Banff International Research Station
for Mathematical Innovation and Discovery

The Many Strands of the Braid Groups
April 22–27, 2007

MEALS

*Breakfast (Buffet): 7:00–9:00 am, Donald Cameron Hall, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Donald Cameron Hall, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Donald Cameron Hall, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

You are welcome to schedule lectures as you see fit, as long as you adhere to the meal times (noted above), coffee break start and end times (noted below) and take into account the "welcome" on Monday morning, the Banff Centre tour on Monday afternoon and group photo on Tuesday morning.

When your schedule is finalized, please e-mail it to the BIRS Station Manager birsmgr@birs.ca by 12 noon on the Thursday before your arrival. You are also encouraged to e-mail the schedule to all of your participants at: <birs event id>-par@lists.pims.math.ca

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Donald Cameron Hall
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45 Breakast

8:45–9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:15 Opening, by Dale Rolfsen
9:15–9:45 Recollection of X.Lin and his work, by Joan Birman
10:00–10:25 H.Morton, Mutants with symmetry
10:25–11:00 Coffee, 2nd floor lounge, Corbett Hall
11:00–11:45 H.Murakami, On a generalization of the volume conjecture
12:00–12:25 G.Zhang, Concordance crosscap number of a knot

12:30–13:30 Lunch
13:45–14:45 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

14:45–15:30 P.Dehornoy, Alternating normal forms of braids
15:30–16:00 Coffee break
16:00–16:25 A.Clay, Normal subgroups of the braid groups and the Dehornoy ordering
16:45–17:30 J.Guaschi & D.Goncalves, Finite subgroups of the sphere braid groups
17:45–18:10 S.Humphries, Subgroups of braid groups generated by powers of Dehn twists

18:15–19:30 Dinner

Tuesday

7:00–9:00 Breakfast

9:00–9:45 J.Przytycki, Two–braid intersection of Hochschild and Khovanov homologies
10:00–10:25 L.Watson, Knots with identical Khovanov homology
10:25–10:50 Coffee break
10:50–11:35 D.Thurston, Combinatorial Heegard–Floer homology for knots via grid diagrams
11:45–12:30 S.Morrison, Functoriality for Khovanov homology in S^3

12:30–13:30 Lunch
14:30 Group Photo; meet on the front steps of Corbett Hall

14:45–15:30 C.Lescop, Surgery formulae for finite type invariants of rational homology 3–spheres
15:30–16:00 Coffee break
16:00–16:25 W.Menasco, A calculus for Legendrian and transversal knots
16:35–17:00 R.Fenn, Welded braids, links, their configuration spaces and other properties
17:10–17:35 H.Matsuda, A calculus on links via closed braids
17:45–18:10 J.Birman, Lorenz knots, templates and closed braids

18:15–19:30 Dinner

Wednesday

7:00–9:00 Breakfast

9:00–9:45 D.Krammer, A Garside type structure on the Torelli group
10:00–10:25 D.Margalit, Dimension of the Torelli group
10:25–10:50 Coffee break
10:50–11:35 D.Bessis, Periodic elements in spherical type Artin groups
11:45–12:30 J.Gonzalez–Meneses, A project to find a polynomial solution to the conjugacy problem in braid groups

12:30–13:30 Lunch
Free afternoon
17:30–19:30 Dinner

Thursday

7:00–9:00 Breakfast

9:00–9:45 J.Marché, On asymptotics of quantum representations of mapping class groups via skein theory
10:00–10:25 K.Kawamuro, Braid index and algebraic crossing number
10:25–11:00 Coffee break
11:00–11:45 F.Castel, Rigidity of the representations of the braid group in the mapping class group
12:00–12:25 S.Kamada, On braid presentation of knotted surfaces and the enveloping monoidal quandle

12:30–13:30 Lunch

14:45–15:30 S.Lee, Translation numbers in Garside groups
15:30–16:00 Coffee break
16:00–16:25 E.Lee, Super summit property of abelian subgroups of Garside groups
16:45–17:30 I.Marin, Generalized braid groups as Zariski–dense subgroups of GL_N
17:40–18:05 B.Wiest, The conjugacy problem in right–angled Artin groups and their subgroups

