Possibilities and Limitations of Ultrasound Tomography

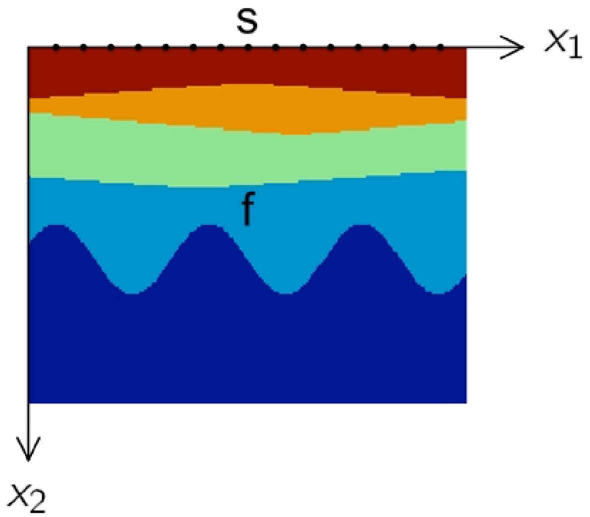
Frank Natterer
University of Münster
Department of Mathematics and Computer Science

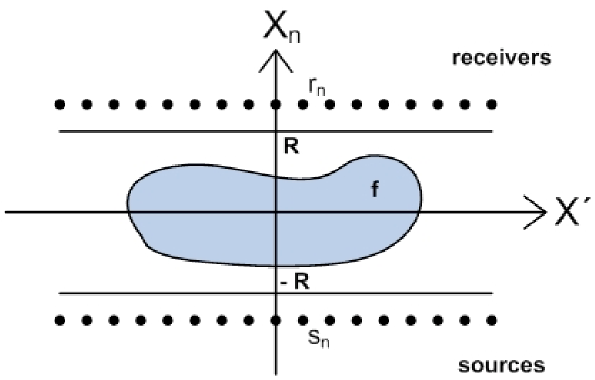
The model problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \Delta u,\ x_2 > 0,\ 0 \le t \le T,\ u = 0\ (t < 0)$$

$$\frac{\partial u}{\partial x_2}(x_1, 0) = q(t)p(x_1 - s), \qquad \mathrm{c}^2 = \frac{c_0^2}{1+f}$$

$$g_s(x_1, t) = u(x_1, 0, t) = (R_s(f))(x_1, t) \text{ seismogram for source } s$$





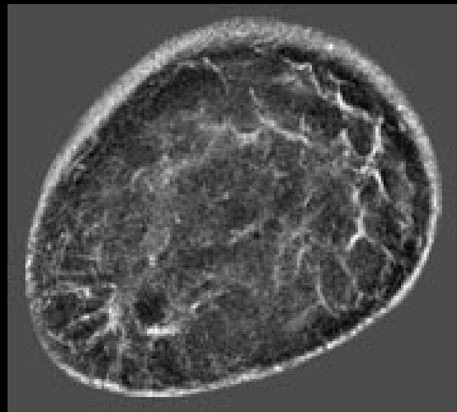
Prototype Scanner
Imaging Tank and Ring Array

BARBARA ANN KARMANOS CANCER INSTITUTE
Wayne State University

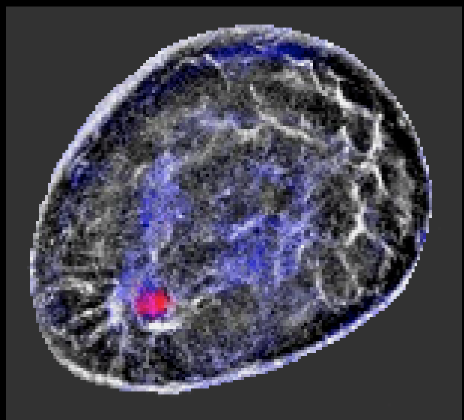
- In plane resolution: 0.5 – 2 mm
- Out of plane resolution: 4 mm
- 2 MHz operating frequency

Imaging Masses: Cancer

Reflection images with
thresholded sound speed and
attenuation images superimposed

