REPORT ON THE WORKSHOP: “DERIVED CATEGORY METHODS IN COMMUTATIVE ALGEBRA”

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This is our report on the Research in Teams Workshop “Derived Category Methods in Commutative Algebra” held at Banff International Research Station (BIRS), 1–8 June 2008.

BACKGROUND

Derived category methods have proved to be very successful in ring theory, in particular in commutative algebra. Evidence is provided by [1,4–8,11,12,15,16,19,20,23,24,27], to list some work of considerable importance.

Surprisingly, there is no accessible introduction or reference to the applications of derived category methods in commutative algebra, or in general ring theory for that matter. To be an effective practitioner of these methods, one must be well-versed in a series of research articles and lecture notes, including unpublished ones: [2,3,9,10,13,14,16–18,22,25,28,29]. To get an overview of their applications in commutative algebra, the list grows further. The purpose of the BIRS workshop was to make progress on a book manuscript—authored by L.W. Christensen, H.-B. Foxby, and H. Holm—that will remedy this deficiency.

As implied in the discussion above, the book has no direct competition. Many books cover applications of classical homological algebra in (commutative) ring theory, but only a few books address derived category methods and their applications in this field: In *Homological Algebra* [9] by Cartan and Eilenberg, resolutions of complexes and derived functors are briefly discussed in the final chapter; no applications are given. In Weibel’s *An introduction to homological algebra* [30], derived categories are introduced in the final chapter; a few applications to ring theory are included as exercises. Derived categories are also covered in *Methods of Homological Algebra* [21] by Gelfand and Manin, but applications to ring theory are not. A very thorough construction of derived categories is given in *Categories and Sheaves* [26] by Kashiwara and Schapira. However, the aim of [26] is sheaf theory, so beyond the construction of derived categories, there is barely any overlap with this book. Finally, Christensen’s *Gorenstein Dimensions* [10] has an appendix on derived category methods. It provides a rudimentary and incomplete survey of technical results without proofs. The fact that it has, nevertheless,
become a frequently cited reference betrays a significant gap in the existing literature.

**HISTORY OF THE BOOK PROJECT**

As one of the pioneers in the applications of derived category methods in commutative algebra, Foxby has previously circulated two sets of lecture notes on the topic [13,17].

In 2006 Christensen and Foxby started the current project. The book in progress offers a systematic development of hyperhomological algebra. This includes the construction of the derived category of a general (associative) ring and a careful study of the functors of importance in ring theory. To demonstrate the strength and utility of the theory, and to motivate the choice of topics, the book includes an extensive course in central homological aspects of commutative ring theory. This part includes many recent results, which were discovered by means of derived category methods, and gives valuable new insight into the theory of commutative rings and their modules.

Based on four peer reviews, Springer-Verlag offered to publish the book, and a contract was signed in late 2007.

For health related reasons, Foxby has been unable to work on the project for some time. To ensure timely completion of the book, Christensen and Foxby decided to add a third author, and in April 2008 Holm accepted to joint the project.

**AIM AND RESULTS OF THE WORKSHOP**

The workshop at BIRS had two purposes. To introduce the new coauthor Holm to the project, and to complete a first rough draft of the manuscript—taking into account the extensive comments in the reports solicited by Springer-Verlag.

The first three days of the workshop were spent on a major reorganization of the manuscript based on the referees’ suggestions and feedback from students and colleagues. This reorganization serves two purposes:

- To make the book more useful as a reference to derived category methods also for researchers in non-commutative algebra.
- To structure the applications to commutative algebra in a fashion more familiar to researchers in that field.

This process was an excellent way to introduce Holm to the scientific as well as the technical and administrative aspects of the book project.

The balance of the workshop was spent on discussions and “prototyping” aimed at merging contributions from the different authors into a coherent text. This includes

- Laying down principles for indexing and cross-referencing
- Standardizing formulations of mathematical statements
- Homogenizing levels of abstraction between chapters
- Homogenizing levels of details in proofs
As the workshop participants live on different continents, and in different time zones, face-to-face meetings as provided by this workshop are of utmost importance for solving scientific as well as editorial problems. We thank BIRS sincerely for providing us with this opportunity.

REFERENCES


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