Valuation of Energy Storage: An Optimal Switching Approach

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Unlike financial “paper” assets, commodities must be physically stored.

Storage infrastructure is a major component of the energy industry.

Large-scale needs; very capital intensive.

Storage allows intertemporal transfer.

Financially, storage is a straddle on calendar prices.

Can be used to speculate: commodity prices fluctuate, aim is to buy low and sell high (and store in between).
Examples

- Natural Gas Storage: salt domes, pipelines, depleted reservoirs, aquifers.
- This is already a multi-billion industry with active trading.
- Poised for further growth with rolling-out of Liquified Natural Gas on world-wide basis.
- Hydroelectric Pumped Storage.
- The most scalable method of storing electricity; about 75% efficiency; 38 plants in the US, worldwide capacity of almost 50 GWh.
- Resource management. Metal/fossil fuel is “stored” in the ground, with one-way inventory depletion. Further exploration permits possibility of “replenishment”.

Pricing Energy Storage
Complex Problem:

- Commodity prices are stochastic
- Strong seasonality effects
- Possibility of both forward and spot trades
- Engineering constraints/exogenous events
- Margin requirements on borrowed funds
- Inventory-dependent Injection/withdrawal rates
- Storage costs/switching costs
We focus on the timing optionality within a real-options framework.

The presence of inventory makes the problem highly path-dependent!

Concentrate on the gas storage application.

The two key state variables are $(G_t)$ gas prices (stochastic).

$(C_t)$ current inventory of gas (a function of manager’s policy).
(\(G_t\)) is exogenously given and is a \(d\)-dimensional Markov process.

- Inventory constraints: \(c_{\text{min}} \leq C_t \leq c_{\text{Max}}\).
- At each instant \(t\), choose an operating regime: \(u_t\) — rate for amount of gas to inject/withdraw.
- Transmission constraints: \(a_{\text{min}} \leq u_t \leq a_{\text{Max}}\).

Resulting inventory path \(\bar{C}_t(u)\):

\[
d\bar{C}_s(u) = a_u(s)\bar{C}_s(u)\, ds, \quad \bar{C}_0(u) = c.
\]

- Fixed horizon \(T\): typically the facility is rented from the owner and must be returned at a later date.

\[\Rightarrow\] Set of admissible policies \(u \in \mathcal{U}(c)\).
Control Problem

- The manager maximizes total revenue on $[0, T]$.
- When operating regime is $i$, rate of revenue is $\psi_i(t, G_t, C_t)$.
- When operating regime is changed from $i$ to $j$, switching costs $K_{i,j}(G_t, C_t)$ are paid.
- Let $V(t, g, c, i)$ denote maximum expected future profit given the initial conditions.
- Wish to find

$$V(0, g, c, i) = \sup_{u \in U} E \left[ \int_0^T \psi_u(t)(G_t, C_t) \, dt - \sum_{t \leq T} K_{u^{-},u_t} \right].$$

- Bellman Principle:

$$V(0, g, c, i) = \sup_{u} E \left[ \int_0^t \psi_u(s)(G_s, C_s) \, ds - \sum_{s \leq t} K_{u^{-},u_t} + V(t, G_t, \bar{C}_t(u), u_t) \right].$$
Because payoffs are \textit{linear} in $u_t$, controls are necessarily of \textit{bang-bang type}, so the only choices are $u_t \in \mathcal{I} \triangleq \{a_{min}, 0, a_{Max}\}$. So a priori have a finite-dimensional control space.

$\implies$ Three possible operational states

- **inject** $dC_t = a_{inj}(C_t)dt$, $\psi_{-1}(G_t, C_t) = -b_{-1}(C_t) - a_{inj}(C_t) \cdot G_t$
- **store** $dC_t = 0$, $\psi_0(G_t, C_t) = -b_0(C_t)$
- **withdraw** $dC_t = -a_{wdr}(C_t)dt$, $\psi_1(G_t, C_t) = -b_1(C_t) + a_{wdr}(C_t) \cdot G_t$

$b_i$'s are the O&M costs, storage costs, transmission inefficiencies, etc.
1-d Exponential OU model:

\[ dG_t = \kappa(\bar{g} - \log G_t)G_t \, dt + \sigma G_t \, dW_t \]

Mean-reverting, log-normal, non-negative (Jaillet et al. 2004).

Terminal condition reflects stipulations for final inventory:

\[ V(T, g, c, i) = -2g \cdot \max(c - c, 0). \]

Gas pressure laws:

\[ a_{wdr}(c) = k_0 \sqrt{c}, \quad a_{inj}(c) = k_1 \sqrt{\frac{1}{c + k_2} - \frac{1}{k_3}}. \]
Related Literature

- **Optimal Switching (w/out inventory)**: Zervos (2003), L. and Carmona (2005), Barrera-Esteve et al. (2006).
The operational flexibility of the manager is a compound timing option. Under mild assumptions can show that will only make a finite number of changes in the optimal policy.

