Impedance-Acoustic Imaging

Otmar Scherzer

Computational Science Center, University Vienna, Austria
&
Radon Institute of Computational and Applied Mathematics (RICAM), Linz, Austria

Photoacoustic Example

In-vivo ligation of the Ramus interventricularis anterior (= LAD) to induce myocardial infarction: 30min ligation, 120min reperfusion

 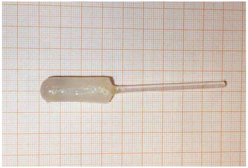

Figure: Tomograph, Probe, Experiment

Photoacoustic

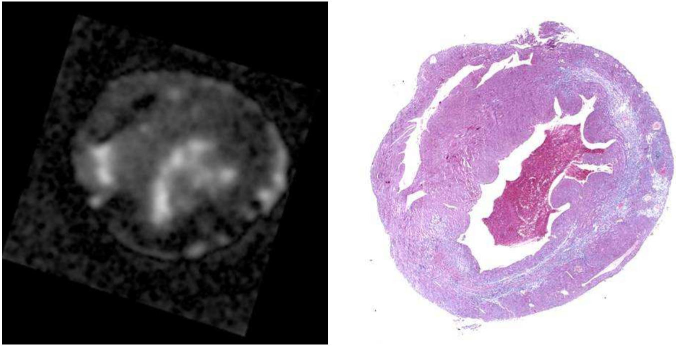

Figure: Photoacoustic Imaging, Histology

Grossauer, Holutta, Jaschke, Nuster, Paltauf, S.
No physical quantity.

Impedance–Acoustic Idea

Physical principle:

1. Induce electrical current
2. Induced current induces thermal heating
3. Induced heating produces ultrasound waves

Reconstruction:

1. Photoacoustic reconstruction $3 \to 2$
2. Conductivity reconstruction $2 \to 1$

Modeling Equations: Electric Potential

$B \subset \mathbb{R}^n$ domain of interest – body, S surface.

- Time dependent voltage $F(x,t) = f(x)\sqrt{g(t)}$ on S.
- Induced electric potential $U(x,t) = u(x)\sqrt{g(t)}$ on B.
  u is quasi-static electric potential.
- $g \sim$ amount of applied electrical power.

Modeling assumption: Electric potential reaches its state of equilibrium immediately

Joule's Law

Relation between

Rate of absorbed electrical power density $\dot{Q}(x,t)$ and the *electric potential*

$$\dot{Q}(x,t) = \sigma(x)|\nabla u(x)|^2 g(t).$$

where u solves quasi-static equation

$$\begin{aligned} \nabla \cdot (\sigma(x)\nabla u(x)) = 0 \qquad & \text{in } B, \\ u(x)|_S = f(x) \qquad & \text{on } S\,. \end{aligned}$$

Rate of change of temperature:

$$\dot{T}(x,t) = \frac{1}{\rho(x,t)c(x)}\dot{Q}(x,t),$$

- $c(x)$ is the specific heat capacity
- $\rho(x,t)$ is the mass density

Model assumption is valid if current is only applied for a short time – we can neglect thermal diffusion.

$\Rightarrow$ Pulsed voltage (which is proportional to g).

Physical Quantities

- (Robinson, Richardson, Green and Preece) The specific heat capacity and density of breast fat: $c = 2.43\mathrm{J}/(\mathrm{gK})$ and $\rho = 0.934\mathrm{g}/\mathrm{cm}^3$
- cube of $1cm$ side-length: $0,934g$. Electrical resistance $R = \sigma^{-1}$length/area $= 250\mathrm{Ohm}$.
- Specific electrical conductivity in adipose tissue: $\sigma = 0.4/(\mathrm{Ohmm})$.
- Pulse of $\Delta t = 1\mu s$ with $\sigma|\nabla u| = I = 3\mathrm{A}$
- Resulting temperature change: $\Delta T = 0.99\,\mathrm{mK}$.

Temperature change seems large enough to produce ultrasound waves.
Physical parameters in accordance with *high frequency surgery*.

Linearized Expansion Equation

Change of temperature is related to change of density and to the change of pressure:

$$\beta(x)\,\dot{T}(x,t) = \frac{1}{v_s^2}\dot{p}(x,t) - \dot{\rho}(x,t),$$

- v_s is the speed of sound,
- $\beta(x)$ is the thermal expansion coefficient,
- change of density is related to velocity

$$\dot{\rho}(x,t) = -\rho_0 \nabla \cdot v(x,t)\ .$$

- Euler equation: $\rho_0 \dot{v}(x,t) = -\nabla p(x,t)$.

