

# Northwest Functional Analysis Seminar

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## 1 Overview of the Field

The field of functional analysis is a central and thriving branch of modern mathematics. Western Canada is particularly strong in the field, as there are researchers in the region who are internationally recognised for their contributions to Banach space geometry, noncommutative harmonic analysis, operator algebras, noncommutative geometry, and operator theory.

The region is home to a number of Tier I Canada Research Chairs in the field of functional analysis, and has a significant number of additional distinguished faculty. A PIMS Collaborative Research Group in Geometric and Harmonic Analysis (led by researchers in Calgary and Edmonton) wrapped up its activities in 2008, and a PIMS Collaborative Research Group in Operator Algebras and Noncommutative Geometry (led by researchers in Victoria, Edmonton, and Regina) commenced its activities in May 2009.

## 2 Scientific Developments

The Northwest Functional Analysis Seminar (NWFAS) is a bi-annual regional scientific meeting of researchers (faculty and postdoctoral) and graduate students in functional analysis. Participants are drawn mainly from universities in Western Canada. At this meeting the scientific program addressed: Banach space geometry, ring theoretical notions in Banach algebras, classification of  $C^*$ -algebras, complex manifolds, dynamical systems, invariant subspace theory, noncommutative geometry, noncommutative harmonic analysis, and applications of functional analysis to seismic imaging and models for rotating fluids.

A novelty of NWFAS is that young researchers form the bulk of the featured speakers, providing them with a venue to communicate their research results and to form contacts with other functional analysts within the region. The program also regularly features lectures by two or three senior researchers who address topics that are currently attracting high levels of interest.

Of the fifteen lectures at this meeting, 5 were presented by graduate students, 3 by postdoctoral researchers, 5 by recent tenure-track appointments, and 2 by senior researchers.

### 3 Participants († indicates speaker)

#### Graduate Students

†Cheng, Hin Hei	University of Alberta
†Deeley, Robin	University of Victoria
†Kinzebulatov, Damir	University of Toronto
McCann, Shawn	University of Calgary
McLean, Doug	University of Calgary
†Popov, Alexey	University of Alberta
†Rivasplata, Omar	University of Alberta
Spektor, Susanna	University of Alberta

#### Postdoctoral Researchers

†Karabash, Illia	University of Calgary (PIMS PDF)
†Nica, Bogdan	University of Victoria (PIMS PDF)
†Yuncken, Robert	University of Victoria (PIMS PDF)
Zhang, Dali	University of Calgary

#### Faculty Researchers

†Belinschi, Serban	University of Saskatchewan
Binding, Paul	University of Calgary
Brenken, Berndt	University of Calgary
Brudnyi, Alex	University of Calgary
Emerson, Heath	University of Victoria
Erljman, Juliana	University of Regina
Farenick, Douglas	University of Regina
Florice, Remus	University of Regina
Haagerup, Uffe	University of Southern Denmark
†Ivanescu, Cristian	Grant MacEwan University
†Killough, Brady	Mount Royal University
†Lamoureux, Michael	University of Calgary
Phillips, John	University of Victoria
Runde, Volker	University of Alberta
†Samei, Ebrahim	University of Saskatchewan
Sourour, Ahmed	University of Victoria
Troitsky, Vladimir	University of Alberta

### 4 Presentation Highlights

#### Banach algebras and ring theory

Alex Brudnyi, in joint work with, A. Sasane, discussed topological conditions on the maximal ideal space of a unital semi-simple commutative complex Banach algebra  $A$  that imply that  $A$  is a projective, free ring. Several examples were presented, most notably the Hardy algebra of bounded holomorphic functions on coverings of a Riemann surface of finite type.

Bogdan Nica followed the notion general stable rank, which encodes the passage from stably free to free finitely generated modules. He explained that the right perspective on the general stable rank is to view it as a member of a quartet of stable ranks—the other three being the Bass, the topological, and the connected stable ranks. Nica's lecture presented some properties of the general stable rank, as well as some exact computations.

### Banach spaces and operators

Vlad Yaskin investigated which Banach spaces embed into  $L_p$  with some  $p < 0$ . Using methods of Harmonic Analysis, he proved that for any two integers  $k$  and  $m$ ,  $0 < k < m < n - 3$ , there is a norm on  $\mathbf{R}^n$  such that the resulting normed space embeds into  $L_{-m}$  but not in  $L_{-k}$ .

Alexey Popov discussed algebras  $\mathcal{A}$  of operators on a Banach space  $X$  having an almost invariant half-space. That is, a subspace  $Y$  of  $X$  of infinite dimension and co-dimension, such that for every  $T \in \mathcal{A}$  there exists a finite-dimensional subspace  $F$  of  $X$  such that  $TY \subseteq Y + F$ . Popov proved that if  $\mathcal{A}$  is generated by a finite number of commuting operators then it also has a common invariant subspace.

