Banff International Research Station

for Mathematical Innovation and Discovery

Transversal and Helly-type theorems in Geometry, Combinatorics and Topology
09w5047,
September 20 to September 25, 2009.

MEALS

*Breakfast (Buffet): 7:00 – 9:30 am, Sally Borden Building, Monday – Friday
*Lunch (Buffet): 11:30 am – 1:30 pm, Sally Borden Building, Monday – Friday
*Dinner (Buffet): 5:30 – 7:30 pm, Sally Borden Building, Sunday – Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverages in those areas.*

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk – Professional Development Centre - open 24 hours)
17:30-19:30	Buffet Dinner
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00-8:45	Breakfast
8:45-9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:15-10:00	Richard Pollack. *Double Permutation Sequences and Arrangements of Planar Families of Convex Sets.*
10:00-10:15	Coffee Break, 2nd floor lounge, Corbett Hall
10:15-10:40	Helge Tverberg . *On two problems from the Asinowski-H-K-T paper 2003.*
10:40-10:55	Break
10:55-11:20	Xavier Goaoc. *The growth rate of families of (geometric) permutations.*
11:30-13:00	Lunch
13:00-14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:30-14:55	Tamon Stephen. *Colourful Simplicial Depth.*

14:55-15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15-15:40	Natan Rubin. *Transversals of Convex Polytopes in 3-space: Combinatorics and Algorithmics.*
15:40-15:55	Break
15:55-16:20	Otfried Cheong. *Isolated line transversals to convex polytopes in R^3.*
16:20-16:35	Break
16:35-17:00	Andras Bezdek. *On some finite packing problems.*
17:30-19:30	Dinner

Tuesday

7:00-9:00	Breakfast
9:15-10:00	Luis Montejano. *Topological Transversals to a Family of Convex Sets.*
10:00-10:20	Coffee Break, 2nd floor lounge, Corbett Hall
10:20-10:45	R.N. Karasev. *Theorems of Borsuk-Ulam type for flats and common transversals.*
10:45-10:50	Break
10:50-11:15	Michel Pocchiola. *A relative homotopy theorem for arrangements of double pseudolines.*
11:30-13:30	Lunch
14:30-14:55	Ricardo Strausz. *A generalisation of Barany-Carathedory Theorem in dimension 3.*
14:55-15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15-15:40	David Csaba Toth. *Containment queries for colorful simplices.*
15:40-15:45	Break
15:45-16:10	W. Kuperberg. *Packing densities of convex cones.*
17:30-19:30	Dinner

Wednesday

7:00-9:00	Breakfast
9:15-10:00	R.N. Karasev. *Cohomology multiplication in topological Tverberg-like theorems.*
10:00-10:20	Coffee Break, 2nd floor lounge, Corbett Hall
10:20-10:45	Jorge Ramirez Alfonsin. *Transversals to the convex hull of all k-set of Discrete Subsets of R^n.*
10:45-10:50	Break
10:50-11:15	David Larman . *Skeleta and Shadow Boundaries of Convex Bodies.*
11:30-13:30	Lunch
	Free Afternoon
17:30-19:30	Dinner

Thursday

7:00-9:00	Breakfast
9:15-10:00	Andreas Holmsen. *Some new results concerning T(k)-families in the plane.*
10:00-10:20	Coffee Break, 2nd floor lounge, Corbett Hall
10:20-10:45	Ferenc Fodor. *The T(4) property of families of unit disks.*
10:45-10:50	Break
10:50-11:15	Valeriu Soltan. *Helly-type Results on Common Supports of Convex Bodies.*
11:30-13:30	Lunch
14:30-14:55	Deborah Oliveros. *K-Tolerance Helly Theorem and covering numbers for hypergraphs.*
14:55-15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15-15:40	Pablo Soberon. *Piercing numbers for balanced families.*
15:40-15:45	Break
15:45-16:10	Antoine Deza. *More bounds on the diameter of convex polytopes.*
16:10-16:15	Break
16:15-16:40	Gergely Ambrus. *Recent developments on the polarization problem.*
17:30-19:30	Dinner

Friday

7:00-9:00	Breakfast
9:15-10:00	Karoly Bezdek. *Tarski's plank problem revisited.*
10:00-10:20	Coffee Break, 2nd floor lounge, Corbett Hall
10:20-10:45	Aladar Heppes. *Transversals in superdisjoint T(3)-families of translates.*
10:45-10:50	Break
10:50-11:15	Javier Bracho. *Line transversals to intervals and descendants.*
11:30-13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

BIRS Workshop 09w5047
Transversal and Helly-type theorems in Geometry, Combinatorics and Topology

Abstracts

September 20 to September 25, 2009

Arseniy Akopyan

Some generalizations of a diameter of sets and Jung problem.