18:15–19:30 Dinner

20:00–21:30 Problem session

Friday

7:00–9:00 Breakfast

9:00–9:25 E.Kin, The ratio of the topological entropy to the volume for pseudo–Anosov braids
9:40–10:05 T.Kohno, Loop spaces of configuration spaces and link invariants
10:05–10:30 Coffee break
10:30–10:55 E.Yurasovskaya, String links and orderability
11:00–11:25 D.Rolfsen, Ordered groups and pseudo-Aosov maps

11:30–13:30 Lunch
Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms,

Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **David Bessis** (Ecole Normale Supérieure, Paris, France)
Title: *Periodic elements in spherical type Artin groups*
Abstract: In the braid group on n strings, the classification of periodic elements (elements with a central power) follows from a classical theorem of Kerekjarto. We generalize this to the other spherical type Artin groups and obtain a complete description of periodic elements, their conjugacy classes and their centralizers. A key ingredient is a categorical reformulation of a theorem by Bestvina.

Speaker: **Joan Birman** (Columbia University, New York, USA)
Title: *Lorenz knots, templates and closed braids*
Abstract: Lorenz knots were first defined in a 1983 paper that Bob Williams and I wrote. They arise as the periodic orbits in the flow associated to solutions to a particular ODE in 3-space which has since become a paradigm for chaos. They are of renewed interest right now because of work by Etienne Ghys, who proved that the identical family of knots (and their defining 'template') are as the closed orbits in the classical modular flow on the complement of the trefoil knot.

Speaker: **Fabrice Castel** (Université de Dijon, France)
Title: *Rigidity of the representations of the braid group in the mapping class group*
Abstract: In 1995, Perron and Vannier proved that the morphism from the braid group into the mapping class group of an orientable surface, that sends the generators of the braid group on Dehn twists, is injective. We show that under some restrictions on the genus, the embeddings between these two groups arise all from the embedding defined by Perron and Vannier. Using the rigidity of such embeddings, one can for instance compute the group of automorphisms of the braid group as well as the group of automorphisms of the mapping class group. The proof of the theorem is based on Nielsen-Thurston theory and of a simultaneous action of the mapping class group on itself, on the complex of curves and on the complex of subsurfaces.

Speaker: **Adam Clay** (University of British Columbia, Vancouver, Canada)
Title: *Normal subgroups of the braid groups and the Dehornoy ordering*
Abstract: The braid groups admit a left-ordering, discovered by Dehornoy, which is discrete as an ordering. I will show that normal subgroups interact with the Dehornoy ordering in such a way that "nearly all" normal subgroups of the braid groups are densely ordered with respect to this ordering. In particular, some popular normal subgroups–such as the commutator subgroup and kernels of the Burau representations–can be easily analyzed. This is joint work with Dale Rolfsen.

Speaker: **Alissa Crans** (Loyola Marymount University, Los Angeles, USA)—talk cancelled due to illness
Title: *Analogues of self-distributivity*
Abstract: This is joint work with Scott Carter, Mohamed Elhamdadi, and Masahico Saito. Self-distributive binary operations have appeared extensively in knot theory in recent years, specifically in algebraic structures called 'quandles.' A quandle is a set equipped with two binary operations satisfying axioms that

capture the essential properties of the operations of conjugation in a group. The self-distributive axioms of a quandle correspond to the third Reidemeister move in knot theory. Thus, quandles give a solution to the Yang-Baxter equation, which is an algebraic distillation of the third Reidemeister move. We formulate analogues of self-distributivity in the categories of coalgebras and Hopf algebras and use these to construct additional solutions to the Yang-Baxter equation.

Speaker: **Patrick Dehornoy** (Université de Caen, France)
Title: *Alternating normal forms of braids*
Abstract: We describe new types of normal forms for braid monoids, Artin-Tits monoids, and, more generally, all monoids in which divisibility has some convenient lattice properties ("locally Garside monoids"). We show that, in the case of braids, one of these normal forms turns out to coincide with the normal form introduced by Burckel and deduce that the latter can be computed easily. This approach leads to a new, simple description for the canonical well-order of B_n^+ in terms of that of B_{n-1}^+ which, in turn, leads to unprovability statements for certain games involving braids.

