# Banff International Research Station

## for Mathematical Innovation and Discovery

# 08w2133: Singular phenomena in nonlinear optics, hydrodynamics and plasmas

## Friday, October 24, 2008 to Sunday, October 26, 2008

### MEALS

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall (*included in workshop*)

***For meal options at the Banff Centre, there are buffets (breakfast: 7:00-9:30am; lunch: 11:30am-1:30pm; dinner: 5:30-7:30pm), Gooseberry's Deli, located in the Sally Borden Building, and The Kiln Cafe, located beside Donald Cameron Hall. There are also plenty of restaurants and cafes in the town of Banff, a 10-15 minute walk from Corbett Hall.***

### MEETING ROOMS

**All lectures are held in Max Bell 159. LCD projector, overhead projectors and blackboards are available for presentations** *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

### SCHEDULE

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| **Friday** | |
| **16:00** | Check-in begins (Front Desk - Professional Development Centre - open 24 hours) |
| **Saturday** | |
| **07:00-08:30** | Breakfast |
| **08:30** | Walter Craig, *Bounds on Kolmogorov spectra for the Navier - Stokes equations* |
| **09:00** | Catherine Sulem, *Water waves over a random topography* |
| **09:30** | Alexander Balk, *Generation of zonal jets by the Rossby wave turbulence* |
| **10:00-10:30** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:30** | Vladimir E. Zakharov, *Freakons: New type of Solitons* |
| **11:00** | Philippe Guyenne, *Numerical simulation of 3D overturning waves* |
| **11:30** | Benno Rumpf, *Quasisolitonic turbulence in the Majda-McLaughlin-Tabak equation* |
| **12:00-13:30** | Lunch |
| **13:30** | Michael Weinstein, *Ground State Selection and Energy Equipartition in the Nonlinear Schroedinger / Gross Pitaevskii Equation* |
| **14:00** | Pierre Raphaël, *Standing ring solutions to a super critical NLS* |
| **14:30** | Boaz Ilan, *Lattice solitons, orbital instabilities, and the band-gap interface* |
| **15:00-15:30** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **15:30** | Martin V. Goldman, *Weak electron-phase space holes* |
| **16:00** | Mark Hoefer, *Dispersive Regularization of Degenerate Rarefaction Wave Interactions: Matter-Wave Interference* |
| **16:30** | Pavel M. Lushnikov, *Regularization of collapse in cellular dynamics* |
| **17:00-17:15** | Break |
| **17:15** | Brenton LeMesurier, *Modelling Pulses in Molecular Chains and Conservative Time-Discrete Hamiltonian Systems* |
| **17:45** | Henry Warchall, *Funding Opportunities in the Mathematical Sciences at the National Science Foundation* |

**Sunday**

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|---|---|
| **07:00-8:30** | Breakfast |
| **08:30** | Robert Krasny, *Regularized Point Vortex Simulations of Vortex Sheet Roll-Up* |
| **09:00** | Xinwei Yu, *On the energy conservation of ideal MHD equations* |
| **09:30** | Natalia Vladimirova, *Propagation and quenching in a reactive Burgers-Boussinesq system* |
| **10:00-10:15** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:15** | Becca Thomases, *Mixing Transitions and Oscillations in Low-Reynolds Number Viscoelastic Fluids* |
| **10:45** | Vladimir Mezentsev, *Full vectorial modelling of femtosecond bullets for laser inscription of photonic structures* |
| | Checkout by 12 noon. |

** 2-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 15:00 on Sunday, although participants are still required to checkout of the guest rooms by 12 noon. There is no coffee break on Sunday afternoon, but self-serve coffee and tea are always available in the 2nd floor lounge, Corbett Hall. **

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## 08w2133: Singular phenomena in nonlinear optics, hydrodynamics and plasmas
## Friday, October 24, 2008 to Sunday, October 26, 2008

