

Extremal Kähler metrics

June 28 – July 3 2009

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

- 16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
Lecture rooms available after 16:00 (if desired)
- 17:30–19:30 Buffet Dinner, Sally Borden Building
- 20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and small assortment of snacks available on a cash honour-system.

Monday

- 7:00–8:45 Breakfast
- 8:45–9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
- 9:00–10:00** Nefton Pali (Paris), *Degenerate Complex Monge-Ampère Equations*.
- 10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11.30** Joel Fine (Brussels), *Calabi flow and projective embeddings*.
- 11:30–13:00 Lunch
- 13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
- 14:00 Group Photo; meet on the front steps of Corbett Hall
- 14:10–15.10** Gabriele La Nave (Yeshiva, New York), *Kähler-Ricci flow and V-soliton equations: flow versus static equations*.
- 15:10–15:30 Coffee Break, 2nd floor lounge, Corbett Hall.
- 15.30–16.30** Valentino Tosatti (Harvard), *Degenerations of Calabi-Yau metrics*.
- 16.30–17.30** Gábor Székelyhidi (Columbia), *Greatest lower bounds on the Ricci curvature of Fano manifolds*.
- 17:30–19:30 Dinner

Tuesday

- 7:00–9:00 Breakfast
9:00–10:00 Sean Paul (Wisconsin), *Hyperdiscriminant polytopes, Chow Polytopes, and K-energy asymptotics on algebraic manifolds.*
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30 Bo Berndtsson (CTH, Gothenburg), *Curvature of direct image bundles and some applications to Kähler Geometry.*
11:30–13:00 Lunch
13:30–14.30 Xiaohua Zhu (Beijing), *Stability of Kähler-Ricci flow.*
14:30–15:00 Coffee Break, 2nd floor lounge, Corbett Hall.
15.00–16.00 Hajime Tsuji (Sophia, Tokyo), *Semipositivity of relative canonical bundle via Kähler-Einstein currents and Monge-Ampère foliations.*
16.30–17.30 Richard Thomas (London), *Weighted projective embeddings and orbifold cscK metrics.*
17:30–19:30 Dinner

Wednesday

- 7:00–9:00 Breakfast
9:00–10:00 Alberto Della Vedova (Parma), *K-stability and singularities.*
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30 Charles Boyer (New Mexico), *The Kähler cone versus the Sasaki cone.*
11:30–13:30 Lunch
Free Afternoon
17:30–19:30 Dinner

Thursday

- 7:00–9:00 Breakfast
9:00–10:00 Akito Futaki (Tokyo), *Constructing Kähler-Ricci solitons from Sasaki-Einstein manifolds.*
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30 Christina Tønnesen-Friedman (Union College), *Extremal Kähler metrics on projective bundles over a curve.*
11:30–13:00 Lunch
13:30–14.30 Yuiji Sano (Kyushu), *An example of asymptotically Chow unstable manifolds with constant scalar curvature.*
14:30–15:00 Coffee Break, 2nd floor lounge, Corbett Hall.
15.00–16.00 Song Sun (Wisconsin), *Kähler quantization and the space of Kähler metrics.*
16.30–17.30 Bing Wang (Princeton), *Kähler Ricci flow on Fano manifolds.*
17:30–19:30 Dinner

Friday

- 7:00–9:00 Breakfast
9:00–10:00 Santiago Simanca (New Mexico), *The L^2 norm of the second fundamental form of isometric immersions*
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30 Informal Discussions
11:30–13:30 Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Extremal Kähler metrics

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Bo Berndtsson** (CTH, Gothenberg)

Title: *Curvature of direct image bundles and some applications to Kähler Geometry*

Abstract: I will discuss a theorem on positivity of vector bundles that appear as direct images of adjoint bundles of (semi)positive line bundles over a Kähler fibration. Special cases of the theorem are results on the relative canonical bundles of a fibration (corresponding to trivial line bundles) and on variations of Kähler metrics over a fixed manifold (corresponding to trivial fibrations). I will focus on the second aspect of the theorem with applications to the study of geodesics in the space of Kähler metrics and generalized Moser-Trudinger inequalities (the last application is from a recent article by Robert Berman).

Speaker: **Charles Boyer** (New Mexico)

Title: *The Kähler cone versus the Sasaki cone*

Abstract: It is well-known that Sasakian geometry is contact geometry with a transverse Kähler structure. The analogue of the Kähler cone is called the Sasaki cone; however, there are important differences. Instead of certain cohomology classes the Sasaki cone is parameterized by Reeb vector fields lying in the Lie algebra of a certain torus. Nevertheless, both give rise to an important class of metrics, and one can ask about the existence of extremal metrics in the sense of Calabi. In this talk I will discuss both similarities and differences concerning the existence of extremal metrics mainly through the use of examples.