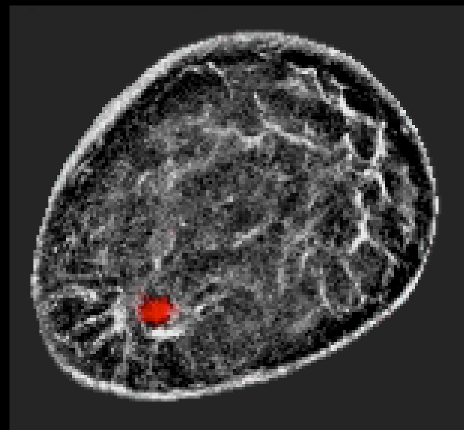


Reflection image



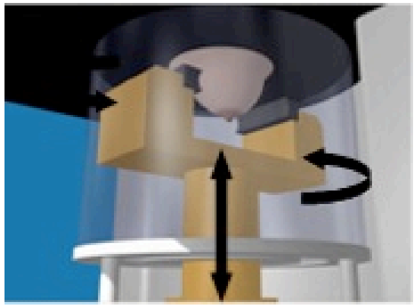
Attenuation (blue) and
sound speed (red) added

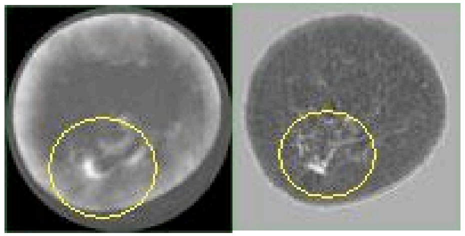
Strong sound speed enhancement
Strong attenuation



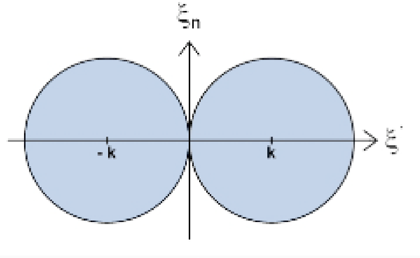
Sound speed added (red)

