

COMBINATORIAL CURVATURE

Suppose that all polygons forming a polyhedral surface are regular and of side length 1. Then $\chi(v) = 2\pi \cdot \phi(v)$, where

$$\phi(v) = 1 - \frac{1}{2} \deg(v) + \sum_{v \sim f} \frac{1}{|f|}$$

Thus, $\sum_v \phi(v) = \chi(S)$.

Introduced by Gromov in the study of hyperbolic groups.

Higuchi Conjecture

If G is a locally finite infinite plane graph, then G has a vertex whose combinatorial curvature is not positive.

$\exists v: \phi(v) \leq 0$.