

**Northwest Functional Analysis Seminar**  
**Banff International Research Station**  
**October 16-18, 2009**

**Schedule of Lectures**

SATURDAY, OCTOBER 17, 2009

Time	Speaker
08.55–09.00	Greetings
09.00–09.30	S. Belinschi
09.40–10.10	O. Rivasplata
10.10–10.40	Coffee Break
10.40–11.10	C. Ivanescu
11.20–11.50	B. Killough
12.00–12.30	E. Samei

Lunch

Time	Speaker
14.30–15.00	A. Brudnyi
15.10–15.40	B. Nica
15.40–16.20	Coffee Break
16.20–16.50	D. Kinzbulatov
17.00–17.30	V. Yaskin

SUNDAY, OCTOBER 18, 2009

Time	Speaker
09.00–09.30	R. Yuncken
09.40–10.10	R. Deeley
10.10–10.40	Coffee Break
10.40–11.10	A. Popov
11.20–11.50	I. Karabash
12.00–12.30	M. Lamoureux
12.30–12.35	Wrap Up

Departure

ORGANISED BY: B. Brenken, H. Emerson, D. Farenick, and V. Troitsky

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**Titles and Abstracts**

BELINSCHI, SERBAN, University of Saskatchewan. *Operator-valued free probability.*

If  $(\Omega, \Sigma, P)$  is a classical probability space, and  $\Sigma'$  is a sub-sigma-algebra of  $\Sigma$ , one can consider the spaces  $L, L'$  of, say, bounded complex-valued functions on  $\Omega$  which are measurable with respect to  $\Sigma$  and  $\Sigma'$ , respectively. On the one hand, it is clear that  $L' \subseteq L$ , and both spaces contain the constant function, and on the other, by the Lebesgue-Radon-Nikodim theorem, that to any  $f$  in  $L$  we can associate a unique  $f'$  in  $L'$ . This setup has been generalized to a noncommutative context already around the middle of the last century: if  $(M, \tau)$  is a von Neumann algebra with normal faithful trace  $\tau$  and  $B \subseteq M$  is a unital von Neumann subalgebra, then there exists a trace-preserving conditional expectation  $E : M \rightarrow B$  which corresponds to the map  $f \mapsto f'$  above.

It was however in the late '80s and early '90s when a significant new type of noncommutative independence over such a subalgebra  $B$ , namely freeness with amalgamation, had been defined by Voiculescu. In this talk we will present some of the ideas behind, and some of the applications of, freeness with amalgamation. We will enumerate some of the (very many) open problems in the field, and conclude with an interpretation of classical normal distribution from this perspective.

BRUDNYI, ALEX, University of Calgary. *On the projective freeness of Banach Algebras.*

In the talk I present some joint results with A. Sasane. Let  $R$  be a unital semi-simple commutative complex Banach algebra, and let  $M(R)$  denote its maximal ideal space, equipped with the Gelfand topology. I formulate some topological conditions on  $M(R)$  for  $R$  to be a projective free ring, that is, a ring in which every finitely generated projective  $R$ -module is free. I will illustrate the result by several examples, notably the Hardy algebra of bounded holomorphic functions on coverings of a Riemann surface of finite type, and also some algebras of stable transfer functions arising in control theory.

DEELEY, ROBIN, University of Victoria. *Geometric  $K$ -homology with coefficients.*

Inspired by work of Baum and Douglas, we introduce a model for  $K$ -homology with coefficients. Fundamental to this model is the replacement

of  $\text{spin}^c$  manifold theory with  $\text{spin}^c$   $z/k$ -manifold theory. Briefly, a  $z/k$ -manifold is a pair of manifolds, one with boundary and one without boundary, with the property that the boundary of the former decomposes into  $k$ -copies of the latter. Using these objects, we produce a geometric model for  $K$ -homology with coefficients in  $z/k$ . We then use inductive limits to obtain models for any countable abelian coefficient group.