TechniScan

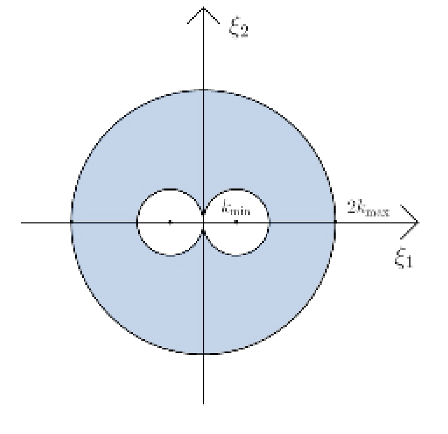




Coverage in Fourier domain



Transmission

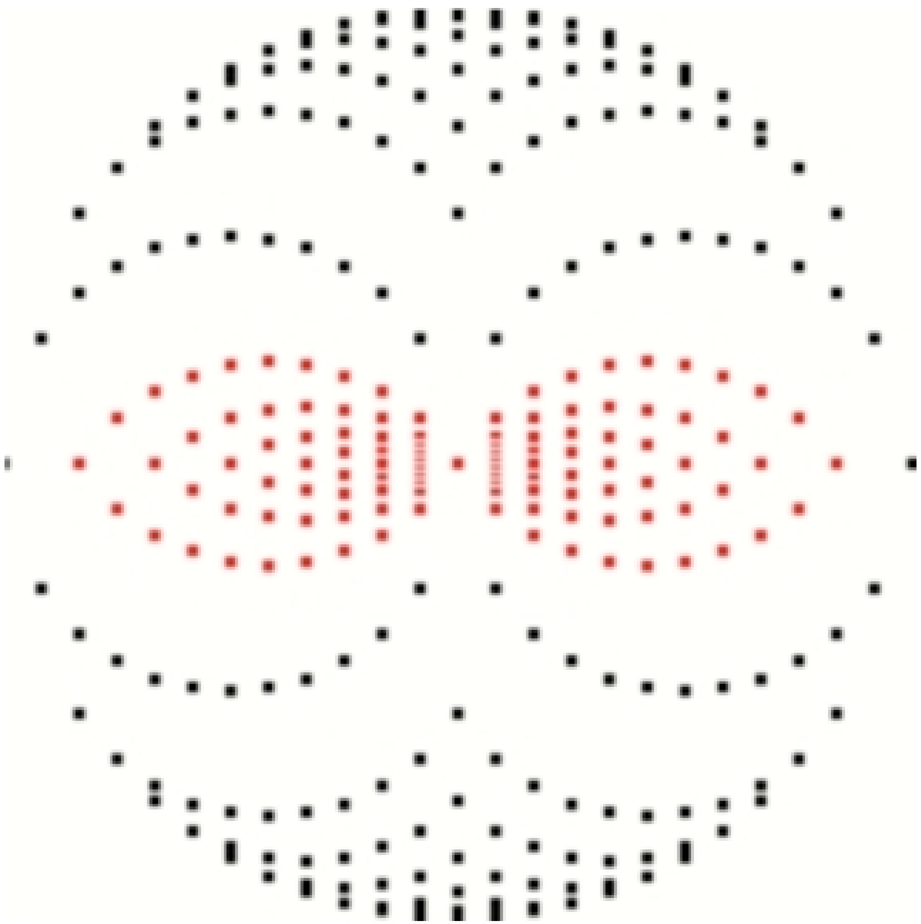


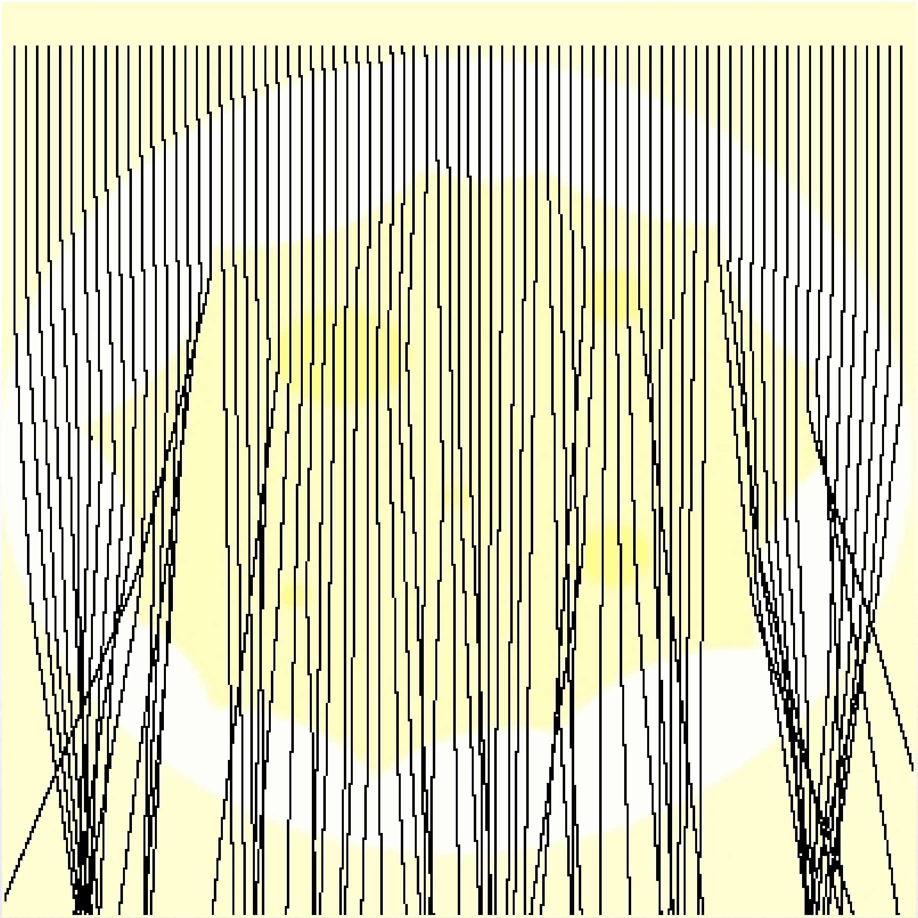
Reflection

$$\hat{f}(\sigma + \rho, a(\sigma) - a(\rho)) \qquad \hat{f}(\sigma + \rho, a(\sigma) + a(\rho))$$

$$a(\sigma) = \sqrt{k^2 - |\sigma|^2}$$

Discrete set of points in Fourier domain





Rays for
Salt Lake City
breast phantom

Kaczmarz' method (nonlinear)

Solve $R_s(f) = g_s$ for all sources s.

