



# Banff International Research Station

for Mathematical Innovation and Discovery

## New Topological Contexts for Galois Theory and Algebraic Geometry March 9–14, 2008

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

### MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

### SCHEDULE

#### Sunday

- 16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)  
Lecture rooms available after 16:00 (if desired)
- 17:30–19:30** Buffet Dinner, Sally Borden Building
- 20:00** Informal gathering in 2nd floor lounge, Corbett Hall  
Beverages and small assortment of snacks available on a cash honour-system.

#### Monday

- 7:00–8:45** Breakfast
- 8:45–9:00** Introduction and Welcome to BIRS by Alitha D'Ottavio, Max Bell 159
- 9:00–10:00** Hess: Hopf-Galois extensions in monoidal model categories
- 10:00–10:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11:30** ABS: Algebraic  $K$ -theory and traces for structured ring spectra, I
- 11:30–13:00** Lunch
- 13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
- 14:00** Group Photo; meet on the front steps of Corbett Hall
- 14:15–15:15** Montgomery: Hopf Galois theory, I
- 15:15–15:45** Coffee Break, 2nd floor lounge, Corbett Hall.
- 15:45–16:30** Hovey: Semisimple ring spectra
- 16:45–17:30** Blumberg: Localization for  $THH$  and  $TC$  of schemes
- 17:30–19:30** Dinner

## Tuesday

- 7:00–9:00 Breakfast  
9:00–10:00 Montgomery: Hopf Galois theory, II  
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall  
10:30–11:30 Jardine: Étale  $K$ -theory: a modern view  
11:30–13:30 Lunch  
13:30–14:30 ABG:  $\infty$ -categories and applications, I  
14:30–15:00 Coffee Break, 2nd floor lounge, Corbett Hall  
15:00–15:45 ABS: Algebraic  $K$ -theory and traces for structured ring spectra, II  
16:30–17:15 Sagave: DG-algebras and derived  $A_\infty$ -algebras  
17:30–19:30 Dinner  
19:30–??? Poster Session

## Wednesday

- 7:00–9:00 Breakfast  
9:00–10:00 Rognes: Topological logarithmic structures I  
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall  
10:30–11:30 BM: Topological André-Quillen cohomology and related topics  
11:30–13:30 Lunch  
Free Afternoon  
17:30–19:30 Dinner

## Thursday

- 7:00–9:00 Breakfast  
9:00–10:00 Goerss: Realizing families of Landweber exact homology theories  
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall  
10:30–11:30 Lawson: Topological Automorphic Forms  
11:30–13:30 Lunch  
13:30–14:30 ABG:  $\infty$ -categories and applications, II  
14:30–15:00 Coffee Break, 2nd floor lounge, Corbett Hall  
15:00–15:45 Devinatz: Towards the finiteness of the homotopy groups of the  $K(n)$ -local sphere  
16:30–17:15 Rognes: Topological logarithmic structures II  
17:30–19:30 Dinner  
19:30–??? May:  $E_\infty$  ring spaces and  $E_\infty$  ring spectra

## Friday

- 7:00–9:00 Breakfast  
9:00–11:30 Problem Session  
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall  
11:30–13:30 Lunch  
Checkout by 12 noon.

\*\* 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*



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### ABSTRACTS

(in alphabetic order by speaker surname)

Speakers: **Matthew Ando, Clark Barwick & David Gepner (ABG)** (Universities of Illinois at Urbana-Champaign, Oslo, Sheffield)

Title:  *$\infty$ -categories and applications, I & II*

Abstracts:

I Infinity Categories and Applications – Models for the  $\infty$ -category of  $\infty$ -categories.

$\infty$ -categories provide an excellent foundation for abstract homotopy theory, offering several advantages over the more traditional approach via Quillen model categories. After discussing some of their merits, we turn to definitions and models for higher categories and specifically  $(\infty, 1)$ -categories, including simplicial categories, Segal categories, complete Segal spaces, and quasicategories. Finally, we focus on quasicategories, which are in some sense the smallest and most streamlined of the known models.

II Post-Prandial Infinity-Categories – Third Half.

