

Metric Space \mathcal{X}

-4-

$$I = [0, 1] \subset \mathbb{R}$$

$$\mathcal{W} = \{ f: I^2 \rightarrow I: \text{symmetric, measurable} \}$$

$$G = ([n], \text{edge weight } \omega) \mapsto A_G \in \mathcal{W}:$$

$$I = I_1 \cup \dots \cup I_n, \quad \lambda(I_i) = 1/n$$

$$\text{for } x \in I_i, y \in I_j: \quad A_G(x, y) = \omega(i, j)$$

$$\text{Cut-norm: } \|f\|_{\square} = \sup_{S, T \subseteq I} \left| \int_{S \times T} f(x, y) dx dy \right|,$$

"Relabeling" vertices:

measure-preserving $\phi: I \rightarrow I$

$$f^{\phi}(x, y) = f(\phi(x), \phi(y)), \quad f^{\phi} \in \mathcal{W}$$

$$\text{Distance } \delta_{\square}(f, g) = \inf_{\phi} \|f - g^{\phi}\|_{\square}$$

$$\mathcal{X} = \mathcal{W} / \{ \delta_{\square} = 0 \}$$