Update:

$$f \longleftarrow f - \alpha (R_s'(f))^*(R_s(f) - g_s)$$

Compute the adjoint by time reversal:

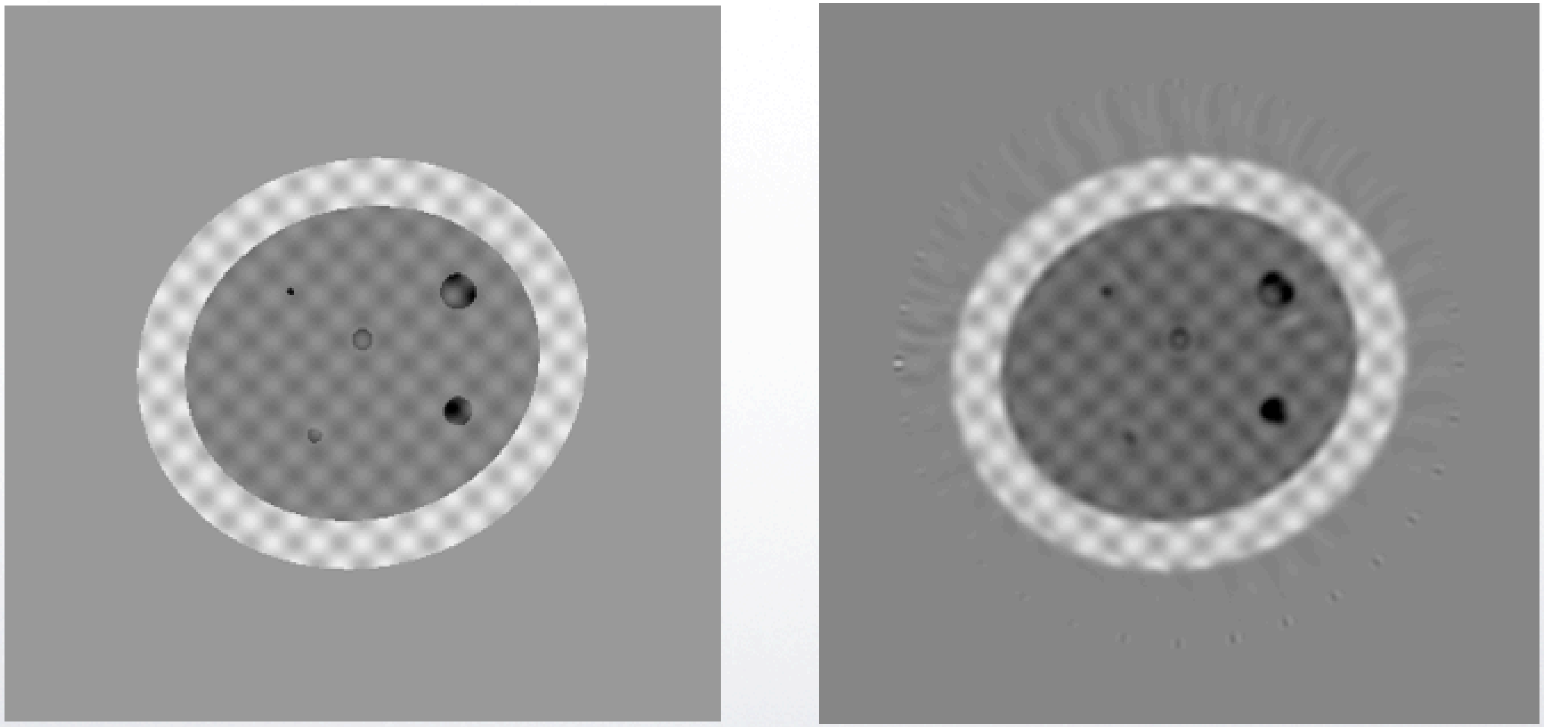
$$(R_s'(f))^* r)(x) = \int_0^T z(x,t) \frac{\partial^2 u(x,t)}{\partial t^2} dt$$

