tool for computing L^2 -Betti numbers of groups*

Thursday

- 7:00–8:45 Breakfast
8:45–9:30 Igor Belegradek, *The craft of warped products*
9:45–10:30 Karin Melnick, *An embedding theorem for automorphism groups of Cartan geometries*
10:30–11:00 Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45 Frank Quinn, *K -theory of virtually cyclic groups*
11:45–1:30 Lunch
1:30–2:15 Qayum Khan, *Splitting and fibering 5-manifolds over the circle*
2:30–3:15 Alexander Dranishnikov, *Asymptotic dimension of Coxeter groups*
3:15–3:45 Coffee Break, 2nd floor lounge, Corbett Hall
3:45–4:30 Steve Ferry, *Volume growth, de Rham cohomology, and the Higson compactification*
4:45–5:30 Stratos Prassidis, *Detecting linear groups*
5:30–7:30 Dinner

Friday

- 7:00–9:00 Breakfast
11:30–1:30 Lunch

Checkout by 12 noon.



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ABSTRACTS

Speaker: **Arthur Bartels** (Universität Münster)

Title: *Topological rigidity for aspherical manifolds with word-hyperbolic fundamental groups*

Abstract: We prove the L -theoretic version of the Farrell-Jones conjecture for word-hyperbolic groups. As an application we obtain topological rigidity for closed aspherical manifolds with word-hyperbolic fundamental group if the dimension is at least 5, i.e., the Borel conjecture holds in this case.

This is joint work with Wolfgang Lück.

Speaker: **Igor Belegradek** (Georgia Tech)

Title: *The craft of warped products*

Abstract: (Multiple and iterated) warped products is a major source of examples in Riemannian geometry, and replicating most warped product curvature computations is just a tedious exercise for a bright calculus student. On the other hand, coming up with new warped product constructions (such as the ones of Farrell-Jones or Gromov-Thurston) may require specialized intuition and laborious experimentation. This talk aims to demystify the process. At the end we shall work out a warped product computation that yields a Mostow-type rigidity for hyperplane complements in finite volume complex hyperbolic manifolds.

Speaker: **Mladen Bestvina** (University of Utah)

Title: *Compactifications of Teichmüller space and outer space*

Abstract: I'll review the constructions of Thurston's boundary of Teichmüller space and Culler-Morgan's boundary of Outer space. I'll then go through a list of desirable properties of boundaries and discuss whether they hold or fail (or whether they are unknown) for the two boundaries above.

Speaker: **Jonathan Block** (University of Pennsylvania)

Title: *Searching for Novikov*

Abstract: We discuss some examples of Novikov type theorems and Novikov type conjectures in complex geometry.

Speaker: **Sylvain Cappell** (Courant Institute - NYU)

Title: *Characteristic classes, monodromy and Hodge theory*

Abstract: We compare the roles of Atiyah-Meyer formulae to measure the contribution of monodromy in topological and Hodge-theoretic settings. This involves various new characteristic classes which have analogies to those entering into topological rigidity.

Speaker: **Jim Davis** (Indiana University)

Title: *Remarks on the Farrell-Jones Conjecture*

Abstract: The talk will attempt to survey the Farrell-Jones Conjecture, the Novikov Conjecture, codimension one splitting phenomena, and connections with topological and equivariant rigidity, using crystallographic groups as a model case.

Speaker: **Alexander Dranishnikov** (University of Florida)

Title: *Asymptotic dimension of Coxeter groups*

Abstract: Asymptotic dimension of a right-angled Coxeter group does not exceed the dimension of the Davis complex. It would be nice to extend this result to all Coxeter groups.

Speaker: **Steve Ferry** (Rutgers University)

Title: *Volume growth, de Rham cohomology, and the Higson compactification*

Abstract: We construct a variant of (real) DeRham cohomology and apply it to prove that the integral cohomology of the Higson compactification of R^n has uncountably generated n th integral cohomology. Earlier work of Dranishnikov-Ferry-Weinberger shows that the Novikov Conjecture follows from the mod p acyclicity of the Higson compactification. Our construction suggests that the integral cohomology of the Higson compactification may have a real vector space structure.

Speaker: **David Fisher** (Indiana University)

Title: *Quasi-isometric embeddings of symmetric spaces*

Abstract: Let X and Y be irreducible symmetric spaces of higher rank. The main point of this talk is to describe some rather surprising examples of quasi-isometric embeddings of certain X 's into certain Y 's. These embeddings occur in cases where X does not isometrically embed in Y . I will also discuss some partial results limiting possible behavior of quasi-isometric embeddings and indicate some open questions. This is joint work with Kevin Whyte.

Speaker: **Nigel Higson** (Penn State University)

Title: *The C^* -algebra assembly map for groups that act on, or embed in, Hilbert space.*

Abstract: In this survey talk I shall describe elements of the proof of the Baum-Connes conjecture for groups that admit a proper and isometric action on Hilbert space, as well as the additional steps that prove the Novikov conjecture for groups that embed uniformly into Hilbert space.

