Upper Bound for $m \approx \mu n^2$

$G \in \mathcal{G}_{n,m}$. RL: ε-reg.

$|V_i| \approx \frac{n}{k}$

λ-coloring $c : V(G) \to [\lambda]$ gives $C : [k] \to 2^{[\lambda]}$:

$$C(i) = \{a \in [\lambda] : |c^{-1}(a) \cap V_i| > \varepsilon |V_i|\}$$

\# possible C's $\leq (2^\lambda)^k = O(1)$. Fix most frequent C.

By regularity: $ij \in E(R) \Rightarrow C(i) \cap C(j) = \emptyset$

$$\alpha_A = \frac{1}{k} |\{i : C(i) = A\}|, \quad A \subseteq [\lambda]$$

$$\log(P_G(\lambda)) \lesssim \sum_i |V_i| \log(|C(i)|) = n \sum_A \alpha_A \log(|A|)$$

$$\left.\begin{array}{l} \sum \alpha_A = 1 \\ \sum\limits_{A \cap B = \emptyset} \alpha_A \alpha_B \gtrsim m/n^2 \approx c \\ \text{Maximize } \sum\limits_A \alpha_A \log(|A|) \end{array}\right\} OPT(c, \lambda)$$