$$\frac{\partial^2 z}{\partial t} = c^2(x) \Delta z \text{ for } x_2 > 0$$

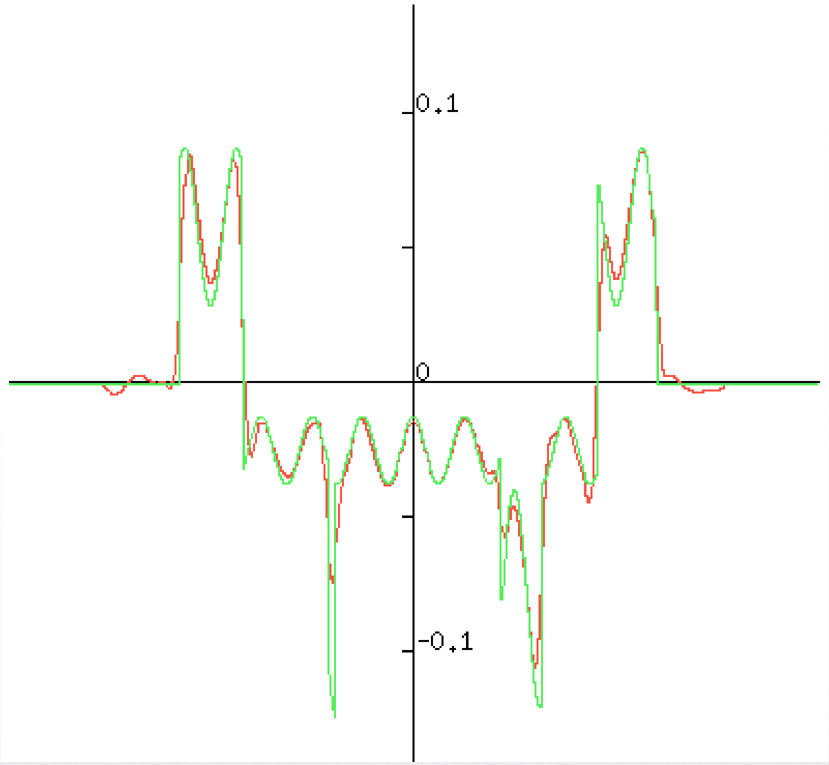
$$\frac{\partial z}{\partial x_2} = r \text{ on } x_2 = 0$$

$$z = 0,\ t > T$$

Frequency 300kHz, 3 sweeps, smallest tumor 2 mm



Horizontal cross section through smallest tumor



Condition for the initial approximation:

$$-\Delta u_0 - k^2(1+f_0)u_0 = \delta(x-s)$$

$$-\Delta u - k^2(1+f_0)u = -k^2(f-f_0)u + \delta(x-s).$$

First step of iteration:

$$-\Delta u - k^2(1+f_0)u = -k^2(f-f_0)u_0 + \delta(x-s).$$

Highly necessary condition for convergence:

$$|\text{phase}(u) - \text{phase}(u_0)| < \pi.$$

WKB-approximation:

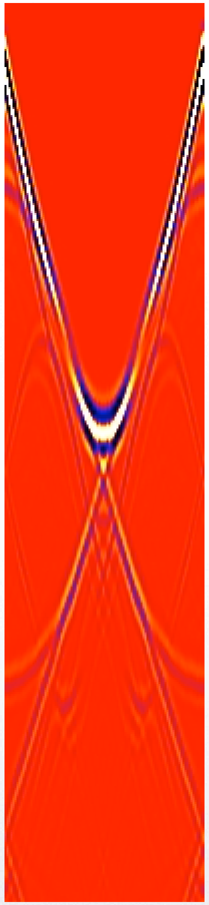
$$u \approx A \exp{(ik\Phi)} \qquad u_0 \approx A_0 \exp{(ik\Phi_0)}$$