**ABSTRACTS**
**(in alphabetic order by speaker surname)**

Speaker: **Alexander Balk** (University of Utah - Department of Mathematics )
Title: *Generation of zonal jets by the Rossby wave turbulence*
Abstract: We will consider the problem of alternating zonal jets, like Jupiter's stripes. Mathematically similar flows appear also in tokamak plasmas, where these zonal flows turn out to be barriers to the anomalous transport. We will see that the generation of these singular flows is implied by the conservation of the extra invariant. We will also review some recent developments in the study of this invariant. (1) Is the extra invariant an artifact of the Fourier dynamics, like it happens with the North-South momentum? We will see that the extra invariant is real and note its representation in the physical space. (2) The extra invariant is mainly based on large scale modes (even to a greater degree than the energy). A small dissipation has almost no effect on the extra invariant, while destroying the enstrophy conservation.

Speaker: **Walter Craig** (McMaster University - Department of Mathematics and Statistics)
Title: *Bounds on Kolmogorov spectra for the Navier - Stokes equations*
Abstract: Let $u(x,t)$ be a (possibly weak) solution of the Navier - Stokes equations on all of $\mathbb{R}3$, or on the torus $\mathbb{R}3/\mathbb{Z}3$, with or without a divergence-free forcing term. The *energy spectrum* of $u(\cdot,t)$ is the spherical integral

$$E(\kappa,t) = \int_{|k|=\kappa} |\hat{u}(k,t)|^2 dS(k), \qquad 0 \le \kappa < \infty,$$

or alternatively, a suitable approximate sum. An argument involving scale invariance and dimensional analysis given by Kolmogorov in 1941, and subsequently refined by Obukov, predicts that in three dimensions, solutions of the Navier - Stokes equations at large Reynolds number and exhibiting fully developed turbulent behavior should obey

$$E(\kappa,t) \sim C\kappa^{-5/3},$$

at least in an average sense. I will explain a global estimate on weak solutions in the norm $|\mathcal{F}\partial_x u(\cdot,t)|_\infty$ which gives bounds on a solution's ability to satisfy the Kolmogorov law. The result gives rigorous upper and lower bounds on the inertial range, and in the unforced case an upper bound on the time of validity of the spectral regime. These results are joint work with Andrei Biryuk.

Speaker: **Frederic Dias** (ENS Cachan - Centre de Mathe'matiques et de Leurs Applications)
Title: *TBA*
Abstract: TBA

Speaker: **Martin V. Goldman** (University of Colorado at Boulder - Department of Physics)

Banff, October, 2008

**Weak electron-phase space holes***

Martin V. Goldman and David L. Newman
Physics Department
University of Colorado at Boulder, CO

In the small potential limit[1], the 1D Vlasov-Poisson eqns admit stationary self-consistent solutions involving an analytic unipolar potential, $\phi(x)$. The electric field $E = -\partial_x\phi$, is bipolar. Electrons are trapped by $-e\phi$ in a shallow phase-space hole inside a separatrix at energy, $W = mv^2/2 - e\phi(x) = 0$. The stationary electron energy distribution is defined piecewise in terms of a symmetrized untrapped distribution, $f_u$, and a trapped distribution, $f_T$,

as, $f(W) = \left\{ \begin{array}{l} f_u(W),\ W > 0 \\ f_T(W),\ W < 0 \end{array} \right\}$, where f(W) is continuous across the separatrix but *not* smooth.

The trapped and untrapped electron densities are assumed to be separately expandable in Taylor series in $\sqrt{\phi}$. If the finite non-vanishing first derivative, $\partial_W f_T(W)\big|_{W=0}$ is assumed >> $\partial_W f_u(W)\big|_{W=0}$, the potential, $\phi(x) = \phi_{max}\,\mathrm{sech}^4(x/a)$. The half-width, $\Delta x \propto a$, depends only on a linear screening integral over the untrapped zero-order distribution, $f_u(mv^2/2)$. $\phi_{max}$ depends on both $f_u$, and $\partial_W f_T(W)\big|_{W=0}$. $\phi_{max}$ can be made small enough to justify the expansions.