Speaker: **Alberto Della Vedova** (Parma)

Title: *K-stability and singularities*

Abstract: To check the K-stability of a smooth polarized manifold (M,L) is need to compute the Futaki invariant of every test configuration for (M,L) . This leads, in principle, to consider degenerations of M also to very singular schemes. In this talk we show that is sufficient to consider smooth test configurations with reduced simple normal crossing central fibers, and we discuss some applications. The talk is based on a joint work with Arezzo and La Nave.

Speaker: **Joel Fine** (Brussels)

Title: *Calabi flow and projective embeddings*

Abstract: Let $X \subset \mathbb{C}P^N$ be a smooth projective variety. I will describe a flow, called balancing flow, which tries to move the embedding of X until it is balanced (has zero centre of mass in $\mathbb{C}P^N$). I will then explain how this flow is related to Calabi flow. If $L \rightarrow X$ is a Kähler manifold with ample line bundle, high powers L^k give embeddings $X \rightarrow \mathbb{C}P^N$ in larger and larger projective spaces. For each k , we evolve the embedding via balancing flow and restrict the Fubini-Study metric to X to get a sequence of metric flows. Provided the initial embeddings are correctly chosen, this sequence of flows converges to Calabi flow, for as long as the latter exists. This can be seen as a parabolic analogue of a result of Donaldson, which says that when a cscK metric exists in $c_1(L)$, L^k gives balanced embeddings for all large k and the balanced metrics converge to the cscK metric.

Speaker: **Akito Futaki** (Tokyo)

Title: *Constructing Kähler-Ricci solitons from Sasaki-Einstein manifolds*

Abstract: This is a joint work with Mu-Tao Wang.

Speaker: **Gabriele La Nave** (Yeshiva, New York)

Title: *Kähler-Ricci flow and V-soliton equations: flow versus static equations*

Abstract: We will show how the Kähler-Ricci flow can be completely described by an exact elliptic equation of soliton type on some family of principal bundles over the origin manifold (in fact there is a one-to-one correspondence) and how the variation of symplectic quotients of the said family corresponds to some form of surgery. We will then discuss applications to a conjecture of Tian's in the context of performing a Geometric version of the Minimal Model Program (via the Kähler-Ricci flow) and some applications to the convergence of the Kähler-Ricci flow on Fano manifolds.

Speaker: **Nefton Pali** (Paris)

Title: *Degenerate Complex Monge-Ampère Equations*

Abstract: We prove existence and uniqueness of the solutions of degenerate complex Monge-Ampère equations for some cohomology classes at the boundary of the Kähler cone. This is a joint work with Professor Jean-Pierre Demailly.

Speaker: **Sean Paul** (Wisconsin)

Title: *Hyperdiscriminant polytopes, Chow Polytopes, and K-energy asymptotics on Algebraic manifolds*

Abstract: Let (X, L) be a polarized algebraic manifold. I have recently proved that the Mabuchi energy of (X, L) is bounded from below along any degeneration if and only if the Hyperdiscriminant polytope (of X) contains the Chow polytope with respect to the various Kodaira embeddings. In particular I can show that the asymptotic expansion of the Mabuchi energy along any degeneration exists and is logarithmic (previously only known for hypersurfaces), and the coefficient of blow up is an integer – moreover, this integer is given by minimizing the integral linear functional corresponding to the degeneration over the two polyhedra of the title. I define the generalized Futaki invariant to be the difference of the minima.

Speaker: **Yuiji Sano** (Kyushu)

Title: *An example of asymptotically Chow unstable manifolds with constant scalar curvature*

Abstract: In this talk, I will explain about an example of polarized manifolds which admit constant scalar curvature metrics, but are not asymptotically Chow semistable. Our example is a non-symmetric toric Fano manifold with Kähler-Einstein metrics discovered by Nill and Paffenholz very recently. This talk will be based on joint works with Futaki and Ono (arXiv:0811.1315) and with Ono and Yotsutani (arXiv:0906.3836).

Speaker: **Santiago Simanca** (New Mexico)

Title: *The L^2 norm of the second fundamental form of isometric immersions*

Speaker: **Song Sun** (Wisconsin)

Title: *Kähler quantization and the space of Kähler metrics*

Abstract: In this talk we will show that for a projective manifold, the space of Kähler metrics is the metric limit of a sequence of finite dimensional symmetric spaces. As a result, this yields alternative proofs of some metric properties of the space of Kähler metrics.