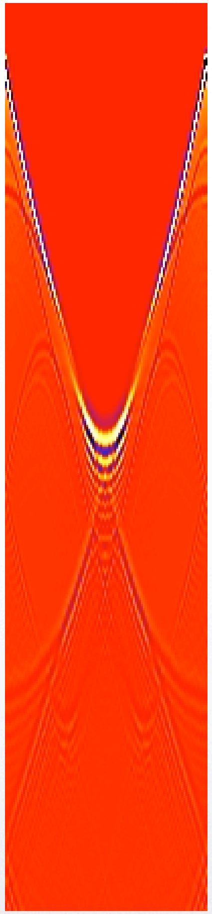
$$\Phi \approx \Phi_0 + \frac{1}{2}\int (f - f_0) ds$$

$$\text{phase}(u) - \text{phase}(u_0) \approx \frac{k}{2}\int (f - f_0) ds$$

$$|\int (f - f_0) ds| < \frac{2\pi}{k} = \lambda$$

data for
100 kHz



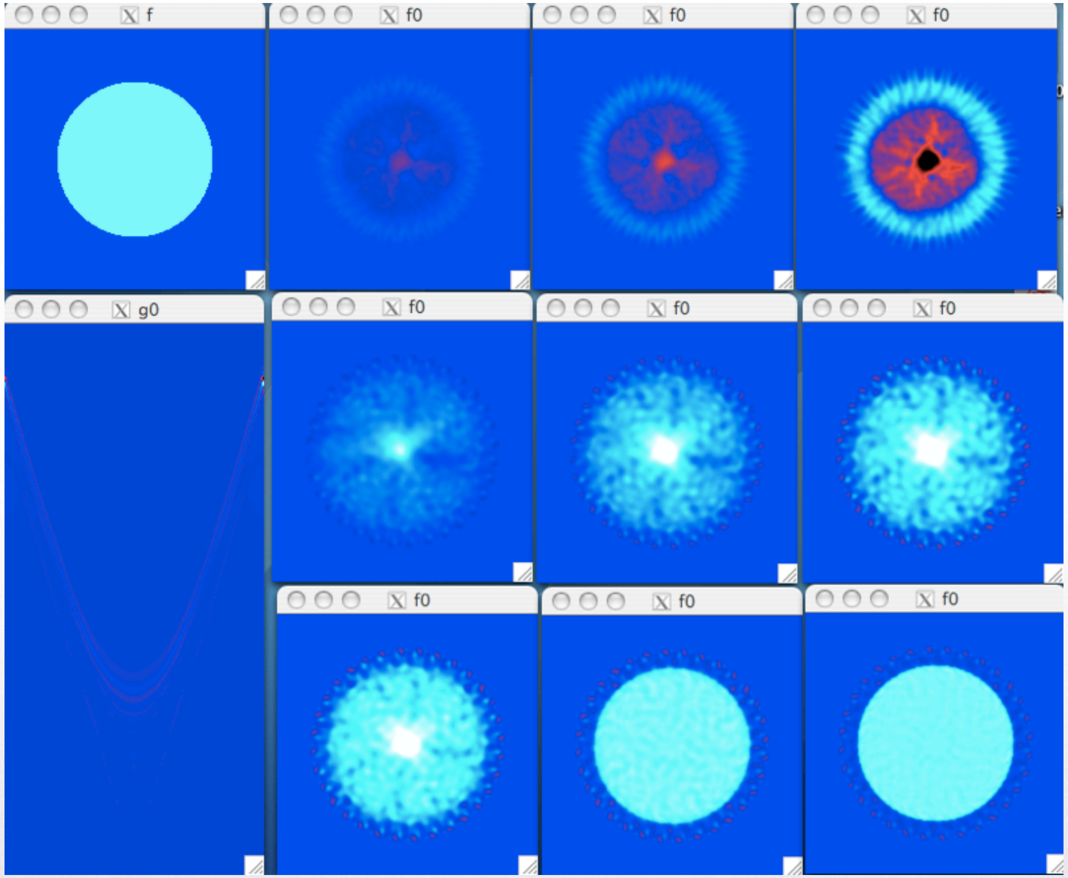


data for
300 kHz

Method of false frequency

original

data