Abstract: Let M be an metric space and for each p-element set $W \subset M$ there exists a q-element subset $U \subset W$ of diameter 1. Then M can be divided into parts $M_1, M_2, \ldots, M_{p-1}$ in a such way that $diamM_1 + diamM_2 + \ldots + diamM_{p-1} \leq [\frac{p-1}{q-1}]$.

Gergely Ambrus

Recent developments on the polarization problem

Abstract: The polarization problem states that for any system $u_1, \ldots, u_n$ of unit vectors in an n-dimensional real Hilbert space, there exists a unit vector v, such that $(u_1, v) \ldots (u_n, v) \geq n^{-n/2}$. In the talk we shall give a brief overview of the recent results. In particular, we pose a natural, stronger conjecture, and transform both problems to a geometric setting. By this means it turns out, that the strong polarization problem serves as the "proper" real analogue of the complex plank problem, which was proved by K. Ball in 2001.

Andras Bezdek

On some finite packing problems.

Abstract:

Karoly Bezdek

Tarski's plank problem revisited

Abstract: In the 1930's, Tarski introduced his plank problem at a time when the field of Discrete Geometry was about to born. It is quite remarkable that Tarski's question and its variants continue to generate interest in the geometric and analytic aspects of coverings by planks in the present time as well. The talk is of a survey type with some new results and with a list of open research problems on the discrete geometric side of the plank problem.

Ted. Bisztriczky

The $T(5)$ property of families of overlapping unit disks.

We consider a finite family $\mathcal{F}$ of unit disks in the plane with the properties:
$T(k)$: Any k-element subfamily of $\mathcal{F}$ has a (line) transversal, and
$O(d)$: The distance between the centres of any two elements of $\mathcal{F}$ is greater than d.
It is well known that $\mathcal{F}$ has a transversal in each of the following cases:

$$k = 3 \text{ and } d > 2(\sqrt{2})(\text{sharp})$$
$$k = 4 \text{ and } d > 4/\sqrt{3}(\text{sharp})$$
$$\text{and } \; k = 5 \text{ and } d \geq 2.$$

In this preliminary report, we present arguments that $\mathcal{F}$ has a transversal in the case that $k = 5$ and $d = \sqrt{3}$. Joint work with K. Böröczky and A. Heppes.

Otfried Cheong

Isolated line transversals to convex polytopes in R^3.

Abstract: A line ℓ is a *transversal* to a family F of convex polytopes in R^3 if it intersects every member of F. If, in addition, ℓ is an isolated point of the space of line transversals to F, we say F is

a *pinning* of ℓ. We show that any minimal pinning of a line by polytopes in R^3 such that no face of a polytope is coplanar with the line has size at most eight. If in addition the polytopes are disjoint, then it has size at most six. We completely characterize configurations of disjoint polytopes that form minimal pinnings of a line.

Antoine Deza

More bounds on the diameter of convex polytopes.

Abstract: Let $\Delta(d,n)$ be the maximum possible edge diameter over all polytopes defined by n inequalities in dimension d. The conjecture of Hirsch, formulated in 1957, states that $\Delta(d,n)$ is not greater than $n-d$. No polynomial bound is known for $\Delta(d,n)$, the best one being quasipolynomial and due to Kalai and Kleitman in 1992. Goodey showed in 1972 that $\Delta(4,10)=5$ and $\Delta(5,11)=6$. Recently, Bremner and Schewe proved that $\Delta(4,11)=\Delta(6,12)=6$. In this follow-up work, we show that $\Delta(4,12)=7$ and present evidence that $\Delta(5,12)=\Delta(6,13)=7$.

Ferenc Fodor

The T(4) property of families of unit disks.

Abstract: For a family $\mathcal{F}$ of n mutually disjoint unit disks in the plane, we show that if any four disks are intersected by a line then there is a line that intersects at least $n-1$ disks of $\mathcal{F}$.

This is joint work with T. Bisztriczky (University of Calgary) and D. Oliveros (UNAM).

Xavier Goaoc

The growth rate of families of (geometric) permutations.