Combination

of the equations for rate of changes of pressure and change of density yields:

$$\frac{1}{v_s^2}\ddot{p}(x,t) - \Delta p(x,t) = \frac{\beta(x)}{\rho_0 c(x)}\sigma(x)|\nabla u(x)|^2\dot{g}(t).$$

Summary

Take $g = \delta$ (impulse), then

$$\begin{aligned} \nabla \cdot (\sigma \nabla u(x)) &= 0 \quad \text{in } B, \\ u(x)|_S &= f \quad \text{on } S, \end{aligned}$$

and

$$\begin{aligned} \ddot{p}(x,t) - \Delta p(x,t) &= 0 \quad \text{in } \mathbb{R}^n, \\ p(x,0) &= \sigma(x)|\nabla u(x)|^2 \chi_B(x) \text{ in } \mathbb{R}^n, \\ \dot{p}(x,0) &= 0 \text{ in } \mathbb{R}^n . \end{aligned}$$

Reconstruction algorithm for σ

- Time reversal algorithm $p \to \tilde{u}$,

  $$\tilde{u}(x) := \sigma(x)|\nabla u(x)|^2 \ .$$

  Not the emphasize of this talk.

- Reconstruct σ from $\tilde{u}$ with an iteration method.

Similar problems derived for different measurement setups by [Ammari et al, Nachman et al].

Formal Newton Algorithm

u_σ solves

$$\boxed{\nabla \cdot (\sigma \nabla u) = 0, \qquad u|_S = f.}$$

Directional derivative of u with respect to σ:

$$v_\tau := \lim_{h \to 0} \frac{u_{\sigma + h\tau} - u_\sigma}{h}$$

is the solution of

$$\boxed{\nabla \cdot (\sigma \nabla v_\tau) = -\nabla \cdot (\tau \nabla u_\sigma), \qquad v_\tau|_S = 0.}$$

2 Step Newton

Let

$$E = \sigma |\nabla u_\sigma|^2$$

be the reconstructed energy density and

$$E'(\sigma)\tau = \tau |\nabla u_\sigma|^2 + 2\sigma \nabla u_\sigma \cdot \nabla v_\tau .$$

$\sigma_n \approx \hat{\sigma}$ (true solution), then a Newton-step consists in solving

$$E'(\sigma_n)\Delta = E - \sigma_n |\nabla u_{\sigma_n}|^2$$

and the update $\sigma_{n+1} = \sigma_n + \Delta$.
Computationally expensive inversion of $E'(\sigma) \Rightarrow$

$$E'(\sigma)\tau = (M_\sigma + P_\sigma)\tau,$$

with

$$M_\sigma \tau := \tau |\nabla u_\sigma|^2 \quad \text{and} \quad P_\sigma \tau := 2\sigma \nabla u_\sigma \cdot \nabla v_\tau .$$

Multiplication operator can be inverted computationally easily, but maybe has to be regularized.

Iterative Method, similarly to Ammari et al.

Given E, f, and σ_n,

- calculate ∇u_{σ_n},
- set $\tau := \frac{E}{|\nabla u_{\sigma_n}|^2} - \sigma_n$,
- calculate the solution v_τ of the linearized problem,
- update $\sigma_{n+1} := \frac{E - 2\sigma \nabla u_{\sigma_n} \cdot \nabla v_\tau}{|\nabla u_{\sigma_n}|^2}$.

Data and Photoacoustic Back-Projection $T = 4$

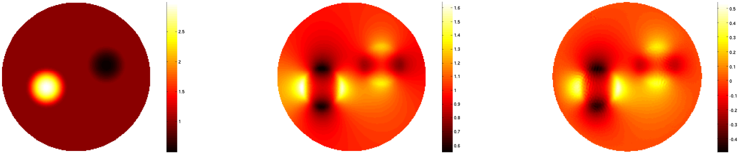

Figure: Exact conductivity, $\sigma|\nabla u_\sigma|^2$, reconstructed at $T = 4$

Reconstruction

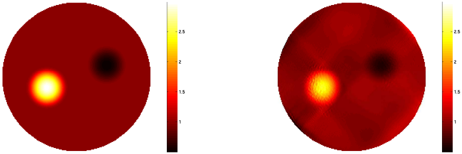

Figure: Exact and reconstructed conductivity σ

References

1. Ammari, H. and Bonnetier, E. and Capdeboscq, Y. and Tanter, M. and Fink, M.: Electrical impedance tomography by elastic deformation, SIAM Appl. Math. (2008)
2. Ammari, H. and Capdeboscq, Y. and Kang, H. and Kozhemyak, A.: Mathematical models and reconstruction methods in magneto-acoustic imaging, EJAM (2009)
3. Nachman, A. and Tamasan, A. and Timonov, A., Conductivity imaging with a single measurement of boundary and interior data, Inverse Problems (2007)
4. Gebauer, B. and Scherzer, O.: Impedance-acoustic tomography, SIAM Appl. Math. (2008)

Thanks

The work is supported by the Austrian Science Fund, NFN Project Photoacoustic Imaging, Projects P10501 and P10505.

Thanks to Günther Paltauf, KFU Graz for support with physical quantities.

Thank you for your attention