Banff International Research Station
for Mathematical Innovation and Discovery

L-functions, ranks of elliptic curves, and random matrix theory
July 8–13, 2007

MEALS

*Breakfast (Buffet): 7:00–9:00 am, Donald Cameron Hall, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Donald Cameron Hall, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Donald Cameron Hall, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Donald Cameron Hall
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00	Rubinstein: Probability models for elliptic curves
10:00	Coffee Break, 2nd floor lounge, Corbett Hall
10:30	Darmon: Survey of special values of L-functions, BSD, and Heegner points
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:30	Keating: Random matrix theory I
15:30	Coffee Break, 2nd floor lounge, Corbett Hall
16:00	Kohnen: Siegel modular forms I
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:30	Mao: Shimura correspondence and computation of L-values
10:30	Coffee Break, 2nd floor lounge, Corbett Hall
11:00	Darmon: Shintani lifts, p-adic families, and derivatives of quadratic twists
12:00–13:30	Lunch
14:00	Kohnen: Siegel modular forms II
15:00	Coffee Break, 2nd floor lounge, Corbett Hall
15:30	Keating: Random matrix theory II
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00	Watkins: Non-trivial vanishings of odd quadratic twists
10:00	Coffee Break, 2nd floor lounge, Corbett Hall
10:30	Delaunay: Odd rank quadratic twists of elliptic curves
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:30	Kisilevsky: Ranks of Elliptic Curves in Families of Cubic Extensions
10:30	Coffee Break, 2nd floor lounge, Corbett Hall
11:00	Rubin: Ranks of elliptic curves in families of quadratic twists
12:00–13:30	Lunch
14:00	Kohnen: Values of spinor zeta functions at the central point
15:00	Coffee Break, 2nd floor lounge, Corbett Hall
15:30	Miller, Duenez, and Huynh: Finite conductor models for zeros of elliptic curves
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00	Lecture (open)
10:00	Coffee Break, 2nd floor lounge, Corbett Hall
10:30	Wrap-up discussion
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

Speaker: **Henri Darmon** (McGill University)
Title: *Shintani lifts, p-adic families, and derivatives of quadratic twists*
Abstract: Let E be an elliptic curve over Q of prime conductor p and let f be its associated modular form of weight two on $\Gamma_0(N)$. The Shimura correspondence associates to f a modular form g of weight $3/2$ on $\Gamma_0(N)$, and for certain negative discriminants $-D$, the D-th fourier coefficient $c(D)$ of g encodes the central critical values $L(f, -D, 1)$ of the $-D$-th quadratic twist of f. In particular, when the sign in the functional equation for $L(f, -D, s)$ is -1, the coefficient $c(D)$ vanishes, and one expects $L'(f, -D, 1)$ to be related to the height of a point $P(D)$ on the $-D$-th twist of E. We will explain how Hida's theory of p-adic families of modular forms can be used to construct a generating series (arising froma p-adic family of mouldar forms of half-integral weight) whose D-th coefficient encodes the p-adic formal group logarithm of $P(D)$ for these discriminants. This is a report on joint work (still in progress) with Gonzalo Tornaria.

Speaker: **Henri Darmon** (McGill University)
Title: *Survey of special values of L-functions, BSD, and Heegner points*
Abstract:

Speaker: **Christophe Delaunay** (Universite Claude-Bernard (Lyon I))
Title: *Odd rank quadratic twists of elliptic curves*
Abstract: I will explain, in this talk, how to study the regulators of elliptic curves of rank 1 in a family of quadratic twists of a fixed elliptic curve. In particular, I will formulate some conjectures on the average size of these regulators. I will also present an efficient algorithm to compute explicitly the invariants of the odd quadratic twists of an elliptic curve (the regulators, the order of the Tate-Shafarevich groups etc.) and discuss the numerical data obtained using this method and how it compares with the predictions. I will also discuss the question of the number of extra rank in a family of odd rank quadratic twists.

Speaker: **Jon Keating** (University of Bristol)
Title: *Random matrix theory I and II*
Abstract: I will briefly review the calculation of the moments of the characteristic polynomials of random matrices. These moments will then be used to determine the value distribution of the characteristic polynomials in the range relevant to the application to elliptic curves.

Speaker: **Hershy Kisilevsky** (Concordia University)
Title: *Ranks of elliptic curves in families of cubic extensions.*
Abstract: Let $E/\mathbb{Q}$ be an elliptic curve defined over the rational field $\mathbb{Q}$. We examine the Mordell-Weil rank of $E(K)$ as K ranges over certain families of cubic extensions of $\mathbb{Q}$.

Speaker: **Winfried Kohnen** (Universitat Heidelberg)
Title: *Siegel modular forms I and II*
Abstract: We will introduce the basic concepts of the theory of Siegel modular forms, like the Siegel

modular group and its action on the generalized upper half space, reduction theory, examples of Siegel modular forms, Hecke operators and L-functions. Time permitting, I will also shortly talk about the Ikeda lifting theorem.

Speaker: **Winfried Kohnen** (Universitat Heidelberg)
Title: *Values of spinor zeta functions at the central point*
Abstract: We will report on a conjecture of Boecherer relating central critical values of twisted spinor zeta functions of Siegel cusp forms of genus 2 to the square of an average sum of Fourier coefficients. Numerical evidence for the conjecture will be given.

Speaker: **Zhengyu Mao** (Rutgers University Newark)
Title: *Shimura correspondence and computation of L-values*
Abstract: Let $f(z)$ be a new form of weight $2k$ and D a fundamental discriminant. One can associate to $f(z)$ a set of weight $k+1/2$ forms through Shimura correspondence in a generalized sense. We describe some constructions of these half integral weight forms and application in computation of L-values $L(f, D, k)$.

Speakers: **Miller, Duenez, and Huynh** (Brown University, University of Texas at San Antonio, and University of Bristol)
Title: *Finite conductor models for zeros of elliptic curves*
Abstract: The behavior of zeros near the central point for families of elliptic curves support the Katz-Sarnak Density Conjecture in the limit as the conductors tend to infinity. Explicitly, for suitable statistics their behavior is the same as that of eigenvalues of a subgroup of $U(N)$ as $N \to \infty$. It is a fascinating question to determine a good theory for finite conductors. We know some general features any such theory must have. In addition to knowing its limiting behavior, it should incorporate our observations on excess rank, as well as the observed repulsion of zeros near the central point. In this talk we discuss these observations and attempts at finding an appropriate model for finite conductors.

Speaker: **Karl Rubin** (UC Irvine)
Title: *Ranks of elliptic curves in families of quadratic twists.*
Abstract: In this talk we will survey known lower bounds for the number of elliptic curves of rank at least r in a family of quadratic twists, for small values of r. Specifically, if E is an elliptic curve over $\mathbf{Q}$, we will discuss constructions that lead to lower bounds of the form $N(x, r) \geq x^A$, where $N(x, r)$ is the number of squarefree integers d with $|d| < x$ such that the twist of E by d has rank at least r.

Speaker: **Michael Rubinstein** (Univeristy of Waterloo)
Title: *Probability models for elliptic curves*
Abstract: I will present a survey of probability models for elliptic curves and discuss the conjectures these lead to.

Speaker: **Mark Watkins** (University of Bristol)
Title: *Non-trivial vanishings of odd quadratic twists*
Abstract: We report on the computation of odd parity quadratic twists whose Heegner point is torsion. We compare the rank data with that of Rubinstein.