Abstract: Let $[n]=\{1,...,n\}$ and let $P(m,k,n)$ denote the maximum size of a family of permutations on $[n]$ that has at most k distinct restrictions(*) to any m elements of $[n]$. How does the asymptotic behavior of $P(m,k,n)$, as n goes to infinity, depend on m and k? I will describe some results in this direction. This is joint work with Otfried Cheong (KAIST, Korea) and Cyril Nicaud (Univ. Marne-La-Vallee, France).

(*) Let s be a permutation on $[n]$ and X a subset of $[n]$. The restriction of s on X is the permutation t on X such that the orders induced on X by s and t are the same. Equivalently, for any $u,v\in X$ we have $t^{-1}(u)<t^{-1}(v)$ if and only if $s^{-1}(u)<s^{-1}(v)$.

Aladár Heppes

Transversals in superdisjoint $T(3)$-families of translates.

Abstract: Let K denote an oval, a centrally symmetric compact convex domain with non-empty interior. A family of translates of K is said to have property $T(k)$ if to every subset of at most k translates there exists a common line transversal intersecting all of them. Two translates, K_i and K_j are said to be φ-disjoint, $\varphi>0$, if the concentric φ-enlarged copies of K_i and K_j are disjoint. It is well known that in a φ-disjoint family of congruent discs $T(3)\Rightarrow T$ if $\varphi>\sqrt{2}$, and $T(3)\not\Rightarrow T$ if $\varphi<\sqrt{2}$. In this talk finite φ-disjoint $T(3)$-families of translates of an oval, different from the disk, will be investigated.

Andreas Holmsen

Some new results concerning T(k)-families in the plane.

Abstract: A $T(k)$-family is a finite family of convex sets in the plane such that every k admit a line transversal. It is known that every $T(k)$ family has a partial line transversal of size at least $a(k)|F|$ where $a(k)$ is a function that tends to 1 as k tends to infinity. Previous bounds on this function are due to Katchalski-Liu (1980) and Eckhoff (2008). I will present some new (and sharper) bounds on $a(k)$.

R.N. Karasev

Theorems of Borsuk-Ulam type for flats and common transversals.

In this talk we discuss some results on the topology of the real Grassmannian and its canonical vector bundle. These topological assertions are mostly formulated in terms of the cohomology index of the antipodal Z_2-action on the sphere space of the canonical bundle.

One corollary of these topological results is a theorem, that establishes the existence of a k-flat transversal for a family $\mathcal{F}$ of $d+1$ convex compact sets in R^d, provided that for any $K \in \mathcal{F}$ the intersection $K \cap \partial(\operatorname{conv} \bigcup \mathcal{F})$ has no antipodal points, and any $d-k$-dimensional linear image of $\bigcup \mathcal{F}$ is convex. Omitting the requirement of convexity of any $d-k$ dimensional image, we obtain the existence of an equidistant k-flat, instead of the transversal.

A corollary on partitioning d measures in R^d by a single hyperplane into parts of prescribed measure (compare the "ham-sandwich" theorem) is also given, generalizing some results of F. Breuer ("Uneven splitting of ham sandwiches." Discrete and Computational Geometry, 2009, DOI:10.1007/s00454-009-9161-7).

R.N. Karasev

Cohomology multiplication in topological Tverberg-like theorems.

In this talk we discuss the topological proofs of some theorems in discrete and convex geometry, focusing mostly on the Tverberg theorem and its relatives.

The main tool of proof is finding the obstruction to existence of a counterexample. In the case, where this obstruction is formulated as the Euler class of a (possibly equivariant) vector bundle, new results can be obtained by $\times$-multiplying the obstructions.

For example, by $\times$-multiplying the obstruction of the topological Tverberg theorem by the obstruction in the Brouwer fixed point theorem, we obtain the following.

The "dual" Tverberg theorem. *Suppose a family $\mathcal{F}$ of $(d+1)n$ hyperplanes in general position is given in R^d, where n is a prime power. Then $\mathcal{F}$ can be partitioned into n subfamilies of $d+1$ hyperplanes each so that all the simplexes, formed by the respective subfamilies, have a common point.*

Another example in obtained by "almost" $\times$-multiplying the obstruction in the topological Tverberg theorem by an appropriate power of the Euler class of the canonical bundle over the Grassmannian.

The Tverberg transversal theorem. *Let $0 \le m \le d-1$, let r_i $(i = 0, \dots, m)$ be powers of the same prime $r_i = p^{k_i}$. If p is odd, let $d-m$ be even.*

Let for each $i = 0, \dots, m$ f_i map continuously an $(r_i - 1)(d-m+1)$-dimensional simplex $\Delta_i = \Delta^{(r_i-1)(d-m+1)}$ to R^d.