IVANESCU, CRISTIAN, Grant MacEwan College. *The Cuntz semigroup and the classification of simple separable projectionless  $C^*$ -algebras*

The Cuntz semigroup was originally defined as a semigroup of classes of positive elements. Recently the definition was modified to increase its applications in the classification theory. I describe the Cuntz semigroup, specifically the new version (also known as the stable version) of the Cuntz semigroup and other related results that are useful in the classification program. I present an interesting class of  $C^*$ -algebras, namely the class of simple separable projectionless  $C^*$ -algebras. The pull back of the Cuntz semigroup from the inductive limit, the existence theorem and the uniqueness theorem are also discussed.

KARABASH, ILLIA, University of Calgary. *Spectral analysis of a linearized equation arising in the liquid film theory.*

The parabolic equation with forward-backward diffusion

$$(1) \quad h_t + Lh = 0, \quad Lh = \varepsilon \partial_\theta (\sin \theta \partial_\theta h) + \partial_\theta h, \quad 0 < t < T,$$

arises as a linearization of a model describing the time evolution of a thin film on the inner surface of a rotating cylinder. The last feature states that solution should be periodic in  $\theta$ . It turned out that periodical conditions have no sense in the weighted  $L^2$ -space that makes  $L$  self-adjoint. Periodical conditions can be partially saved in the space  $L^2(0, 2\pi)$ . However, in  $L^2(0, 2\pi)$ , the operator  $L$  becomes “highly non-self-adjoint” in the terminology of E.B. Davies.

The talk is devoted to the spectral analysis of the operator  $L$  in the space  $L^2(0, 2\pi)$ . We find the natural domain and prove that the set of eigenfunctions is complete, but does not form an unconditional basis. The last of these results is essentially based on the analysis of ill-posedness of the Cauchy problem for equation (1).

KILLOUGH, BRADY, Mount Royal College. *Traces on  $C^*$ -algebras associated to Smale spaces.*

We consider the class of hyperbolic dynamical systems known as Smale spaces. A key feature of such spaces is the existence of local coordinates on which the dynamics are expanding/contracting. For a given Smale space,  $(X, \varphi)$ , there exists a unique  $\varphi$ -invariant probability measure that maximizes the entropy, known as the Bowen measure. Moreover, the Bowen measure is locally a product of measures supported on the local expanding/contracting

sets. There is also a well-known construction that takes a Smale space and produces three  $C^*$ -algebras, associated with three natural equivalence relations on the space. Integration against the Bowen measure (or its expanding/contracting part) yields a trace on each of these three algebras. We present a result relating these integration traces to an asymptotic of the usual trace of a bounded operator on a Hilbert space.

KINZEBULATOV, DAMIR, University of Toronto. *Almost periodic holomorphic functions on coverings of complex manifolds.*

H. Bohr's theory of almost periodic functions has numerous applications to various areas of mathematics. Two branches of this theory, both extending the classical setting of almost periodic functions on reals, were particularly rich in interesting and deep results: holomorphic almost periodic functions on tube domains and almost periodic functions on topological groups. This talk is devoted to a natural link between these two concepts: holomorphic almost periodic functions on coverings of complex manifolds, their function-theoretic properties and the 'sprouts' of the theory of analytic sheaves on the corresponding Bohr compactifications of the coverings. This is joint work with Alexander Brudnyi.

LAMOUREUX, MICHAEL P., University of Calgary. *Properties of Gabor multipliers for physical modeling.*

We present techniques developed for numerical modeling of wave propagation, and source-signature removal in seismic imaging, based on a class of linear operators known as Gabor multipliers. These operators are localized Fourier multipliers, whose actions is selectively localized by an element of a partition of unity. We discuss boundedness and stability properties for these operators, approximations to PDEs and pseudodifferential operators, and an approximate functional calculus.