These weak stationary holes yield[1] a reasonable interpretation of spacecraft observations of half-width versus speed for a large number of measured bipolar field structures as well as for the shape of the bipolar waveforms. Electrostatic Vlasov simulations suggest that such stationary solutions can be attractors, dynamically accessed during the nonlinear saturation of weak (kinetic) electron-electron instabilities. Vlasov simulations show that there are a variety of other weak phase space hole states which appear to act as attractors. Additional classes of analytic stationary potentials and weak phase space holes are discussed.

* Work supported by NASA, DOE and NSF

1 Goldman, M. V., D.L. Newman and A. Mangeney, PRL **99**, 145002 (2007)

Speaker: **Philippe Guyenne** (University of Delaware - Department of Mathematical Sciences)
Title: *Numerical simulation of 3D overturning waves*
Abstract: We present numerical simulations of 3D overturning water waves over bottom topography. The numerical model solves the fully nonlinear potential flow equations using a 3D high-order boundary element method combined with an explicit time integration scheme, expressed in a mixed Eulerian-Lagrangian formulation. Results on wave profiles and kinematics will be presented. Comparisons with 2D results as well as with theoretical predictions will also be shown.

Speaker: **Mark Hoefer** (Columbia University - Department of Applied Physics and Applied Mathematics (formerly of the National Institute of Standards and Technology))
Title: *Dispersive Regularization of Degenerate Rarefaction Wave Interactions: Matter-Wave Interference*
Abstract: In an Eulerian fluid, shock waves do not propagate into a region of zero density. Instead, degenerate rarefaction or expansion waves describe the fluid behavior. The interaction of two such waves does however generate gradient singularities which must be regularized. A dissipative regularization for this problem leads to two counter-propagating shock waves moving away from the initial interaction point. This talk will present a dispersive regularization for this problem using the Whitham averaging method which gives rise to an oscillatory interaction region described by a modulated train of solitons decaying to small amplitude linear waves. Viewed in the context of dispersive shock waves (DSWs), the interaction region can be thought of as two expanding DSWs placed back-to-back. It is demonstrated by comparison of the asymptotic results with numerical simulations and experiments that this interaction region corresponds to the macroscopic, quantum mechanical interference of matter-waves in a Bose-Einstein condensate.

Speaker: **Boaz Ilan** (University of California, Merced - School of Natural Sciences)
Title: *Lattice solitons, orbital instabilities, and the band-gap interface*
Abstract: Positive soliton solutions of nonlinear Schrodinger equations with periodic and irregular-lattice potentials are investigated. Using rigorous, asymptotic and computational methods we show that the solitons are (un)stable precisely whenever they satisfy (violate) the power-slope (Vakhitov-Kolokolov) and lesser-studied spectral condition. Violations of the power-slope and spectral conditions induce focusing and drift instabilities, respectively. This unified approach predicts the strength of the (in)stabilities as well. These results are elucidated by computation of soliton dynamics with periodic, defect, and quasi-crystal lattice structures. In addition, we derive explicit formulae for the profile and power of solitons near the band-gap interface.

Joint works with Gadi Fibich, Yonatan Sivan, and Michael Weinstein.

Speaker: **Robert Krasny** (University of Michigan - Department of Mathematics)
Title: *Regularized Point Vortex Simulations of Vortex Sheet Roll-Up*
Abstract: Vortex sheets are weak solutions of the incompressible fluid equations describing velocity fields with a tangential discontinuity. Physically, a vortex sheet represents a thin shear layer in slightly viscous flow. The initial value problem for vortex sheets is ill-posed due to Kelvin-Helmholtz instability. Moore showed that a curvature singularity forms in finite time in a perturbed vortex sheet. To go past the critical time, it is necessary to regularize the principal value integral defining the sheet velocity. This talk presents regularized point vortex simulations of the problem. For a large value of the regularization parameter, the sheet rolls up into a smooth spiral past the critical time. As the regularization parameter is reduced, we observe the onset of chaotic dynamics due to resonances in the vortex core.

Speaker: **Brenton LeMesurier** (College of Charleston - Department of Mathematics)
Title: *Modelling Pulses in Molecular Chains and Conservative Time-Discrete Hamiltonian Systems*
Abstract: A new approach is described for generating time discretizations for a large class of Hamiltonian systems which exactly conserve the energy and other quadratic conserved quantities of the corresponding differential equations.