Then every simplex Δ_i has r_i points $x_{i1}, x_{i2}, \dots, x_{ir_i} \in \Delta_i$ with pairwise disjoint supports so that all the points $f_i(x_{ij})$ are contained in a single m-flat.

These (and several similar, if the time limit allows) results will be discussed.

W. Kuperberg

Packing densities of convex cones.

Abstract: We consider the family C of convex bodies K in R^3 each of which is a cone over a convex plane disk, and packing densities of all members of this family. If the admissible packings allow translations of K, or translations of K and $-K$, the symmetric image of K, only, then there is a supremum smaller than 1 and an infimum greater than 0 for the packing densities of all K in C, and these extreme values are attained at certain members of C. Since these densities are affine

invariants, the packing density of each K in C depends only on its base. All four problems of finding the convex plane regions that produce cones whose packing densities of this sort are extreme remain open. The analogous four problems on covering are open as well. In this talk we give a motivation for considering arrangements of convex bodies consisting of translates of K and $-K$ only, and we discuss some partial results and related ideas.

David Larman

Skeleta and Shadow Boundaries Of Convex Bodies.

For a convex body K in Euclidean space, the s-skeleton of K is the set of points of K which are not at the centre of an $s+1$ dimensional disc contained in K. So, for a polytope, the 1-skeleton is the usual set of edges and vertices. I shall describe many of the unsolved problems relating to the s-skeleton. A result which, in many ways, sparked the surge of interest in polytopes, was the simplex algorithm. This guarantees, for a polytope, the existence of an edge path of finite length, between any two vertices. But is there a similar result for any convex body? It was proved, long ago, that there is a path, in the 1-skeleton, between any two exposed points of any convex body. Whether we can ensure that the length of the path is finite, remains open. One possible approach, is to extend the known results on shadow boundaries.

Luis Montejano

Topological Transversals to a Family of Convex Sets.

Abstract: We say that F has a topological n-transversal of index (m, k), $0 \leq n < m$, $0 \leq k \leq d-m$, if there are, homologically, as many transversal m-planes to F as m-planes through a fixed n-plane in R^{m+k}. The purpose of my lecture is to talk about some results concerning topological transversals and to use them, together with the Lusternik-Schnirelmann category and several versions of the colorful Helly Theorem of Lovasz, to obtain geometric results that, until know, can not be obtained by us only with geometric tools.

Deborah Oliveros

k-Tolerance Helly Type Theorem and covering numbers for hypergraphs.

Abstract: I will present some Helly type results for covering numbers in graphs and hypergraphs and some aplications of this results to classical Helly type theorems in convex sets as well as some aplications to piercing numbers.

This is a joint work with Luis Montejano.

Michel Pocchiola

A relative homotopy theorem for arrangements of double pseudolines.

Abstract: I will show that any two arrangements of double pseudolines of the same size are homotopic via a finite sequence of mutations during which the only moving curves are the curves that belong to the set difference of the two arrangements. The proof is based on an enhanced version of the Pumping Lemma of Habert and Pocchiola [1] of independant interest. A second application of this enhanced version of the Pumping Lemma will be discussed.

[1] L. Habert and M. Pocchiola. Arrangements of Double Pseudolines. In Proceedings of the 25th Annu. Sympos. Comput. Geom., pages 314-323, June 2009, Aahrus, Denmark.

Richard Pollack

Double Permutation Sequences and Arrangements of Planar Families of Convex Sets.

Abstract: We (re)introduce Double Permutation Sequences, which provide a combinatorial encoding of arrangements of convex sets in the plane. We also recall the notion of a topological affine plane and several (some new) of its properties. In particular, that there is a universal topological affine plane P (i.e. any finite arrangement of pseufolines is isomorphic to some arrangement of finitely many lines of P).

Most of this work is joint with Jacob E. Goodman and some involves numerous other people, among whom are Raghavan Dhandapani, Andreas Holmsen, Shakhar Smorodinsky, Rephael Wenger, and Tudor Zamfirescu.

Jorge Ramirez Alfonsin

Transversals to the convex hull of all k-set of Discrete Subsets of R^n.

Abstract: Let $k, d, h \geq 1$ be integers with $d \geq h$. What is the maximum positive integer n such that every set of n points in R^d has the property that the convex hull of all k-set have a transversal $(d-h)$-plane? What is the minimum positive integer n such that every set of n points in general position in R^d has the property that the convex hull of all k-set do not have a transversal $(d-h)$-plane? In this talk, we discuss these two questions. We define a special Kneser hypergraph and, by using the well-known h-Helly property, we relate our second question with the chromatic number of such hypergraphs. Moreover, we establish a connection (when $h = 1$) with the so called Kneser's conjecture.