300kHz

100kHz

300kHz

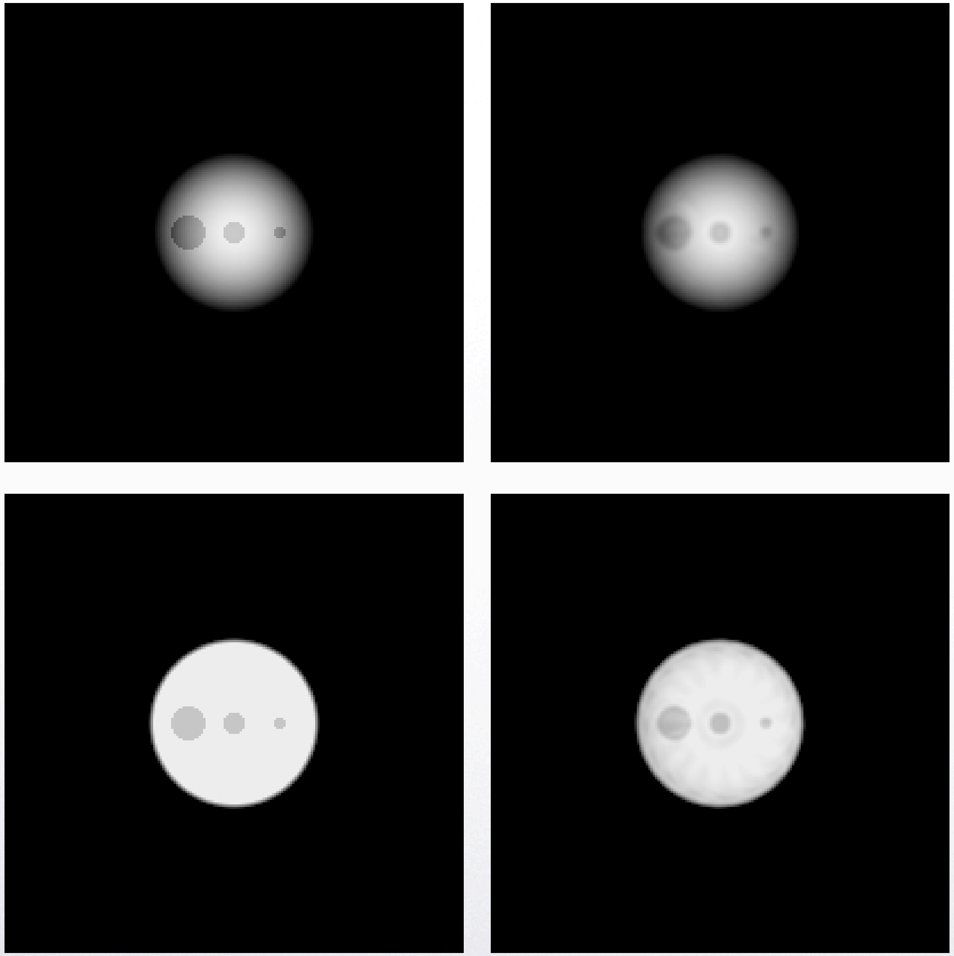
Questions:

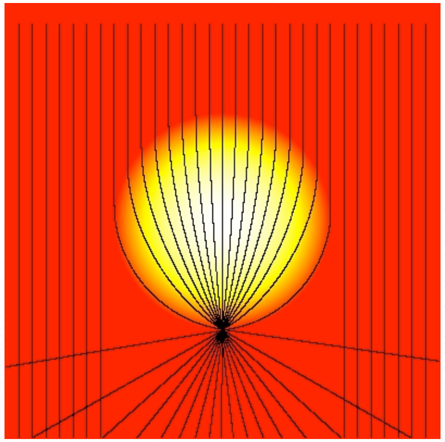
How does the algorithm work in the presence of caustics?

How does the algorithm work for trapped rays?

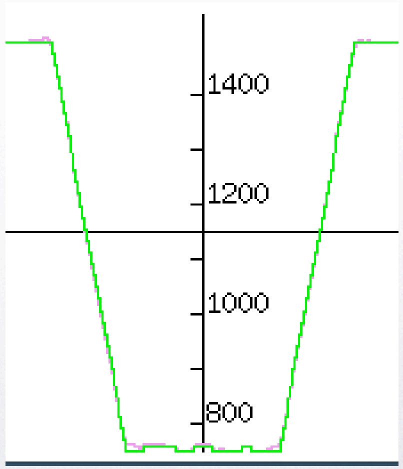
Answer:

The algorithm doesn't even realize the presence of caustics and trapped rays.





