Computability, Reverse Mathematics and Combinatorics
December 7–12, 2008

Last updated on December 9, 2008

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:50	Hirst, Two variants of Ramsey's theorem
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:20	Simpson, The Gödel Hierarchy and Reverse Mathematics
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:15–15:00	Coffee Break, 2nd floor lounge, Corbett Hall
15:00–15:50	Keisler, Nonstandard Arithmetic, Reverse Mathematics, and Recursive Comprehension
16:00–16:50	Chong, Π^1_1 conservation of the COH Principle over models of $B\Sigma_2$
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:50	Montalban, On the Strength of Fraïssé's conjecture.
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:20	Kohlenbach, Tao's correspondence principle, a finitary mean ergodic theorem and conservation results for Ramsey's theorem for pairs
11:30–13:00	Lunch
13:30–14:20	Carlson, Combinatorics of Words
14:20–15:00	Coffee Break, 2nd floor lounge, Corbett Hall
15:00–15:50	Kierstead, Recursive and On-line Graph Coloring
16:00–16:50	Weiermann, Well partial orderings and their strength in terms of maximal order types
17:30–19:30	Dinner
20:00–?	Problem Session

Wednesday

7:00–8:50	Breakfast
8:50 - 9:10	Solomon, Classically equivalent definitions of well quasi-orders
9:15 - 9:35	Yokoyama, Non-standard analysis within second order arithmetic
9:40 - 10:00	Marcone, An interaction between reverse mathematics and computable analysis
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30 - 10:50	Kierstead, The survival game
10:55 - 11:15	Kjos-Hanssen, Birth-death processes, bushy trees, and a law of weak subsets
11:20 - 11:40	Cenzer, Space Complexity of Abelian Groups
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:50	Buss, Polynomial Local Search higher in the Polynomial Hierarchy and Bounded Arithmetic
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:20	Hirschfeldt, The Atomic Model Theorem and Related Model Theoretic Principles
11:30–13:00	Lunch
13:30–14:20	Towsner, How Constructive is Furstenberg's Multiple Recurrence Theorem?
14:20–15:00	Coffee Break, 2nd floor lounge, Corbett Hall
15:00–15:50	Jockusch, Bounded diagonalization and Ramseyan results on edge-labeled ternary trees
16:00–16:20	Stephan, Implementing Fragments of ZFC within an r.e. Universe
16:30–16:50	Harizanov, Computability and orders on structures
17:00–17:20	Schmerl, Grundy colorings of graphs
17:30–19:30	Dinner
20:00–?	Possible Problem Session

Friday

7:00–9:00 Breakfast
9:00–11:30 Informal Discussions, if desired
10:00-11:00 Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30 Lunch
Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

Speaker: **Sam Buss** (Univ. of Calif., San Diego)
Title: *Polynomial Local Search higher in the Polynomial Hierarchy and Bounded Arithmetic*
Abstract: The talk will discuss provably total functions of bounded arithmetic, and recent characterization of the Σ_i^b definable functions of S_2^{k+1} (or T_2^k), for all $i \leq k$. The main tool is extensions of polynomial local search problems to higher levels of the polynomial time hierarchy, where the feasible set is defined by a Π_k^b predicate but the cost and neighborhood functions are definable by polynomial time terms. These higher level PLS problems can be used to determine the truth of Π_k^b properties and also allow "witness doubling". These results can be formalized and then Skolemized with a weak base theory such as S_2^1 — the stronger theory S_2^{k+1} (or T_2^k) is needed only to prove the existence of a solution. The Skolemization allows us to define sets of clauses that are refutable in a depth m propositional refutation system (a Tait style system for propositional logic), but are conjectured not to be provable in a depth $m - 1/2$ system. We discuss open problems and future directions for research. This work is joint with Arnold Beckmann.

Speaker: **Timothy J. Carlson** (Ohio State)
Title: *Combinatorics of Words*
Abstract: We will discuss some results on the infinite combinatorics of words and their connection to idempotent ultrafilters.

