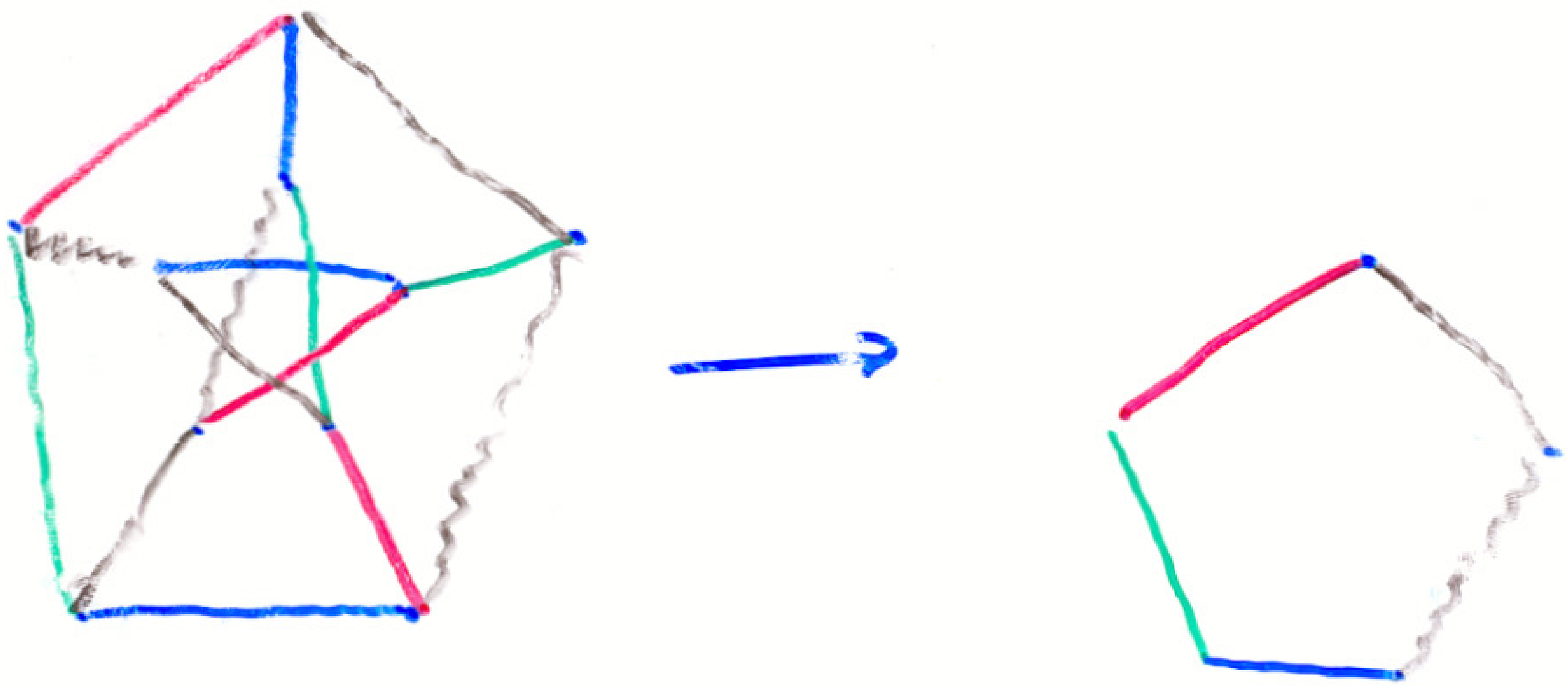
Cut- & cycle-continuous mappings – Robert Šámal



• **cut** = $E(X, V\setminus X)$ for some $X \subseteq V$

• **cycle**: $F \subseteq E(G)$ s.t. $\forall v$ $\deg_F v$ is even

$\mathcal{C}_{top}(G)$ ("incl. $v \in$ Ends, if F is infinite")