Speaker: **Roger Fenn** (University of Sussex, Brighton, GB)
Title: *Welded braids, links, their configuration spaces and other properties*
Abstract: Configuration spaces of the classical braids are well known. A configuration space for welded braids is given with a suggestion for possible invariants.

Speaker: **Daciberg Lima Goncalves** (Universidade de Sao Paulo, Brazil) & **John Guaschi** (Université de Toulouse, France)
Title: *Finite subgroups of the sphere braid groups*
Abstract: It is well known that the sphere braid groups $B_n(S^2)$ have torsion elements. Such elements were characterised by Murasugi. In this talk, we classify the finite subgroups of $B_n(S^2)$. Our work is partly motivated by the study of the generalisation of the Fadell-Neuwirth short exact sequence for pure braid groups to the 'mixed' subgroups of the full braid groups. By giving explicit constructions, we prove that for all $n \geq 3$, $B_n(S^2)$ contains subgroups isomorphic to the dicyclic groups of order $4n$ and $4(n-2)$. It follows that $B_n(S^2)$ contains two non-conjugate copies of the quaternion group of order 8 for all $n \geq 4$ even, one of which lies in the commutator subgroup of $B_n(S^2)$, the other not. Finally we classify the finite subgroups of $B_n(S^2)$: the maximal finite subgroups of $B_n(S^2)$ are either cyclic, dicyclic or binary polyhedral groups (their realisation depending on n). Two corollaries of this classification are: a) the binary tetrahedral group is a subgroup of $B_n(S^2)$ for all $n \geq 4$ even; b) if n is odd then the finite subgroups of $B_n(S^2)$ are cyclic or dicyclic.

Speaker: **Juan Gonzalez-Meneses** (University of Seville, Spain)
Title: *A project to find a polynomial solution to the conjugacy problem in braid groups*
Abstract: This is a joint work with Joan S. Birman and Volker Gebhardt. We present a project to find a polynomial solution to the conjugacy decision problem and the conjugacy search problem in braid groups, whose outline is the following. First we need to determine the geometric type of the braids involved, that is, to classify a given braid as periodic, reducible or pseudo-Anosov. In the periodic case, we give a polynomial solution by using some Garside structures of the braid groups and of Artin-Tits groups of type B. In the reducible case, one needs to find the reducing curves, and also to solve the question of finding the generators of the centralizer of a braid. In the pseudo-Anosov case, we show how one can simplify the situation by taking powers of the original braids, and reducing the problem to the conjugacy search problem for "rigid" braids. We will present our achievements, together with the open problems that remain.

Speaker: **Stephen Humphries** (Brigham Young University, Utah, USA)
Title: *Subgroups of braid groups generated by powers of Dehn twists*
Abstract: Let $F =< x_1, \ldots, x_n >$ be the free group on n generators and let $P_n =< A_{12}, \ldots, A_{n-1,n} >$ be the pure braid group with its standard (Dehn twist) generators. We identify F_n with the subgroup

$< A_{1,n+1}, \ldots, A_{n,n+1} >$ of P_{n+1}. We are interested in the related questions: (1) when is a subgroup of F_n which is generated by a set of powers of conjugates of $x_1, \ldots, x_n$, of finite index in F_n; and (2) when is a subgroup of P_n which is generated by a set of powers of conjugates of $A_{12}, \ldots, A_{n-1,n}$, of finite index in P_n? For example, we give necessary and sufficient conditions for a subgroup of P_n of the form $< A_{12}^{e_{12}}, \ldots, A_{n-1,n}^{e_{n-1,n}} >$ to have finite index in P_n. The answer to question (1) involves Schur's theory of S-rings.