Speaker: **Douglas Cenzer** (University of Florida)
Title: *Space Complexity of Abelian Groups*
Abstract: We develop a theory of $LOGSPACE$ structures and apply it to construct a number of examples of Abelian Groups which have $LOGSPACE$ presentations. We show that all computable torsion Abelian groups have $LOGSPACE$ presentations and we show that the groups $\mathbb{Z}$, $Z(p^\infty)$, and the additive group of the rationals have $LOGSPACE$ presentations over a standard universe such as the tally representation and the binary representation of the natural numbers. We also study the effective categoricity of such groups. For example, we give conditions are given under which two isomorphic $LOGSPACE$ structures will have a linear space isomorphism. Joint with Rodney G. Downey, Jeffrey B. Remmel, and Zia Uddin.

Speaker: **Chi Tat Chong** (National University of Singapore)
Title: Π_1^1 *conservation of the COH Principle over models of* $B\Sigma_2$
Abstract: We report on the joint work with Ted Slaman and Yue Yang. The combinatorial principle COH states that every array coded in a model has a set in the model cohesive for the array. Cholak, Jockusch and Slaman have proved that over the base theory RCA_0, COH+Σ_2^0 induction ($I\Sigma_2^0$) is Π_1^1-conservative over $I\Sigma_2^0$, i.e. the stronger theory with COH added does not prove new Π_1^1 statements. We show that this result extends to models of $\mathrm{RCA}_0 + B\Sigma_2^0$.

Speaker: **Valentina Harizanov** (George Washington University)
Title: *Computability and orders on structures*
Abstract: A magma is left-orderable if there is a linear ordering of its domain, which is left invariant with respect to the magma operation. If the ordering is also right invariant, then the magma is bi-orderable. For arbitrary magmas (not necessarily associative), there is a natural topology on the set of all left orders, and this space is compact. We will focus on computable orderable groups, in particular, free groups, and computability theoretic complexity of their orders.

Speaker: **Denis Hirschfeldt** (University of Chicago)
Title: *The Atomic Model Theorem and Related Model Theoretic Principles*

Abstract: The Atomic Model Theorem states that every complete countable atomic theory has a countable atomic model. In this talk I will describe joint work with Richard A. Shore and Theodore A. Slaman on the computability theoretic and reverse mathematical strength of this and related results on atomic models and type omitting.

Speaker: **Jeffry L. Hirst** (Appalachian State University)
Title: *Two variants of Ramsey's theorem*
Abstract: This talk will explore the computability theory and reverse mathematics of some versions of Ramsey's theorem, including Ramsey's theorem on trees (TT^n_k) and the polarized Ramsey's theorem (PT^n_k). Here are statements of those theorems:

TT^n_k: Let $2^{<\mathbb{N}}$ denote the full binary tree and $[2^{<\mathbb{N}}]^n$ denote all n-tuples of comparable nodes in $2^{<\mathbb{N}}$. If $f : [2^{<\mathbb{N}}]^n \to k$, then we can find a $c < k$ and a subtree S such that S is order isomorphic to $2^{<\mathbb{N}}$, and $f(\sigma) = c$ for every n-tuple σ of comparable nodes in S.

PT^n_k: If $f : [\mathbb{N}]^n \to k$, then we can find a $c < k$ and a sequence $H_1, H_2, \dots H_n$ of infinite sets such that $f(\{x_1, x_2, \dots, x_n\}) = c$ for every nonrepeating n-tuple $(x_1, x_2, \dots, x_n) \in H_1 \times \cdots \times H_n$.

Speaker: **Carl Jockusch** (UIUC)
Title: *Bounded diagonalization and Ramseyan results on edge-labeled ternary trees*
Abstract: I will discuss recent joint work with Rod Downey, Noam Greenberg, and Kevin Milans. We show that the class of weakly 1-random sets is not strongly (or Medvedev) reducible to DNR_3, the class of diagonally noncomputable functions taking values in 0,1,2. The key element of the proof is a new Ramseyan result on rooted ternary trees with certain edges having labels in 0,1. In fact we obtain a family of related results on this topic.

Speaker: **H. Jerome Keisler** (UW–Madison)
Title: *Nonstandard Arithmetic, Reverse Mathematics, and Recursive Comprehension*
Abstract: In the paper "Nonstandard Arithmetic and Reverse Mathematics" (Bull. Symb. Logic 2006), it was shown that each of the five basic theories of second order arithmetic that play a central role in reverse mathematics has a natural counterpart in the language of nonstandard arithmetic. This lecture will survey the results in that paper, and then give an even more natural counterpart of the weakest the basic theories, the theory RCA_0 of Recursive Comprehension.