Recursively define $V^k(t, g, c, i)$ for $k = 0, 1, \ldots, 0 \leq t \leq T$, $g \in \mathbb{R}^d$, $c \in [c_{min}, c_{max}]$ and $i \in \{-1, 0, 1\}$:

\[
V^0(t, g, c, i) \triangleq \mathbb{E}\left[\int_t^T \psi_i(s, G_s, \tilde{C}_s(c, i)) \, ds \bigg| G_t = g\right],
\]

\[
V^k(t, g, c, i) \triangleq \sup_{\tau \in S_t} \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} \psi_i(s, G_s, \tilde{C}_s(c, i)) \, ds \right.
\]
\[
+ \max_{j \neq i} \left\{ -K_{i,j} + V^{k-1}(\tau, G_\tau, \tilde{C}_\tau(c, i), j) \right\} e^{-r(\tau-t)} \bigg| G_t = g\right].
\]

Iterative Optimal Stopping Problems.
Proposition

1. \( V^k \) is equal to the value function for the storage problem with at most \( k \) regime switches allowed.

2. An optimal finite strategy \( u^* = (\tau_1^*, \xi_1^*, \tau_2^*, \xi_2^*, \ldots) \) for \( V^k(0, g, c, i) \) exists and is: \( \tau_0^* = 0, \xi_0^* = i \), and for \( \ell = 1, \ldots, k \)

\[
\tau_{\ell}^* \triangleq \inf \left\{ s \geq \tau_{\ell-1}^* : V^\ell(s, G_s, C_s(u^*), i) \right\}
\]

\[
= \max_{j \neq i} \left( -K_{i,j} + V^{\ell-1}(s, G_s, C_s(u^*), j) \right) \wedge T,
\]

\[
\xi_{\ell}^* \triangleq \arg \max_{j \neq i} \left\{ -K_{i,j} + V^{\ell-1}(\tau_{\ell}^*-1, G_{\tau_{\ell}^*}-1, C_{\tau_{\ell}^*}(u^*), i) \right\}.
\]

3. \( \lim_{k \to \infty} V^k(t, g, c, i) = V(t, g, c, i) \) pointwise, uniformly on compacts.
QVI Approach

The classical analytic theory (Øksendal and Sulem, 2005) implies that the value function is also the (unique viscosity) solution of the Quasi-Variational Inequality

\[
\min \left\{ -V_t - \mathcal{L}_G V(t, g, c, i) + a_i(c) \cdot \partial_c V(t, g, c, i) \right.
\]
\[
- \psi_i(g, c) + r V(t, g, c, i), \quad V(t, g, c, i) - \max_{j \neq i} (V(t, g, c, j) - K_{i,j}) \right\} = 0.
\]

\[V(T, g, c, j)\] given.

Assuming a smooth \(V\) this can then be implemented with a free boundary pde solver.

The problem (1) is degenerate (convection-dominated).
Numerical Algorithm
Bermudization

- Discretize time: decisions are now made only at $t = m\Delta t$.
- Then to make a decision today compute

$$V(t, G_t, C_t, i) = \max_j \left( -K_{i,j} + \mathbb{E}[\psi_{t+1}(G_t, C_t, j) \right.$$

$$+ e^{-r\Delta t} \cdot V(t+1, G_{t+1}, \tilde{C}_{\Delta t}(C_t, j), j)| \mathcal{F}_t] \right).$$

- Can apply Dynamic Programming backward in time once can do the conditional expectation against the Markov state $G$. 

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Numeric Implementation
Tsitsiklis-van Roy and Longstaff-Schwartz Pricing Methods for Optimal Switching

Admissible action set is $j \in I$

Regress $\{ V(g_k, T+1, j) + \psi(g, j) \}$ against $\{g\}$
Get $f: g \rightarrow E^g[ V(g_k, T+1) + \psi(g,j) ]$