Speaker: **Seiichi Kamada** (Hiroshima University, Japan)
Title: *On braid presentation of knotted surfaces and the enveloping monoidal quandle*
Abstract: We introduce a method to describe a knotted surface in 4-space by a sequence of braids, Alexander's and Markov's theorem in dimension 4. It is natural to regard such a sequence as an element of the enveloping monoidal quandle in the sense of Kamada and Matsumoto.

Speaker: **Keiko Kawamuro** (Rice University, Houston, USA)
Title: *Braid index and algebraic crossing number*
Abstract: I will discuss a conjecture that the maximal Bennequin number of a knot is realized at its minimal braid representatives.

Speaker: **Eiko Kin** (Tokyo Institute of Technology, Japan)
Title: *The ratio of the topological entropy to the volume for pseudo–Anosov braids*
Abstract: We consider two invariants of pseudo-Anosov mapping classes. One is the dilatation of pseudo–Anosov homeomorphisms and the other comes from the volume of mapping tori. Both invariants measure a kind of complexity of pseudo–Anosov mapping classes. The mapping class group on the n–punctured disk is identified with the n–braid group up to full twist braids, and it makes to sense to speak of the dilatation and the volume for pseudo–Anosov braids. We are interested in a relation of these two invariants, the dilatation and the volume. In this talk we focus on the ratio of the logarithm of the dilatation namely the (topological) entropy to the volume. We show that there is a constant $c > 0$ such that the ratio of the entropy to the volume for the pseudo–Anosov 3–braids is greater than c. We also extend this result for a family of pseudo–Anosov braids with many strands. This is a joint work with Mitsuhiko Takasawa (Tokyo Institute of Technoloty).

Speaker: **Toshitake Kohno** (University of Tokyo, Japan)
Title: *Loop spaces of configuration spaces and link invariants*
Abstract: It is know by F. Cohen and S. Gitler that the homology of the loop spaces of configuration spaces of ordered points in the Euclidean space is a graded algebra defined by infinitesimal pure braid relations. Based on this result we give a description of a link homotopy invariant as an integral of de Rham cohomology class of the loop space of a configuration space.

Speaker: **Daan Krammer** (University of Warwick, GB)
Title: *A Garside type structure on the Torelli group*
Abstract: In 1969, Garside solved the word and conjugacy problems for braid groups. We now say that he proved braid groups to be Garside groups. In 1998 another Garside structure on the braid group was discovered by Birman-Ko-Lee (BKL).

A well-known class of groups generalising braid groups are the surface mapping class groups. The Torelli group of a surface is the subgroup of the mapping class group of those elements which act trivially on the first homology $H_1(S, Z)$ of the surface.

I will present a Garside type structure on the Torelli group. It depends on the choice of a lexicographic total ordering on $H_1(S, Z)$. It is a close relative of the BKL Garside structure on the braid group.

It is not precisely a Garside structure for a number of reasons:

(1) Rather than as a group, it should be regarded as a groupoid whose object set looks a lot like a topological space;

(2) The distinguished path between two points in general has an infinite number of intermediate stops in a mild way.

Still, the most important properties of Garside groups, such as the grid property, still hold.

Speaker: **Eon-Kyung Lee** (Sejong University, Seoul, Korea)
Title: *Super summit property of abelian subgroups of Garside groups*
Abstract: Garside groups provide a lattice-theoretic generalization of braid groups and finite type Artin groups. In the talk, we show that for every abelian subgroup H of a Garside group, some conjugate $x^{-1}Hx$ consists of super summit elements. Using this property, we show that the centralizer of H is a finite index subgroup of the normalizer of H. Combining with the results on translation numbers in Garside groups, we obtain an easy proof of the algebraic flat torus theorem for Garside groups.

Speaker: **Sangjin Lee** (Konkuk University, Korea)
Title: *Garside groups and translation numbers*
Abstract: The translation number of an element in a combinatorial group is defined as the asymptotic word length of the element. The discreteness properties of translation numbers have been studied for geometric groups such as biautomatic groups and hyperbolic groups. The Garside group is a lattice-theoretic generalization of braid groups and Artin groups of finite type. In this talk, we discuss recent results on the discreteness properties of translation numbers in Garside groups, and their applications to the conjugacy problem.