NICA, BOGDAN, University of Victoria. *On the general stable rank for Banach algebras.*

Roughly speaking, the general stable rank encodes the passage from stably free to free finitely generated (f.g.) modules. The fortunate case when every stably free f.g. module is actually free—a property that is sometimes called the “Hermite property”—corresponds to the general stable rank being equal to 1. The right perspective on the general stable rank is to view it as a member of the “quartet of stable ranks”—the other three being the Bass, the topological, and the connected stable ranks. It is, however, the least understood of the four. In this talk, we present some properties of the general stable rank, as well as some exact computations. We study its properties, and we provide exact computations in the context of Banach algebras.

POPOV, ALEXEY, University of Alberta. *Almost invariant subspaces of operators and operator algebras.*

Let  $X$  be a Banach space and  $T$  a bounded operator on  $X$ . A subspace  $Y$  of  $X$  is said to be almost invariant under  $T$  if  $TY \subseteq Y + F$  for some finite-dimensional subspace  $F$  of  $X$ . This notion is only of interest if  $Y$  is both of infinite dimension and of infinite codimension; such subspaces are referred to as half-spaces.

In this talk we will discuss various properties of operators with almost invariant subspaces. In particular, we show that: (i) there exist operators without invariant half-spaces that have almost invariant half-spaces; (ii) if a closed algebra  $A$  of operators has a common almost invariant half-space then the dimensions of errors corresponding to different operators in  $A$  are uniformly bounded; (iii) if  $A$  is generated by a finite number of commuting operators and has a common almost invariant half-space then it also has a common invariant half-space; (iv) an operator and the closed algebra generated by this operator may have different collections of almost invariant half-spaces.

RIVASPLATA, OMAR, University of Alberta. *The smallest singular value of random matrices containing null entries.*

Consider a large matrix whose entries are independent real-valued random variables; then what can we say about the behaviour of its smallest singular value? In this talk we will discuss recent results about the smallest singular value for random matrices some of whose entries might be zero. The methods employed are linked to the aspect ratio of matrices, meaning the ratio of number of columns to number of rows. We will discuss estimates on the smallest singular value of “tall” random matrices, those whose aspect ratio is dominated by a small positive constant, and also for “almost square” random matrices, those with aspect ratio close to 1. We will also discuss progress on similar estimates for square matrices.

SAMEI, EBRAHIM, University of Saskatchewan. *The co-representations of the von Neumann algebra generated by the left regular representation.*

Let  $G$  be a locally compact group, and let  $VN(G)$  be the von Neumann algebra generated by the left regular representation of  $G$ . We study the co-representations of  $VN(G)$  and show that they are unitary for large classes of groups including SIN-groups, maximally almost periodic groups, and totally disconnected groups.

This is a joint work with Michael Brannan (Queen’s University).

YASKIN, VLADYSLAV, University of Alberta. *On embeddings of normed spaces in  $L_{-k}$ .*

The notion of embedding of a normed space in  $L_{-k}$  was introduced by A. Koldobsky. In this talk we will discuss this notion, its geometric interpretation, and some related questions. In particular, it is an open question whether a space  $(\mathbb{R}^n, \|\cdot\|)$  that embeds in  $L_{-k}$  necessarily embeds in  $L_{-m}$  for  $0 < k < m < n - 3$ . We solve a related problem. We show that there is a normed space  $(\mathbb{R}^n, \|\cdot\|)$  that embeds in  $L_{-m}$  but not in  $L_{-k}$ .

YUNCKEN, ROBERT, University of Victoria. *On index theory for  $SL(3, \mathbb{C})$*

The motivation for this talk is index theory for semisimple Lie groups. We are interested in possible methods for attacking the long-standing Baum-Connes Conjecture with coefficients for the groups  $SL(n, \mathbb{C})$ , for  $n > 2$ . More specifically, we are interested in a convenient construction of the infamous gamma element in KK-theory for such groups. However, the underlying theme of the talk will be the application of noncommutative harmonic analysis to index theory. For the groups  $SL(n, \mathbb{C})$ ,  $C^*$ -algebraic structures will be introduced which facilitate the analysis required for index theory problems. An explicit construction of the gamma element for  $SL(3, \mathbb{C})$  will be described.