$\longrightarrow f(g; j)$ is Continuation Value

Select best action $i^*(g; i)$ using $f$

TvR: $V(g_k, T, i) := f(g, i^*)$

LSM: $V(g_k, T, i) := V(g_k, T+1, i^*) + \psi(g; i^*)$
Simulation Methods

- Simulate $\{g^n_t\}_{n=1}^N$ and work with path values $v(s, g^n_s)$.
- Given future path values $\{v(t + 1, g^n_{t+1}; j)\}$ and associated rewards $\{\psi_{t,t+1}(g^n_t; j)\}$, regress their sum onto $\{g^n_t\}$ to find out the continuation value $\tilde{E}(g^n_t; j)$ for each action $j$.
- Find best action $i^*$ for each path.
- TvR then sets $v(t, g^n_t; i) = \tilde{E}(g^n_t; i^*)$.
- LSM propagates back
  $v(t, g^n_t; i) = v(t + 1, g^n_{t+1}; i^*) + \psi_{t,t+1}(g^n_t, i^*)$.
- In LSM the value function is computed exactly as long as policy decisions are made correctly along the path.
With storage have inventory $C_t$ which depends on the past control $u_s$, $s \leq t$. Dynamic Programming proceeds backwards. Suppose that $v(t + 1, g^n_{t+1}, c; i)$ were known for all $c$. Then can find $v(t, g^n_t, c, i)$ as above.

Interpolate to construct the new $v(t, g, \cdot; i)$ as function of $c$.

Make a grid in the $C$-variable.

If the grid size is $N^c$, then have $N^c$ optimal switching problems. \Rightarrow Expensive.

Can no longer propagate back like in LSM.
Second Attempt

- Instead do quasi-simulation of $C_t$.
- If can *guess* correctly today’s action $\tilde{i}$ and know inventory tomorrow, then have inventory today and can propagate.
- Perform bivariate regression of path values $\{v(t, g^n_{t+1}, c^n_{t+1}; i)\}$ against $(g^n_t, c^n_{t+1})$.
- Try to back-out $c^n_t$ such that $\tilde{C}_{\Delta t}(c^n_t, \tilde{i}) = c^n_{t+1}$.
- Attempt to do LSM and fall back onto TvR when cannot.
- One large bivariate optimal switching problem: BLSM scheme.
Overall BLSM Algorithm I

1. Select a set of bivariate basis functions \((\tilde{B}_j)\) and algorithm parameters \(\Delta t, M = T / \Delta t, N, N_b\).

2. Generate \(N\) paths of the price process: \(\{g_{m\Delta t}^n, m = 0, 1, \ldots, M, n = 1, 2, \ldots, N\}\) with fixed \(g_0^n = g_0\). Generate a random terminal \(c_T^n(i)\).

3. Initialize the pathwise values \(v(T, g_T^n, c_T^n(i), i)\).

4. Moving backward with \(t = m\Delta t, m = M, \ldots, 0\) repeat:
   - **Guess Current C:** generate \((c_{m\Delta t}^n(i))\) by guessing the optimal decision \(\hat{j}^n(m\Delta t, i)\) and solving \(\tilde{C}_{\Delta t}(c_{m\Delta t}^n(i), \hat{j}^n(m\Delta t, i)) = c_{(m+1)\Delta t}^n(\hat{j}^n(m\Delta t, i))\).
   - **Regression Step:** do the bivariate regression to find
     \[
     \tilde{E} : (g, c, k) \mapsto \sum_{j=1}^{N_b} \bar{\alpha}_j \tilde{B}_j(g, c; m\Delta t, k)
     \]
     \[
     \simeq \mathbb{E} \left[ \psi_{m\Delta t}(m\Delta t, g, c) + e^{-r\Delta t} \cdot v((m + 1)\Delta t, G_{(m+1)\Delta t}, c, k) \big| G_{m\Delta t} = g \right]
     \]

of the value tomorrow given today’s prices and tomorrow’s inventory.
Overall BLSM Algorithm II

- **Optimal Decision Step**: find the optimal decision by evaluating $	ilde{E}(g_{m\Delta t}^n, \tilde{C}_\Delta t(c_{m\Delta t}^n(i), j))$ above for different $j$’s.
- **Update Step**: compute $v(m\Delta t, g_{m\Delta t}^n, c_{m\Delta t}^n(i), i)$ via ▶ (LSM) if correctly guessed $\hat{j}^n(m\Delta t, i)$ or via ▶ (TvR) if not.
- **Switching Sets**: the points

$$\{ (g_{m\Delta t}^n, c_{m\Delta t}^n) : n \text{ is such that } \hat{j}^n(m\Delta t, i) = i \}$$

define the empirical action set for policy $i$.