### C\*-algebras

Cristian Ivanescu spoke on the Cuntz semigroup, which has recently received intensive study in the Elliott classification program. In his lecture, Ivanescu presented an existence theorem and the uniqueness theorem in connection with the classification up to isomorphism of simple separable projectionless C\*-algebras.

Brady Killough lectured on the class of hyperbolic dynamical systems known as Smale spaces. There is also a well-known construction that takes a Smale space and produces three C\*-algebras, each associated with a natural equivalence relation on the space. Integration against the Bowen measure (or its expanding/contracting part) yields a trace on each of these three algebras. Killough presented a result relating these integration traces to an asymptotic of the canonical trace of a bounded operator on a Hilbert space.

### Function theory

Damir Kinzebulatov spoke on his joint work with Alex Brudnyi devoted to the study of holomorphic almost periodic functions on coverings of complex manifolds, their function-theoretic properties, and the ‘sprouts’ of the theory of analytic sheaves on the corresponding Bohr compactifications of the coverings. This work provides a natural link between (i) holomorphic almost periodic functions on tube domains and (ii) almost periodic functions on topological groups.

### Noncommutative geometry

Robert Yuncken spoke on an application of noncommutative harmonic analysis to index theory that aims to provide a convenient construction of the infamous gamma element in KK-theory for semisimple Lie groups. After introducing the relevant C\*-algebraic structures for the special case of the groups  $SL(n, \mathbf{C})$ , Yuncken described an explicit construction of the gamma element for  $SL(3, \mathbf{C})$ .

Robin Deeley, inspired by work of Baum and Douglas, introduced a geometric model for  $K$ -homology with coefficients in  $\mathbf{Z}_k$ . Fundamental to this model is the replacement of  $\text{spin}^c$  manifold theory with  $\text{spin}^c \mathbf{Z}_k$  manifold theory. Deeley then used inductive limits to obtain models for any countable abelian coefficient group.

### Noncommutative harmonic analysis

Ebrahim Samei, in joint work with Michael Brannan, proved that certain co-representations of the von Neumann algebra generated by the left regular representation of a locally compact group are in fact unitary representations for large classes of groups that includes SIN-groups, maximally almost periodic groups, and totally disconnected groups.

Yin Hei Chen lectured on a geometric property, called the  $H$ -separation property, of closed subgroups  $H$  in a locally compact group  $G$ . Chen explained how this property is related to the duality of subgroups in an operator algebraic setting.

### Noncommutative probability theory

Serban Belinschi opened the meeting with a lecture on operator-valued free probability, concentrating on the ideas behind, and some of the applications of, freeness with amalgamation. He described some of the (very many) open problems in the field, and concluded with an interpretation of classical normal distribution from this perspective.

Omar Rivasplata, in joint work with Alexander Litvak, found estimates on the least singular value of a random matrix. He considers large matrices of size  $N \times n$ , with  $N \geq n$ , with entries being independent random variables. Unlike in previous studies, he allows some of the entries to be zeros.

### **Applications**

Illia Karabash described his spectral analysis of a partial differential operator  $L$  arising in the theory of rotating liquid films. The periodic conditions cannot be formulated in weighted  $L^2$ -spaces in a manner that makes  $L$  self-adjoint. However, Karabash explained how periodic conditions can be partially saved in the space  $L^2(0, 2\pi)$ , even though the operator  $L$  becomes “highly non-self-adjoint,” and he identifies the natural domain of  $L$  and proves that the set of eigenfunctions is complete, but does not form an unconditional basis.

Michael Lamoureux presented techniques developed for numerical modeling of wave propagation and source-signature removal in seismic imaging, based on a class of linear operators known as Gabor multipliers. He discussed boundedness and stability properties for these operators, approximations to PDEs and pseudodifferential operators, and an approximate functional calculus.

## **5 Funding**

In addition to the support provided by BIRS, the PIMS Collaborative Research Group in Operator Algebras and Noncommutative Geometry provided financial support for travel expenses to graduate students, to postdoctoral researchers, and to new and as yet unfunded faculty members.

## **6 Outcome of the Meeting**

This meeting of the Northwest Functional Analysis Seminar was the fourth, the first three having taken place at BIRS in 2003, 2005, and 2007. Like the first three meetings, the 2009 meeting was very successful in its aim to play a formative role in developing the profiles of young researchers and graduate students. In addition to a strong scientific program, the seminar provided the only venue in 2008 and 2009 at which the region’s researchers in functional analysis came into contact collectively at one meeting.