An essential feature is a procedure for constructing discrete approximations of partial derivatives in a way that mimics essential properties of derivatives, in particular for the quadratic forms of most "momenta".

The approach is applied to a class of systems which includes models of energetic pulse propagation in protein due to Davydov, Scott et al. These models have the integrable nonlinear Schrödinger equation as a continuum limit, with sech pulse solutions. Extensions will be shown to systems like coupled systems of discrete nonlinear Schrödinger equations and oscillators. The discrete models have self-focusing effects not seen in the 1D cubic NLS.

The resulting time-discrete systems serve as numerical methods for Hamiltonian DE's, but are also of possible intrinsic interest as fully discrete dynamical system models, with solutions respecting symmetries and invariants of the Hamiltonian in the spirit of Noether's Theorem.

Speaker: **Pavel M. Lushnikov** (University of New Mexico - Department of Mathematics and Statistics)
Title: *Regularization of collapse in cellular dynamics*
Abstract: Biological cells interact through chemotaxis when cells secret diffusing chemical (chemoattractant) and move towards gradient of chemoattractant creating effective nonlocal attraction between cells. Macroscopic description of cellular density dynamics through Keller-Segel model has striking qualitative similarities with nonlinear Schrödinger equation including critical collapse in two dimensions and supercritical in three dimensions. Critical collapse has logarithmic corrections to $(t_0 - t)^{1/2}$ scaling law of self-similar solution. Microscopic motion of eucaryotic cells is accompanied by random fluctuations of their shapes. We derive a nonlinear diffusion equation coupled with chemoattractant from microscopic cellular dynamics in dimensions one and two using excluded volume approach. Nonlinear diffusion coefficient depends on cellular volume fraction and it provides regularization (prevention) of cellular density collapse. A very good agreement is shown between Monte Carlo simulations of the microscopic Cellular Potts Model and numerical solutions of the macroscopic equations for relatively large cellular volume fractions.

Speaker: **Vladimir Mezentsev** (Aston University - Electronic Engineering)
Title: *Full vectorial modelling of femtosecond bullets for laser inscription of photonic structures*
Abstract: Full vectorial analysis of nonlinear propagation of femtosecond laser pulse is presented. Full set of Maxwell's equations coupled to standard Drude model of the generated plasma is modelled. The results are compared with the orthodox models based on paraxial envelope approximation.

Speaker: **Pierre Raphaël** ( IMT, Toulouse, France)
Title: *Standing ring solutions to a super critical NLS*
Abstract: I will discuss the description of the singularity formation for some focusing nonlinear Schrödinger equations $iu_t + \Delta u + u|u|^{p-1} = 0$ in $\mathbb{R}^N$. I will start by reviewing the analysis of the stable "log-log" blow up regime for the $L^2$ critical case $p = 1 + \frac{4}{N}$ as derived by Frank Merle and myself. I will then explain how this analysis can be adapted to provide the existence of standing ring bow up solutions in the super critical case. In particular, for $p = 5$ and *any* $N \geq 2$, we prove the existence of radially symmetric blow up solutions which concentrate their $L^2$ mass on the unit sphere of $\mathbb{R}^N$ and the stability of this singularity formation in the radial class. I will put this result in perspective with the recent works by Fibich, Gavish and Wang on non standing ring blow up solutions to the super critical (NLS). This is joint work with Jeremie Szeftel (CNRS and Princeton University).

Speaker: **Benno Rumpf** (Chemnitz University of Technology - Faculty of Natural Sciences)
Title: *Quasisolitonic turbulence in the Majda-McLaughlin-Tabak equation*
Abstract: The Majda-McLaughlin-Tabak equation is a simple onedimensional model system for turbulence. In my talk, I'll discuss the formation of collapses and of quasisolitons from a weakly turbulent background in this system.