The language L_2 of second order arithmetic has a sort for the natural numbers and a sort for sets of natural numbers, while the language ${}^*L_1$ of nonstandard arithmetic has a sort for the natural numbers and a sort for the hyperintegers. In nonstandard analysis one often uses first order properties of hyperintegers to prove second order properties of integers. An advantage of this method is that the hyperintegers have more structure than the sets of integers. The method is captured by the Standard Part Principle (STP), a statement in the combined language $L_2 \cup {}^*L_1$ which says that a set of integers exists if and only if it is coded by a hyperinteger. We say that a theory T' in $L_2 \cup {}^*L_1$ is conservative with respect to a theory T in L_2 if every sentence of L_2 provable from T' is provable from T.

For each of the basic theories $T = \mathsf{WKL}_0, \mathsf{ACA}_0, \mathsf{ATR}_0, \Pi^1_1\text{-}\mathsf{CA}_0$ in the language L_2 of second order arithmetic, the 2006 paper gave a theory U of nonstandard arithmetic in the language ${}^*L_1$ such that:

(1) $U + \mathsf{STP}$ implies T and is conservative with respect to T.

The nonstandard counterpart for RCA_0 in that paper does not have property (1), but instead has a weakened form of the STP. In this lecture we give a new nonstandard counterpart of RCA_0 which does have property (1). That is, we give a theory U of nonstandard arithmetic in ${}^*L_1$ such that $U + \mathsf{STP}$ implies and is conservative with respect to RCA_0.

If time permits, some related open questions will be discussed.

Speaker: **Hal Kierstead** (Arizona State University)
Title: *Recursive and On-line Graph Coloring*
Abstract: I will survey results on recursive and on-line graph coloring. A recursive graph is a graph whose

vertex and edge sets are both recursive. The basic question is whether for a class $\mathcal{C}$ of graphs there exists a function f such that every recursive k-colorable graph $G \in \mathcal{C}$ has a recursive $f(k)$-coloring. In order to prove positive results, early researchers strengthened the requirements for the effective presentation of graphs under consideration. One method was to restrict to *highly recursive* graphs. A graph is highly recursive if it is recursive, every vertex has finite degree, and its degree function is recursive. Another method was to consider digraphs, and in particular posets. Here the additional structure of the orientation of an edge provides useful information. Later, more sophisticated methods from graph theory led to positive results for the original problem.

At the same time, computer scientists began considering the problem of on-line coloring. An on-line graph coloring algorithm receives the vertices of a graph one at a time. When a vertex is received, the algorithm also learns the edges between it and the previous vertices. At this time it must irrevocably color it. There is an obvious, although not exact, correspondence between on-line and recursive coloring algorithms. However, there are many interesting results concerning on-line coloring finite graphs that have no recursive version, because the performance is measured not only in terms of chromatic number, but also in terms of the number of vertices. Moreover, there are some real world motivations for considering on-line coloring.

Speaker: **Hal Kierstead** (Arizona State University)
Title: *The Survival Game*
Abstract: The following (p, s, t)-*survival game* plays a critical role in my analysis with Konjevod of on-line Ramsey theory. The game is played by two players Presenter and Chooser. It begins with presenter choosing a positive integer n and fixing a hypergraph $H_0 = (V_0, E_0)$ with n vertices and no edges. The game now proceeds in rounds. Let $H_{i-1} = (V_{i-1}, E_{i-1})$ be the hypergraph constructed in the first $i - 1$ rounds. On the i-th round Presenter plays by presenting a p-subset $P_i \subseteq V_{i-1}$ and Chooser responds by choosing an s-subset $X_i \subseteq P_i$. The vertices in $P_i - X_i$ are discarded and the edge X_i is added to E_{i-1} to form E_i. So $V_i = V_{i-1} - (P_i - X_i)$ and $E_i = E_{i-1} \cup \{X_i\}$. Presenter wins the survival game if for some i, the hypergraph H_i contains a copy of the complete s-uniform hypergraph K_s^t on t vertices. I will discuss a proof that Presenter has a winning strategy for all positive integers p, s, t with $s \leq p$. The case $s = 2$ is an entertaining puzzle, but for larger s the only strategy I know uses finite model theoretic techniques and requires more than $n = A(2^s - 1, t)$ starting vertices, where A is the Ackermann function.