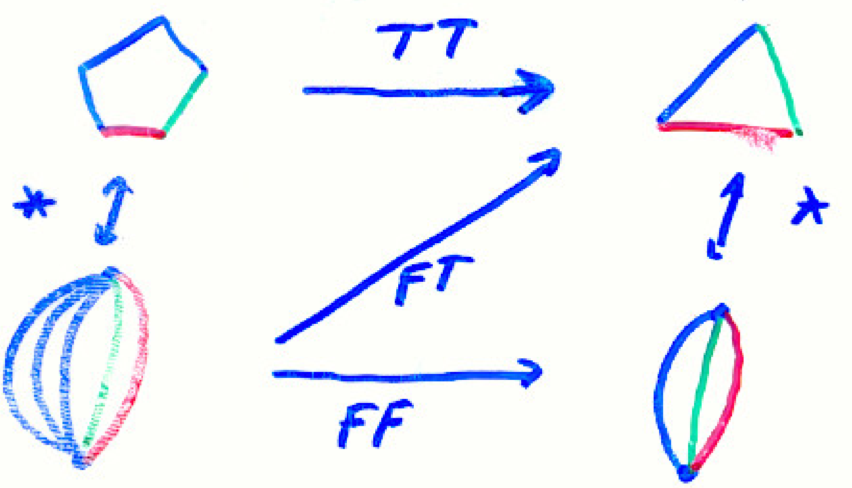
• $f: E(G) \longrightarrow E(H)$ is

- **TT** (**cut-cont.**) iff $f^{-1}(C)$ is a cut $\forall C \overset{\omega}{\subseteq} E(H)$ – cut

- **FF** (**cycle-cont.**) iff — " — cycle — " — cycle (poss. infinite)

- **FT** iff — " — cycle — " — cut

• **ex.**



alternative def.

f is TT iff

$\forall C \dots$ finite cycle in G

$f[C]$ is a cycle in H

$$f[C] = \{e \in E(H),\ |f^{-1}(\{e\}) \cap C| \text{ is odd}\}$$

(Equivalent, as $(\text{cuts})^{\perp} =$ Fin. cycles [illegible])

$$G \xrightarrow{\text{hom}} H \implies G \xrightarrow{TT} H$$

Thm [Hopkins, Staton; Bondy, Locke]

$$\Delta(G) \le 3, \quad G \not\supseteq K_3 \Rightarrow b(G) \ge \frac{4}{5}$$

G finite

$$b(G) = \frac{\mathrm{MAXCUT}(G)}{|E(G)|}$$

Thm [DeVos, Š.]

$$\Delta(G) \le 3, \quad G \not\supseteq C_3, C_4, \dots, C_{16} \Rightarrow G \xrightarrow{TT} C_5$$

G finite

$$G \xrightarrow{hom} \text{Clebsch}$$

— or G infinite (compactness)

$$G \xrightarrow{TT} H \iff G \xrightarrow{\text{hom}} \Delta(H)$$

$$\Delta(H) = \left(P(V(H)),\ \{AB,\ A \triangle B \in E(H)\}\right)$$

infinite H: we may use P or P_{fin}

finite case: $G \xrightarrow{TT} K_n \implies \chi(G) \leq 2^n$

infinite: $G \xrightarrow{TT} K_{\omega_0} \iff \chi(G) \leq \omega_0$

$G \xrightarrow{FF} H$, H has a "nice collection of cycles" (CDC)

$\Rightarrow$ G has a nice coll. of cycles

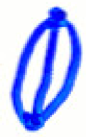
finite graphs

Conj. [Jaeger] G bridgeless $\Rightarrow G \xrightarrow{FF} Pt$

(WMA G is 3-regular)

easy case: $\chi'(G) = 3$

$$\Downarrow$$

$G \xrightarrow{FF}$  $\xrightarrow{FF} Pt$

hard case ("snarks") $\chi'(G) = 4$

.... the only known examples are "built from" the Petersen graph Pt

Question G 3-regular, infinite, 4-edge-connected

1) Is G 3-edge-colorable ?

2) $G \xrightarrow{FF} Pt$?