Speaker: **Lizhen Ji** (University of Michigan)

Title: *Novikov conjectures for linear and related groups*

Abstract: An important problem in topological rigidity is the Borel conjecture. A stable version of the Borel conjecture is implied by the integral version of Novikov conjecture in L -theory. In this talk, I will discuss the integral Novikov conjecture in several theories for some natural classes of linear groups and other related groups.

Speaker: **Qayum Khan** (Vanderbilt University)

Title: *Splitting and fibering 5-manifolds over the circle*

Abstract: We explore an extension of the Cappell-Weinberger theorem for splitting 5-manifolds along two-sided 4-submanifolds. The emphasis will be on the application to fibering 5-manifolds mapping to the circle. These new examples have fibers F with the following technical properties: (1) the algebraic K - and L -theory obstructions to fibering vanish and are sufficient, (2) the second homology of the fundamental group G of F does not vanish, and (3) the group G has exponential growth.

Speaker: **Peter Linnell** (Virginia Tech)

Title: *L^2 -Betti numbers and the Atiyah conjecture*

Abstract: I will briefly describe some results on the Atiyah conjecture and its relation to L^2 -Betti numbers and embeddings of group rings into Artinian rings. Then I will sketch some proofs. There are various versions of the Atiyah conjecture; here is one of them. Let $\mathcal{N}(G)$ denote the group von Neumann algebra of the group G . There is a well-defined dimension $\dim_{\mathcal{N}(G)}$ associated to every $\mathcal{N}(G)$ -module, which is either a non-negative real number or infinity. Suppose G is a group for which the finite subgroups have bounded order and d is the least common multiple of the finite subgroups. Then one version of the Atiyah conjecture states that $d(\dim_{\mathcal{N}(G)} M)$ is either an integer or infinity for every $\mathcal{N}(G)$ -module M that is

induced up from $\mathbb{C}G$. This implies that $\mathbb{C}G$ is embeddable in a semisimple Artinian ring. The case G is a congruence subgroup (groups of matrices congruent to the identity matrix modulo some prime) will especially be considered.

Speaker: **Karin Melnick** (Yale University)

Title: *An embedding theorem for automorphism groups of Cartan geometries*

Abstract: Cartan geometries are a framework in which to study differential-geometric structures that is well-adapted to questions about homogeneity. I will present a theorem that says roughly that, for H a Lie subgroup of the automorphism group of a Cartan geometry, and $S < H$ preserving a finite measure, the Zariski closure of the adjoint representation of S on the Lie algebra of H embeds in the adjoint representation of the structure group of the Cartan geometry. Consequences include tight bounds on the real rank and nilpotence degree of automorphism groups. For the class of parabolic Cartan geometries, which includes conformal, CR, and projective structures, the theorem can be used to show that when a group of automorphisms attains the maximal rank, then the Cartan geometry is complete and locally homogeneous.

This is joint work with Uri Bader and Charles Frances.

Speaker: **Igor Mineyev** (University of Illinois at Urbana-Champaign)

Title: *The symmetric join functor and metric structures on hyperbolic groups*

Abstract: This talk will concern (a) refined geometric group theory and (b) metric structures of hyperbolic groups and their boundaries. Both are stated as objectives of this workshop, and both are related to rigidity questions.

The symmetric join of an arbitrary metric space X is the formal union of topological intervals (called lines) connecting each ordered pair of points in X . The symmetric join of X happens to admit a metric extending the metric on X ; with respect to this metric each line in the symmetric join of X is a geodesic path.

Each hyperbolic group admits a nice metric and its ideal boundary admits a conformal structure, both similar to the ones in the $\text{CAT}(-1)$ case. These metrics allow defining the notion of hyperbolic dimension for hyperbolic groups, and this in turn allows posing rigidity questions for hyperbolic groups. The symmetric join functor behaves especially nicely when applied to hyperbolic groups and complexes.

We will discuss several open problems in geometry, topology, and geometric group theory which are related to the above two topics.

Speaker: **Nicolas Monod** (Université de Genève)

Title: *Superrigidity and $\text{CAT}(0)$ spaces*

Abstract: We prove a superrigidity theorem for actions of irreducible lattices on $\text{CAT}(0)$ spaces. Since such spaces generalize for instance symmetric spaces and buildings, one obtains in particular a new and self-contained proof of Margulis' superrigidity theorem for uniform irreducible lattices in non-simple groups. As an application, there is a "arithmeticity/non-linearity" alternative.

The proofs rely on simple geometric arguments, including a splitting theorem which can be viewed as an infinite-dimensional (and singular) generalization of the Lawson-Yau/Gromoll-Wolf theorem.

Speaker: **Frank Quinn** (VPI)

Title: *K-theory of virtually cyclic groups* Abstract: A review of recent results that K-theory of virtually cyclic groups assembles from finite subgroups and infinite virtually cyclic groups whose finite quotient is a p -group that acts trivially on the kernel. In particular K-theory of the infinite dihedral group assembles from cyclic subgroups.