Speaker: **Bjørn Kjos-Hanssen** (University of Hawaiʻi at Mānoa)
Title: *Birth-death processes, bushy trees, and a law of weak subsets*
Abstract: I will sketch a proof that every set of integers that is Martin-Löf random relative to $0'$ has an infinite subset that computes no Martin-Löf random set. I will then discuss the relation between this result and the still-open question whether Stable Ramsey's Theorem for Pairs implies Weak Weak König's Lemma.

Speaker: **Ulrich Kohlenbach** (Technische Universität Darmstadt)
Title: *Tao's correspondence principle, a finitary mean ergodic theorem and conservation results for Ramsey's theorem for pairs.*
Abstract: In the first part of the talk we discuss the proof theory of a correspondence principle implicit in recent work of T. Tao and apply this principle to study the strength of different finitary versions (one by Tao and another one – inspired by monotone functional interpretation – due to ourselves) of the infinite pigeonhole principle (joint work with J. Gaspar). In the second part we show how recent proof theoretic metatheorems can be used to provide quantitative finitizations even in the absence of compactness. As an example we give a new quantitative form of the mean ergodic theorem for uniformly convex Banach spaces (joint work with L. Leustean). In the third part we calibrate the provable recursive function(al)s of systems that may use fixed sequences of instances of Ramsey's theorem for pairs (joint work with A. Kreuzer).

Speaker: **Alberto Marcone** (Universitá di Udine, Italy)
Title: *An interaction between reverse mathematics and computable analysis*
Abstract: I will discuss some recent joint work with Guido Gherardi, where we used some reverse mathematics ideas and techniques to attack an open problem in computable analysis á la Weihrauch. We introduce the c.a. version of WKL_0 and prove its equivalence with the c.a. version of the Hahn-Banach theorem. Rather then focusing on the result itself, I'll try to shed some light on the relationship between reverse mathematics and computable analysis.

Speaker: **Antonio Montalban** (Univerity of Chicago)
Title: *On the Strength of Fraïssé's conjecture.*
Abstract: Fraïssé's conjecture, which is now Laver's theorem, says that the class of countable linear orderings is well-quasi-ordered by the relation of embeddability. A well-quasi-ordering is a partial ordering with no infinite descending sequences and no infinite antichains. The question of what is the proof theoretic strength of Fraïssé's conjecture has been open for twenty year. Some progress has been made but the question is still open.

In my Ph.D. thesis I proved that Fraïssé's conjecture is equivalent to many statements about embeddability of linear orderings. This shows the statement is robust in the sense that is equivalent to many theorems in a certain area of math, and equivalent to all small variations of these theorems. Only the big five systems are known to be robust.

I will also talk about the plan of attack that Alberto Marcone and I have, which involves ordinal notations up to the ordinal Γ_0.

Speaker: **Jim Schmerl** (UCONN)
Title: *Grundy colorings of graphs*
Abstract: A Grundy coloring of a graph is a special kind of proper coloring. Chromatic numbers of graphs are defined in terms of proper colorings, and Grundy numbers are defined using Grundy colorings. I will discuss these concepts in the context of Reverse Mathematics.

Speaker: **Stephen G. Simpson** (Pennsylvania State University)
Title: *The Gödel Hierarchy and Reverse Mathematics*
Abstract: The Gödel Hierarchy is an array of foundationally significant theories in the predicate calculus. The theories range from weak (bounded arithmetic, elementary function arithmetic) through intermediate (subsystems of second-order arithmetic), through strong (Zermelo/Fraenkel set theory, large cardinals). The theories are ordered by inclusion, interpretability, and consistency strength. Reverse Mathematics is a program which seeks to classify mathematical theorems by calibrating their places within the Gödel Hierarchy. The theorems are drawn from core mathematical areas such as analysis, algebra, functional analysis, topology, and combinatorics. Remarkably, the Reverse Mathematics classification scheme exhibits a considerable amount of regularity and structure. In particular, a large number of core mathematical theorems fall into a small number of foundationally significant equivalence classes (the so-called "big five"). There are close connections with other foundational programs and hierarchies. In particular, concepts and methods from degrees of unsolvability play an important role.