In this talk, I describe the relationship between model categories and infinity-categories. By the coherent nerve construction discussed in part I, one can convert any model category into an infinity category. But when one performs infinity categorical constructions on coherent nerves of model categories, how does one understand the result? The answer comes in the form of the Strictification Theorems, which give models for infinity-categories constructed by homotopy limits and internal homs. Further, there are multi-infinity-category versions of this result as well, which allow for the strictification of weak algebras over weak operads. By combining these results with the theory of operator categories, one can prove a number of interesting results, including the assertion that the space of  $E_n$  structures on the infinity-category of left modules over an  $E_1$  ring spectrum  $A$  is canonically equivalent to the space of  $E_{n+1}$  structures on  $A$  itself.

Speakers: **Christian Ausoni, Morten Brun & Christian Schlichtkrull (ABS)** (Universities of Bonn, Bergen, Bergen)

Title: *Algebraic  $K$ -theory and traces for structured ring spectra, I & II*

Abstract: The aim of this series of lectures is to give an overview of the trace invariants associated to the algebraic  $K$ -theory  $K(A)$  of a connective structured ring spectrum  $A$ . We begin with the definition of  $K(A)$  based on matrices, and we explain how it is related to Waldhausen's  $A$ -theory of spaces by means of spherical group-rings. We then review the Bökstedt trace map to topological Hochschild homology, and its refinement given by the cyclotomic trace to topological cyclic homology. We also present some general results, like the theorems on relative  $K$ -theory of Dundas and McCarthy. In the second part of this lecture, we turn to more specific examples of computations of  $K(A)$ , first with rational coefficients, and then with finite coefficients. Finally, we mention an extension of the Lichtenbaum-Quillen conjectures to this context, due to John Rognes.

Speakers: **Maria Basterra, Mike Mandell (BM)** (University of New Hampshire Durham, Indiana University)

Title: *Topological André-Quillen cohomology and related topics*

Abstract:

Speaker: **Andrew Blumberg** (Stanford University)

Title: *Localization for THH and TC of schemes*

Abstract: In this talk I will give a construction of the topological Hochschild homology and topological cyclic homology of a scheme in terms of the ‘spectral derived category’ of the scheme. I will discuss the application of this construction to show that there is a localization sequence associated to the inclusion of an open subscheme, naturally connected via the cyclotomic trace to the localization sequence of Thomason-Trobaugh in  $K$ -theory. This is joint work with Michael Mandell.

Speaker: **Ethan Devinatz** (University of Washington)

Title: *Towards the finiteness of the homotopy groups of the  $K(n)$ -local sphere*

Abstract: I will replace the notion of finiteness by a related notion of ‘essentially finite rank’ which is relevant for certain homotopy or homology groups of finite  $K(n-2)$ -acyclic spectra which are annihilated by  $p$ . Using the Devinatz-Hopkins continuous homotopy fixed point spectra, I will outline a program for proving that, if  $X$  is such a spectrum, then the homotopy groups of its  $K(n)$ -localization are of essentially finite rank, and I will indicate what progress I have made. One consequence of my work is the result that if  $z$  is an element of the  $p$ -Sylow subgroup of  $S_n$  and is non-torsion in the quotient of this group by its center, then the only units in  $(E_n)_*$  fixed by  $z$  are the units in the Witt vectors.

Speaker: **Paul Goerss** (Northwestern)

Title: *Realizing families of Landweber exact homology theories*

Abstract: In this overview, I would like to revisit and explore the following realization problem: given a continuous family of Landweber exact homology theories, when can it be lifted to a diagram of commutative ring spectra? This is not always possible, but work of Hopkins, Miller, Lurie, Behrens, and Lawson has given us a number of important examples where realization is possible, and I would like to meditate some on these in order to see what makes them work. Because of the relative simplicity and because it is easy to make explicit the role of  $p$ -divisible groups, I will emphasize the example of elliptic homology theories (i.e., topological modular forms) and the Hopkins-Miller realization result.

Speaker: **Kathryn Hess** (EPFL)

Title: *Hopf-Galois extensions in monoidal model categories*

Abstract: Rognes introduced the notion of a Hopf-Galois extension of structured ring spectra, of which the unit map from the sphere spectrum to  $MU$  is an important example. In this talk I will outline a theory of homotopic Hopf-Galois extensions in a monoidal category with compatible model category structure that generalizes the case of structured ring spectra. I will give examples of homotopic Hopf-Galois extensions in other categories. This is joint work with Cédric Bujard.