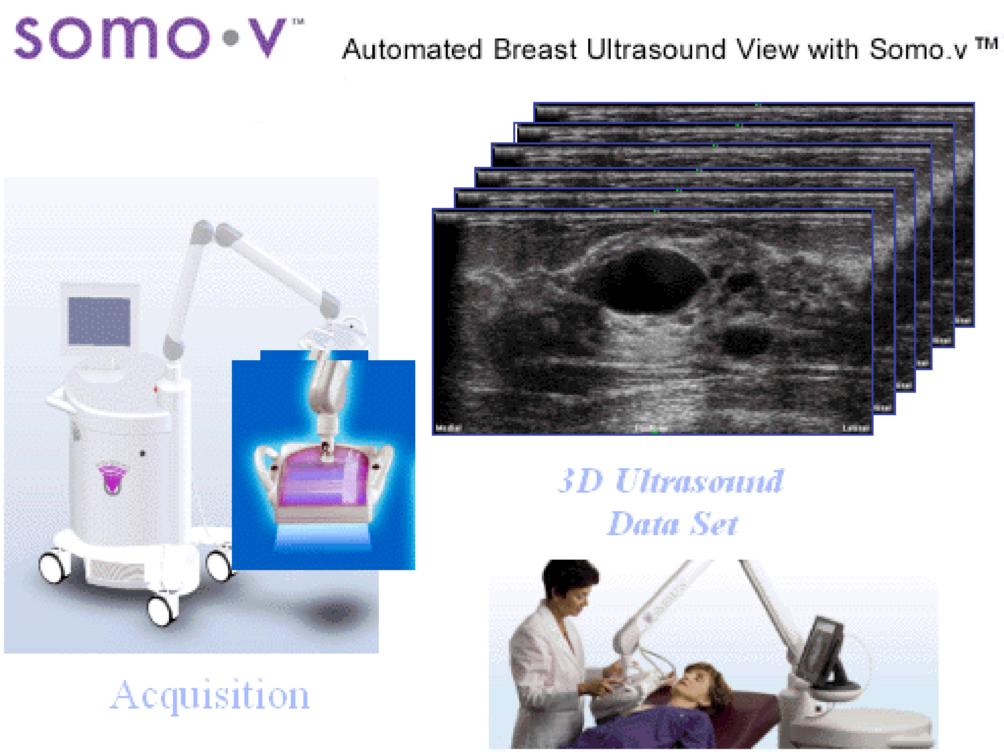
Luneberg lens

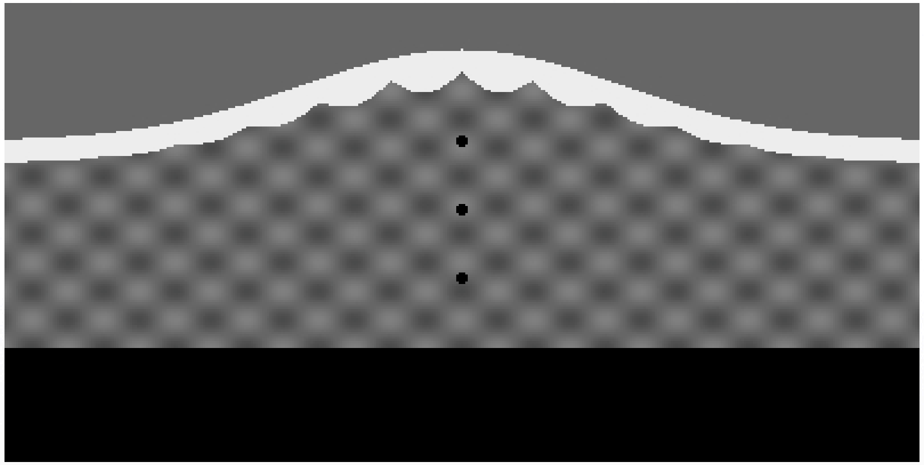


crater

Reflection Imaging

3D scanner of U-Systems





10 cm

diameters of
tumors 2.5 mm.
20 sources on top

5 sweeps of Kaczmarz at
100 kHz (100% bandwidth)

5 sweeps of Kaczmarz at
200 kHz (100% bandwidth)

Conclusions:

Ultrasound tomography close to clinical use

Reconstruction algorithms available for resolution down to 2mm (say)

Low frequency measurements (at 100 kHz, say) or good initial approximation needed

Future: Reflection imaging