5. end Loop

6. Interpolate $V(0, g_0, c, i)$ from the $N$ values $v(0, g_n^0, c_n^0(i), i)$.
Performance

- Complexity is $O(M \cdot N \cdot N_b^3)$.
- Quite fast on “toy problems”, speed comparable to 1-d pde solvers.
- Much faster than the first Mixed Interpolation TvR attempt.
- No results on convergence rate. Expect algorithm variance of $O(N^{-1/2})$.
- Variance strongly affected by choice of basis functions (need intuition about the shape of $V$).
- Number of paths $N$ needed is exponential in number of basis functions $N_b$ used.
- Computing resources: 40,000 paths, 15 basis functions, 400 time-steps takes 30 minutes in Matlab on a desktop.
- Within 2% of “true” value (from pde).
Example from de Jong and Walet (2003):

- $d \log G_t = 17.1 \cdot (\log 3 - \log G_t) \, dt + 1.33 \, dW_t$.
- 8 Bcf capacity: $0 \leq C_t \leq 8$.
- $V(T, g, c, i) = -2 \cdot g \cdot \max(4 - c, 0)$.
- $a_{in} = 0.06 \cdot 365$, $a_{out} = 0.25 \cdot 365$, $r = 0.06$, $T = 1$, $b_i \equiv 0.1$, $K_{i,j} \equiv 0.25$
- Thus, it takes about $8/0.06 = 133$ days to fill the facility and $8/0.25 = 32$ days to empty it.
- $g_0 = 3$, $c_0 = 4$. 
**Table:** Variance of the BLSM scheme as a function of $N$. Standard deviations were obtained by running the algorithm 50 times.

<table>
<thead>
<tr>
<th>No. Paths</th>
<th>Mean</th>
<th>Std. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>8000</td>
<td>14.24</td>
<td>4.81</td>
</tr>
<tr>
<td>16000</td>
<td>11.03</td>
<td>2.08</td>
</tr>
<tr>
<td>24000</td>
<td>10.42</td>
<td>1.48</td>
</tr>
<tr>
<td>32000</td>
<td>10.03</td>
<td>0.940</td>
</tr>
<tr>
<td>40000</td>
<td>10.01</td>
<td>0.698</td>
</tr>
<tr>
<td>pde</td>
<td>9.86</td>
<td>–</td>
</tr>
</tbody>
</table>
function surface showing $V(0.5, g, c, 0; T = 1)$ as a function of current gas price $G_t = g$ and current inventory $C_t = c$. 

**Value Surface**
Optimal Policy Regions

Best Policy showing $i^*(0.5, g, c, 0; T = 1)$ as a function of current gas price $G_t = g$ and current inventory $C_t = c$. 

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Illustrations
Effect of Storage Flexibility on the Value Function. Results obtained using the BLSM algorithm with 40,000 paths.

<table>
<thead>
<tr>
<th>Daily $a_{in}$</th>
<th>Daily $a_{out}$</th>
<th>$V(0, g_0, c_0, 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.25</td>
<td>9.86</td>
</tr>
<tr>
<td>0.03</td>
<td>0.125</td>
<td>6.41</td>
</tr>
<tr>
<td>0.12</td>
<td>0.5</td>
<td>12.96</td>
</tr>
<tr>
<td>0.18</td>
<td>0.75</td>
<td>14.63</td>
</tr>
<tr>
<td>0.12</td>
<td>0.25</td>
<td>12.95</td>
</tr>
</tbody>
</table>
Add-Ons

- Can easily incorporate jumps/seasonality in the model.
- Can add other constraints.
- Use the computed optimal policy in a new simulation to obtain a less biased estimate of $V$.
- Iterate the method to successively improve guesses of optimal policy.
- Can use different bases for different $t$’s, $i$’s.
- Can use other regression tools besides $L^2$: kernel, etc.
PDE Methods vs. Simulation Schemes

PDE Methods:
- Extensive Literature
- Known error rate/stability conditions
- Many speed-ups possible
- Guaranteed structure of optimal policy regions

But:
- hard to handle degenerate C-variable
- Changes to price model may require extensive modification
- Impossible to consider multiple factors
Simulation Schemes:
- Very flexible off-the-shelf capability
- Much easier to scale/add constraints
- Can be easily combined with other simulation engines
- Better probabilistic interpretation

Unfortunately:
- No error analysis
- May be unstable – must fine-tune basis functions
- No structure of optimal policy regions
The major limitation of pde method is curse of dimensionality:

- It is likely that gas prices are described by a factor model (stochastic mean-reversion level, or regime-switching or pure-jump factors).
- In hydroelectric applications, river run-off and precipitation cause exogenous stochastic fluctuations in inventory levels.
- Power supply guarantees: combine a gas storage problem with the need to serve a client base with stochastic demand.
- Margin constraints: loan for buying commodity to store is marked-to-market and subject to margin calls if prices fall too low.
- A lot remains to be done...
References I

Dixit, A., R. S. Pindyck. 1994. 

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Optimal switching with applications to energy tolling agreements. Working paper.