Speaker: **Christine Lescop** (Université of Grenoble, France)
Title: *Surgery formulae for finite type invariants of rational homology 3-spheres*
Abstract: I wish to present four graphic surgery formulae for the degree n part Z_n of the Kontsevich-Kuperberg-Thurston universal finite type invariant of rational homology spheres. Each of these four formulae determines an alternate sum of the form $\sum_{I\subset N}(-1)^{\sharp I}Z_n(M_I)$ where N is a set of disjoint operations to be performed on a rational homology sphere M, and M_I denotes the manifold resulting from the operations in I. The first formula treats the case when N is a set of $2n$ Lagrangian-preserving surgeries (a *Lagrangian-preserving surgery* replaces a rational homology handlebody by another such without changing the linking numbers of curves in its exterior). In the second formula, N is a set of n rational surgeries on the components of a boundary link. The third formula deals with the case of $3n$ surgeries on the components of an algebraically split link. The fourth formula is for $2n$ surgeries on the components of an algebraically split link in which all Milnor triple linking numbers vanish. In the case of homology spheres, these formulae can be seen as a refinement of the Garoufalidis-Goussarov-Polyak comparison of different filtrations of the rational vector space freely generated by oriented homology spheres (up to orientation-preserving homeomorphisms).

Speaker: **Julien Marché** (Université Paris 7, France)
Title: *On asymptotics of quantum representations of mapping class groups via skein theory*
Abstract: We explain a simple proof of the asymptotic faithfulness of quantum representations of the mapping class group of a surface S. The idea is to show that in some sense, the quantum representations converge to the representation $H(S)$, where $H(S)$ is the space of regular functions on the character variety of S in $SL(2,C)$.

Speaker: **Dan Margalit** (University of Utah, USA)
Title: *Dimension of the Torelli group*
Abstract: In joint work with Mladen Bestvina and Kai-Uwe Bux, we prove that the cohomological dimension of the Torelli group for a closed surface of genus g at least 2 is equal to $3g-5$.

Speaker: **Ivan Marin** (Université Paris 7, France)
Title: *Generalized braid groups as Zariski-dense subgroups of GL_N*

Abstract: Embeddings of every (irreducible) spherical-type Artin group in some GL_N have been described in recent years. We show that these embedings have Zariski-dense image, and use this to prove group-theoretical results on Artin groups. In particular we show that these groups are residually torsion-free nilpotent, and compute their Frattini and Fitting subgroups. We also generalize a classical result of D. Long which says that normal subgroups of braid groups which are not included in the center intersect non-trivially. The density result is based on a simple interpretation of these embeddings as monodromy representations, that we shall describe if time permits.

Speaker: **Hiroshi Matsuda** (Columbia University, New York, USA)
Title: *A calculus on links via closed braids*
Abstract: We improve "Markov Theorem Without Stabilization" of Birman and Menasco.

Speaker: **William Menasco** (University at Buffalo, USA)
Title: *A calculus for Legendrian and transversal knots*
Abstract: Using an extended example of the Etnyre-Honda (2,3) cabling of the (2,3) torus knot discuss a calculus of isotopies associated with Legendrian and transversal knots in the standard contact structure of S^3 (Joint work with Douglas Lafountain, University at Buffalo).

Speaker: **Scott Morrison** (University of California, Berkeley, USA)
Title: *Functoriality for Khovanov homology in S^3*
Abstract: (Joint work with Kevin Walker.) I'll tell you what I mean by the Khovanov homology of a knot in S^3 (as opposed to the usual B^3). We can show that Khovanov homology is still functorial in this case, but it takes a bit more work beyond checking the 15 movie moves needed for functoriality in B^3.

Speaker: **Hugh Morton** (University of Liverpool, GB)
Title: *Mutants with symmetry*
Abstract: Mutants with certain extra symmetry, for example the pretzel knots $K(a_1, ..., a_k)$ with k and all a_i odd, can be shown to share many more of their Homfly satellite invariants than is the case for a general mutant. The proofs make use of representation theory of quantum $sl(N)$ modules.