Speaker: **Gábor Székelyhidi** (Columbia)

Title: *Greatest lower bounds on the Ricci curvature of Fano manifolds*

Abstract: On Fano manifolds we study the supremum of the possible t such that there exists a metric in the first Chern class with Ricci curvature bounded below by t . This is the same as the existence time for Aubin's continuity method for finding Kähler-Einstein metrics. For the projective plane blown up in one point we show that this supremum is $6/7$.

Speaker: **Richard Thomas** (Imperial College London)

Title: *Weighted projective embeddings and orbicscK metrics*

Abstract: I will describe joint work with Julius Ross.

We embed polarised orbifolds with cyclic stabiliser groups into weighted projective space via a weighted form of Kodaira embedding. Dividing by the (non-reductive) automorphisms of weighted projective space

then formally gives a moduli space of orbifolds. This can be expressed as a *reductive* quotient and so a GIT problem, thus defining a notion of stability for orbifolds.

The standard package of results extends to this setting: balanced embeddings, asymptotics and cscK metrics, K-stability, slope stability etc. There are also applications to manifolds, to KE, cscK and Sasaki-Einstein metrics.

Speaker: **Christina Tønnesen-Friedman** (Union College)

Title: *Extremal Kähler metrics on projective bundles over a curve*

Abstract: Let $M = P(E) \rightarrow \Sigma$ be the complex manifold underlying the total space of the projectivization of a holomorphic vector bundle E over a compact complex curve Σ of genus ≥ 2 . We address the existence problem of extremal Kähler metrics on such bundles.

As a concrete example, we set up a construction of T^2 symmetric, semi-simple, and rigid extremal Kähler metrics on a $\mathbb{C}P^2$ bundle over Σ . Pursuing polynomial solutions we achieve some concrete almost Kähler examples. For large genus of Σ , these examples are not always positive definite and they can be used to exhibit instability of some Kähler classes. This in turn will lead to non-existence results.

In the general case we show that the splitting of E as a direct sum of stable sub-bundles is necessary and sufficient condition for the existence of extremal Kähler metrics in sufficiently small Kähler classes.

The talk is based on joint work with V. Apostolov, D. Calderbank, and P. Gauduchon

Speaker: **Valentino Tosatti** (Harvard)

Title: *Degenerations of Calabi-Yau metrics*

Abstract: We are interested in the behaviour of families of Ricci-flat Kähler metrics on a compact Calabi-Yau manifold, with Kähler classes approaching the boundary of the Kähler cone. We will give an overview of some examples and results concerning this question, and explain the connections to the theory of degenerate complex Monge-Ampère equations and to birational geometry.

Speaker: **Hajime Tsuji** (Sophia, Tokyo)

Title: *Semipositivity of relative canonical bundle via Kähler-Einstein currents and Monge-Ampère foliations*

Abstract: I would like to explain my recent work on the variation of Kähler-Einstein currents and canonical measures for a projective family of LC pairs. And explain how it connects to algebraic geometry through the moduli spaces of metrized canonical models.

Speaker: **Bing Wang** (Princeton)

Title: *Kähler Ricci flow on Fano manifolds*

Abstract: This is the joint work with Xiuxiong Chen. We define a tamed condition for Kahler Ricci flow on Fano manifolds. If a flow is tamed and the local alpha invariants of the underlying manifold is big, then the flow must converge to a Kahler Einstein metric exponentially fast. The local alpha invariants are algebraic invariants which can be calculated explicitly by algebraic geometry method. Therefore by showing the tamedness condition, we give a new method to find Kahler Einstein metrics. As an application, we show that every two dimensional Kahler Ricci flow is tamed. By calculating the local alpha invariants, we reprove Tian's theorem of Kahler Einstein metrics on Fano surfaces.

Speaker: **Xiaohua Zhu** (Beijing)

Title: *Stability of Kähler-Ricci flow*

Abstract: In this talk, I will prove the following theorem: Let M be a compact Kähler manifold which admits a Kähler-Einstein metric ω_{KE} or a Kähler-Ricci soliton ω_{KS} . Then a Kähler-Ricci flow will converge exponentially to the limit ω_{KE} or ω_{KS} as long as an initial Kähler potential in $[\omega_{KE}]$ or in $[\omega_{KS}]$ is small.