This is a joint work with J. Arocha, J. Bracho and L. Montejano

Natan Rubin

Line Transversals of Convex Polytopes in 3-space: Combinatorics and Algorithmics.

Abstract: In the first part of the talk, we present an upper bound of $O(n^2k^{1+e})$, for any $e > 0$, on the combinatorial complexity of the set $T(P)$ of line transversals of a collection P of k convex polyhedra in R^3 with a total of n facets. Thus, when $k << n$, this is an improvement upon the previously best known bounds, which are nearly cubic in n. Our analysis is curiously related to the three dimensional variant of the following fundamental problem in the geometric transversal theory: Given a collection C of k pairwise-disjoint convex sets (of arbitrary description complexity!) in R^d, bound the maximum number $g_d(k)$ of so called geometric permutations, i.e., the maximum number of distinct orders in which the transversal lines visit individual elements of C.

The second part of the talk deals with a closely related algorithmic problem of ray-shooting at the above collection P of convex polyhedra. We show how to preprocess P into a data structure which uses $O^*(n\, poly(k))$ storage and answers ray-shooting queries in polylogarithmic time, provided that the ray origins are restricted to lie on a fixed line. This is a substantial improvement over previously known techniques which require super-quadratic storage (as a function of n–the number of facets). This result can be generalized to a number of other cases when the lines containing query rays have three degrees of freedom. Once again, handling distinct transversal orders is a key ingredient of the eventual solution.

Joint work with Haim Kaplan and Micha Sharir (SODA 2009, Algorithmica 2009).

Pablo Soberon

Piercing numbers for balanced families.

Abstract: We'll say that a finite family of convex sets has a $(p, q)_r$ property if for every p convex sets there are at least r q-tuples which are intersecting. We'll call the family balanced if every one of the p convex sets is in at least one of the q-tuples. We know that there is a finite upper bound for the piercing number of families with the $(p, q)_1$ property, here we will find piercing numbers for balanced families with some $(p, q)_r$ properties.

Valeriu Soltan

Helly-type Results on Common Supports of Convex Bodies.

Abstract: Following Dawson and Edelstein, we say that a family $\mathcal{F}$ of convex bodies in R^n has the property S provided there is a hyperplane H that supports every member of $\mathcal{F}$. Similarly, $\mathcal{F}$ has the property $S(k)$ if every k-membered subfamily of $\mathcal{F}$ has the property S. We discuss some results and problems related to the Helly-type condition $S(k) \Rightarrow S$.

Tamon Stephen

Colourful Simplicial Depth.

Abstract: The simplicial depth of a point p in R^d with respect to a finite set S of points is the number of $d+1$-sets from S whose convex hull contains p. In statistics, this is a measure of how well p represents S.

A natural generalization is to colour the points of S and consider only the colourful simplices containing p. We exhibit a configuration where p is in the convex hull of each of $d+1$ colours, but is only in d^2+1 colourful simplices. We conjecture that this is minimal and prove a quadratic lower bound. This result sharpens Bárány's Colourful Carathéodory Theorem, and gives an improved lower bound for monochrome simplicial depth.

Parts of this work are joint with A. Deza, S. Huang, T. Terlaky and H. Thomas.

Ricardo Strausz

A generalisation of Barany-Carathedory Theorem in dimension 3.

Abstract: We generalise the famous Barany-Carathedory's theorem to oriented matroids of Euclidian dimension 3. (This is a joint work with J. Bracho and J. Bokowski)

David Csaba Toth

Containment queries for colorful simplices.

Abstract: We study the worst-case complexity of the union of rainbow simplices determined by a set of colorful points in Euclidean space. We also consider possible data structures to support efficient containment queries.

Helge Tverberg

On two problems from the Asinowski-H-K-T paper 2003.

Abstract: The first problem deals with a pair of geometric permutations of disjoint translates $A, B, ...$ of an oval K in the plane. For which K's can one have both permutations $ABCX$ and $BXAC$ simultaneously, i.e. four given sets admit both these permutations. Here Lemma 3 from the paper is important. The second problem deals with a sharpening of Theorem 7 b. Here one wishes to find a more complete characterization of those K for which there are arbitrarily large families of translates of K, admitting three geometric permutations, of the forms WBACXW',WABCXW',WBXACW'.