Speaker: **Hitoshi Murakami** (Tokyo Institute of Technology, Japan)
Title: *On a generalization of the volume conjecture*
Abstract: The volume conjecture says that the large N limit of the N-colored Jones polynomial of a knot evaluated at the N-th root of unity would determine the volume of the knot complement. In this talk we will consider what happens if we change the evaluation.

Speaker: **Jozef Przytycki** (George Washington University, Washington DC, USA)
Title: *Two-braid intersection of Hochschild and Khovanov homologies*
Abstract: We show that Khovanov homology and Hochschild homology theories share common structure. In fact they overlap: Khovanov homology of a $(2, n)$-torus link can be interpreted as a Hochschild homology of the algebra underlining the Khovanov homology. In the classical case of Khovanov homology we prove the concrete connection. In the general case of Khovanov-Rozansky, $sl(n)$, homology and their deformations we conjecture the connection. The best framework to explore our ideas is to use a comultiplication-free version of Khovanov homology for graphs developed by L. Helme-Guizon and Y. Rong and extended here to M-reduced case, and in the case of a polygon to noncommutative algebras. In this framework we prove that for any unital algebra A the Hochschild homology of A is isomorphic to graph cohomology over A of a polygon.

Speaker: **Dale Rolfsen** (University of British Columbia, Vancouver, Canada)
Title: *Ordered groups and pseudo-Anosov maps*
Abstract: This is a report on work in progress regarding finding orderings of groups invariant under a given automorphism. One goal is to show that for a pseudo-Anosov homeomorphism of a surface, there is

an ordering of the surface group invariant under the action of the induced mapping. This would imply the bi-orderability of hyperbolic 3-manifolds which fibre over the circle.

Speaker: **Dylan Thurston** (Barnard College, Columbia University, New York, USA)
Title: *Combinatorial Heegaard-Floer homology for knots via grid diagrams*
Abstract: We give a combinatorial definition of Heegaard-Floer homology. In particular, this yields a very simple algorithm for computing the knot genus. Our method is based on grid diagrams, a representation for knots that, with restrictions on the allowed moves, also yields transverse or Legendrian knots or closed braids up to isotopy.

Speaker: **Liam Watson** (Université du Québec Montréal, Canada)
Title: *Knots with identical Khovanov homology*
Abstract: While it is well know that mutation is not detected by the Jones polynomial, it is presently unknown if mutation of knots preserves Khovanov homology. In this talk we will present a technique for producing pairs of distinct knots that cannot be distinguished by Khovanov homology. As an application, this construction may be applied to produce families of examples of mutant pairs that have identical Khovanov homology.

Speaker: **Bert Wiest** (Université de Rennes, France)
Title: *The conjugacy problem in right-angled Artin groups and their subgroups*
Abstract: We prove that the conjugacy problem in right-angled Artin groups and a large class of their subgroups can be solved in linear time. This concerns in particular all graph braid groups. Some of this talk is joint work with J.Crisp, some with J.Crisp and E.Godelle.

Speaker: **Ekaterina Yurasovskaya** (University of British Columbia, Vancouver, Canada)
Title: *String links and orderability*
Abstract: The group of homotopy classes of string links $H(k)$ has first been described by Nathan Habegger and Xiao-Song Lin in 1990 and provided the main tool to classify links up to link-homotopy. Since then $H(k)$ became an object of interest in itself. I shall discuss $H(k)$ as an example of orderable groups appearing in topology.

Speaker: **Gengyu Zhang** (Tokyo Institute of Technology, Japan)
Title: *Concordance crosscap number of a knot*
Abstract: We define the concordance crosscap number of a knot as the minimum crosscap number among all the knots concordant to the knot. The four-dimensional crosscap number is the minimum first Betti number of non-orientable surfaces smoothly embedded in 4-dimensional ball, bounding the knot. Clearly the 4-dimensional crosscap number is smaller than or equal to the concordance crosscap number. We construct two infinite sequences of knots for which the 4-dimensional one is strictly smaller than the concordance one. In particular, the knot 7_4 is one of the examples.