Speaker: **Reed Solomon** (UCONN)
Title: *Classically equivalent definitions of well quasi-orders*
Abstract: There are several classically equivalent ways to define a well quasi-order. The equivalence of these definitions follows from Ramsey's Theorem for Pairs and we explore the reverse mathematical strength of these equivalences. This work is joint with Alberto Marcone and Peter Cholak.

Speaker: **Frank Stephan (NUS)**
Title: *Implementing Fragments of ZFC within an r.e. Universe*
Abstract: Rabin showed that there is no r.e. model of the axioms of Zermelo and Fraenkel of set theory.

In the present work, it is investigated to which extent one can have natural models of a sufficiently rich fragment of set theory. These models are generated by considering the relation $x \in A_y$ to be generated from a Friedberg numbering $A_0, A_1, A_2, \ldots$ of all r.e. sets and then a member A_x of this numbering is called a set in the given model iff the downward closure of the induced ordering from x is well-founded. It is shown which axioms and basic properties of set theory can be obtained and which cannot be obtained in such a model.
This is joint work with Eric Martin and Pong Wai Yan.

Speaker: **Henry Towsner** (UCLA)
Title: *How Constructive is Furstenberg's Multiple Recurrence Theorem?*
Abstract: On its face, Furstenberg's method for proving Szemeredi's Theorem seems to be as non-constructive as possible, with multiple applications of compactness and a transfinite induction. Yet other proofs, such as those by Szemeredi and Gowers, show that these theorems can be proven by explicit combinatorial means. I will discuss the use of a "partial Dialectica translation" to eliminate the transfinite aspects of the argument, and discuss how conventional unwinding methods suffice to eliminate the other non-constructive aspects.

Speaker: **Andreas Weiermann** (Ghent University)
Title: *Well partial orderings and their strengths in terms of maximal order type*
Abstract: A well partial ordering (wpo) is a partial ordering which is well-founded and which does not admit infinite anti-chains. Famous examples for wpo's are provided by results of Higman, Kruskal, Friedman and Kriz. Every wpo can be extended to a well-ordering on the same domain such that the resulting order type is maximal possible and we may call this order type the maximal order type of the wpo under consideration. The reverse mathematics strengths of assertions about a wpo can typically be measured in terms of its maximal order type. It might therefore be of some interest to get a "formula" providing in natural situations the maximal order type of a wpo. In our talk we will suggest a general principle which yields appropriate maximal order types for the standard trees classes. The conjecture is that it also applies to the tree classes (equipped with an ordering fulfilling a certain gap condition) studied by Harvey Friedman. For classes of trees labeled with two labels some first results have been obtained and we believe that a general result will soon be available. Parts of the talk are based on joint work with A. Montalban, H. Friedman, and M. Rathjen.

Speaker: **Keita Yokoyama** (Tohoku University)
Title: *Non-standard analysis within second order arithmetic*
Abstract: In Tanaka[2], we can find a model theoretic method to do non-standard analysis in WKL_0. By using this method, some popular arguments of non-standard analysis can be carried out in WKL_0. On the other hand, Keisler[1] introduced some systems of non-standard arithmetic which are equivalent in strength to basic systems for Reverse Mathematics. In this talk, I will introduce systems of non-standard second order arithmetic $\mathsf{ns\text{-}ACA}_0$ and $\mathsf{ns\text{-}WKL}_0$ which can be interpreted into ACA_0 and WKL_0, respectively[3]. In these systems, we can do non-standard analysis formally, thus, we can interpret non-standard proofs into second order arithmetic. Using this method, I will show some non-standard proofs in second order arithmetic. Especially, I will show two versions of the Riemann mapping theorem. Furthermore, I will suggest another system of non-standard second order arithmetic.

References

[1] H. Jerome Keisler, *Nonstandard arithmetic and reverse mathematics*, ***The Bulletin of Symbolic Logic***, vol. 12 (2006), no. 1, pp. 100–125.

[2] Kazuyuki Tanaka, *The self-embedding theorem of* WKL_0 *and a non-standard method*, ***Annals of Pure and Applied Logic***, vol. 84 (1997), no. 1, pp. 41–49.

[3] Keita Yokoyama, *Formalizing non-standard arguments in second order arithmetic*, preprint.