Speaker: **Stratos Prassidis** (Canisius College)

Title: *Detecting linear groups*

Abstract: The Farrell-Jones Isomorphism Conjecture for the pseudoisotopy spectrum is known to be true

for discrete linear groups. On the other hand, Lubotzky stated a necessary and sufficient condition for a group to be linear, namely that the group admits a filtration with nice properties. We extend his results to split exact sequences of groups. More precisely, we give conditions for a semidirect product to be linear given that the two factors are linear. The applications include certain poly-free groups. It should be noted, though, that Lubotzky's criterion does not guarantee discreteness.

This is joint work with Fred Cohen and Marston Conder.

Speaker: **Andrew Ranicki** (University of Edinburgh)

Title: *Codimension 1 splitting and the algebraic K-theory of generalized free products*

Abstract: There are three types of codimension 1 submanifolds $M \subset N$ with normal bundle ν and M, N connected:

- (A) ν trivial and M separates N ,
- (B) ν trivial but M does not separate N ,
- (C) ν nontrivial.

In each case the fundamental group $\pi_1(N)$ can be expressed in terms of $\pi_1(M)$ and $\pi_1(N - M)$. Some 35 years ago Waldhausen obtained decomposition theorems for the algebraic K -groups of $Z[\pi_1(M)]$ in terms of $Z[\pi_1(N)]$ and $Z[\pi_1(N - M)]$, assuming that $\pi_1(M)$ injects into $\pi_1(N)$. The decompositions involved exotic Nil groups, which are the obstruction groups for the codimension 1 splitting of homotopy equivalences of CW complexes and manifolds. The first part of the talk will give a brief account of Waldhausen's theorems using chain complexes (which was in fact his original method). The second part of the talk will describe a joint project with Jim Davis (Bloomington) and Qayum Khan (Vanderbilt) on the algebraic K -theory of $Z[G]$ of a group G with a surjection onto the infinite dihedral group D_∞ which features the Nil groups of each of the three types (A), (B), (C). It now turns out that some of these groups for the different types are in fact isomorphic! [Everything has an L -theory version, using the Cappell UNil groups, but there will not be time in the talk to consider L -theory.]

Speaker: **Holger Reich** (University of Dusseldorf Germany)

Title: *Algebraic K-theory and hyperbolic groups*

Abstract: The talk reports on joint work with Arthur Bartels and Wolfgang Lück. We prove the K -theoretic Farrell-Jones conjecture for hyperbolic groups and more generally for subgroups of finite direct products of hyperbolic groups.

Speaker: **John Roe** (Penn State University)

Title: *Analytic structure sets*

Abstract: This talk will discuss a C^* -analogue to the surgery exact sequence. It is joint work with Nigel Higson and Jerry Kaminker.

Speaker: **David Rosenthal** (St. Johns University)

Title: *On the K-theory of groups with finite asymptotic dimension*

Abstract: In joint work with Arthur Bartels, we prove that the assembly maps in algebraic K - and L -theory with respect to the family of finite subgroups is injective for groups G with finite asymptotic dimension that admit a finite model for the universal space for proper G -actions. The result also applies to certain groups that admit only a finite dimensional model for the universal space for proper G -actions. In particular, it applies to discrete subgroups of virtually connected Lie groups.

Speaker: **Roman Sauer** (University of Chicago)

Title: *A new tool for computing L^2 -Betti numbers of groups.*

Abstract: Gaboriau's L^2 -Betti numbers of measured equivalence relations and the introduction of L^2 -Betti numbers for finite von Neumann algebras by Connes and Shlyakhtenko are recent major developments in the theory of L^2 -Betti numbers which connected a topological invariant (L^2 -Betti numbers) to ergodic theory and operator algebras.

The definitions of Gaboriau and Connes-Shlyakhtenko are completely different. We present an alternative approach (homological algebra style) to Gaboriau's theory that fits well with the latter definition.

Using that enables us to establish a new spectral sequence for L^2 -Betti numbers of measured groupoids. The proof combines basic tools from homological algebra (Grothendieck spectral sequence, localization of categories) and ergodic theory (measure disintegration) in a new way.

We present some applications of the spectral sequence which involve no measured groupoids at first glance like: vanishing results and computations of L^2 -Betti numbers of groups, an application to the Hopf-Singer conjecture etc. This is joint work with Andreas Thom.

Speaker: **Lior Silberman** (Harvard University)

Title: *Random groups, expander graphs and fixed points*

Abstract: I will discuss joint work with Assaf Naor on the strong fixed point property of Gromov's random group constructed using a family of expander graphs. I will try to explain why the group has no uniform embedding in Hilbert space, and how our results imply that every linear image of the group is trivial.

Speaker: **Shmuel Weinberger** (University of Chicago)

Title: *Opening remarks*

Abstract: I will welcome all the participants to this conference and charge them with producing beautiful mathematics.