Speaker: **Mark Hovey** (Wesleyan University)

Title: *Semisimple ring spectra*

Abstract: In this talk, we present a definition and partial classification of semisimple ring spectra. This can be looked at as part of the general program of extending ring theory to structured ring spectra. It is also very closely related to the following purely algebraic question: for which graded rings  $R$  can the category of projective  $R$ -modules be triangulated, where the suspension is the shift? This is joint work with Keir Lockridge.

Speaker: **Rick Jardine** (University of Western Ontario)

Title: *Étale  $K$ -theory: a modern view*

Abstract: This lecture shows how to construct the étale  $K$ -theory of a scheme  $S$  and investigate its properties, with modern tools. There is a rather simply defined version of the  $K$ -theory presheaf of spectra  $K$  on the big site for  $S$  that is constructed with Waldhausen’s techniques from big site vector bundles, and which is a presheaf of symmetric spectra. Let  $n$  be a number which is relatively prime to the residue characteristics of  $S$ : then the mod  $n$  étale  $K$ -theory presheaf of spectra is constructed from the

presheaf of spectra  $K$  by taking a stably fibrant model  $FK/n$  for the presheaf  $K/n$  with respect to the étale topology. I shall display descent spectral sequences for mod  $n$  étale  $K$ -theory, including a spectral sequence of Galois descent type for the mod  $n$  étale  $K$ -theory of  $S$  which starts with Galois cohomology of the Grothendieck fundamental group of  $S$ , with coefficients in étale  $K$ -groups. We shall also discuss some standard comparisons with other flavours of  $K$ -theory.

Speaker: **Tyler Lawson** (University of Minnesota)

Title: *Topological Automorphic Forms*

Abstract: In this talk, I will discuss joint work with Mark Behrens on families of Landweber exact homology theories arising from moduli of higher-dimensional abelian varieties with extra structure. I will explain how these moduli give rise to  $p$ -divisible groups and a result of Jacob Lurie that allows realization of them. In particular, I will discuss the case of abelian surfaces with complex multiplication and how these can be related to the case of elliptic homology theories.

Speaker: **Peter May** (University of Chicago)

Title:  *$E_\infty$  ring spaces and  $E_\infty$  ring spectra*

Abstract:  $E_\infty$  ring spectra were defined in 1972, but the term has since acquired several alternative meanings. The same is true of several other terms. The new formulations and constructions are not always known to be equivalent to the old ones and even when they are, the notion of ‘equivalence’ needs discussion: Quillen equivalent categories can be quite seriously inequivalent. Part of the confusion stems from a gap in the modern resurgence of interest in  $E_\infty$  structures.  $E_\infty$  ring spaces were also defined in 1972 and have never been redefined. They were central to the early applications and they tie in to modern applications. We give an informal open ended discussion of the relationships between the original notions and various new ones, explaining what is and what is not known.

Speaker: **Susan Montgomery** (University of Southern California)

Title: *Hopf Galois theory: I & II*

Abstract: My first talk will consist of definitions, examples, and a few basic results about Hopf Galois extensions. In the second lecture, I will try to discuss topics raised by the audience at the first talk.

Speaker: **John Rognes** (University of Oslo)

Title: *Topological logarithmic structures*

Abstract: A logarithmic structure on a commutative  $S$ -algebra  $B$  is a suitable map  $M \rightarrow \Omega^\infty B$  of  $E_\infty$  spaces (with zero). It specifies a topological algebro-geometric object, intermediate between  $\text{Spec}(B)$  and  $\text{Spec}(B[M^{-1}])$ . We report on work in progress on how to define logarithmic versions of  $TAQ$  and  $THH$ . In the case  $B = HA$ , where  $A$  is the valuation ring of a  $p$ -adic number field  $K$  and  $M$  is freely generated by the uniformizer, this agrees with the relative  $THH(A|K)$  of Hesselholt and Madsen. In the case  $B = ku$ , the hope is that this framework provides a setting where the ‘fraction field’ of complex  $K$ -theory makes sense.

Speaker: **Steffen Sagave** (University of Oslo)

Title: *DG-algebras and derived  $A_\infty$ -algebras*

Abstract: Let  $A$  be a differential graded algebra over a commutative ring  $k$ . We show that the homology of  $A$  admits a  $k$ -projective resolution  $E$  coming with a family of higher multiplications. This  $E$  is an instance of a ‘minimal derived  $A_\infty$ -algebra’. The main result is that minimal derived  $A_\infty$ -algebras provide an alternative description of quasi-isomorphism types of dgas.