Speaker: **Catherine Sulem** (University of Toronto - Department of Mathematics )
Title: *Water waves over a random topography*
Abstract: We discuss the problem of nonlinear wave motion of the free surface of a body of fluid over a variable bottom. The object is to describe the character of wave propagation in a long wave asymptotic

regime, under the assumption that the bottom of the fluid region is described by a stationary random process whose variations take place on short length scales. Our principal result is the derivation of effective equations and a consistency analysis. We compute the effects of random modulation of solutions, and give an explicit expression for the scattered component of the solution due to waves interacting with the random bottom.

Speaker: **Becca Thomases** (University of California, Davis)
Title: *Mixing Transitions and Oscillations in Low-Reynolds Number Viscoelastic Fluids*
Abstract: In the past several years it has come to be appreciated that in low Reynolds number flow the nonlinearities provided by non-Newtonian stresses of a complex fluid can provide a richness of dynamical behaviors more commonly associated with high Reynolds number Newtonian flow. For example, experiments by V. Steinberg and collaborators have shown that dilute polymer suspensions being sheared in simple flow geometries can exhibit highly time dependent dynamics and show efficient mixing. The corresponding experiments using Newtonian fluids do not, and indeed cannot, show such nontrivial dynamics. To better understand these phenomena we computationally study the Stokes-Oldroyd-B viscoelastic model in 2D. For low Weissenberg number, flows are "slaved" to the four-roll mill geometry of the fluid forcing. For sufficiently large Weissenberg number, such slaved solutions are unstable and under perturbation transit in time to a structurally dissimilar flow state dominated by a single large vortex, rather than four vortices of the four-roll mill state. The transition to this new state also leads to regions of well-mixed fluid and can show persistent oscillatory behavior with continued destruction and generation of smaller-scale vortices.

Speaker: **Natalia Vladimirova** (University of New Mexico - Department of Mathematics and Statistics)
Title: *Propagation and quenching in a reactive Burgers-Boussinesq system*
Abstract: We investigate the qualitative behavior of solutions of a Burgers-Boussinesq system – a reaction-diffusion equation coupled via force to a Burgers equation – by a combination of numerical and asymptotic techniques. When the force is small the solutions decompose into a traveling wave and an accelerated shock wave moving in opposite direction. When force exceeds some critical value, the solutions are composed of three elementary pieces: a wave fan, a reaction traveling wave, and an accelerating shock, the whole structure traveling in the same direction. With further increase of the force, the wave fan catches up with the accelerating shock wave – the solution drops below reaction threshold and reaction is ceased. The extinction result irrespective of the size of initial data – a major difference with what happens in advection-reaction-diffusion equations where an incompressible flow is imposed.

Speaker: **Henry Warchall**
Program Director in Applied Mathematics
Division of Mathematical Sciences
National Science Foundation
Title: *Funding Opportunities in the Mathematical Sciences at the National Science Foundation*
Abstract: I will describe current opportunities for funding in mathematics and statistics at the National Science Foundation, as well as issues that arise in proposal preparation. There will be ample opportunity for questions from the audience.

Speaker: **Michael Weinstein** (Columbia University - Department of Applied Physics and Applied Mathematics )
Title: *Ground State Selection and Energy Equipartition in the Nonlinear Schroedinger / Gross Pitaevskii Equation*
Abstract: Nonlinear Schroedinger / Gross Pitaevskii (NLS/GP) ?equations are central to the? mathematical description of nonlinear optical and macroscopic quantum systems. We show for NLS/GP systems supporting multiple families of nonlinear bound states ("solitons") that? (i) the generic evolution is toward a nonlinear ground state and (ii) in the weakly nonlinear regime an energy equipartition law holds.

Speaker: **Xinwei Yu** (University of Alberta - Department of Mathematical and Statistical Sciences)
Title: *On the energy conservation of ideal MHD equations*
Abstract: Combining the techniques recently developed in Cheskidov et. al. and a simple observation of the magnetic field equation, we obtain sufficient conditions for the energy conservation of the ideal MHD equations. Our conditions improve previous ones in Caflisch et. al. and Kang-Lee.

Speaker: **Vladimir Zakharov** (Lebedev Physics Institute of the Russian Academy of Sciences / University of Arizona)
Title: *Freakons: New type of Solitons*
Abstract: TBA