

# Multivariable Complex Dynamics

## March 1-6, 2009

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

### SCHEDULE

#### Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

17:30–19:30 Buffet Dinner, Sally Borden Building

#### Monday

7:00–8:45 Breakfast

8:45–9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159

9:00–10:00 *The Cremona group and its finitely generated subgroups*, Serge Cantat

10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall

10:30–11:30 *Lyapunov exponents and bifurcation currents*, François Berteloot

11:30–13:00 Lunch

13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

14:00–15:00 *The Cremona group: old and new*, Igor Dolgachev

15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall

13:30–14:30 *Linearization and globalization*, Kyounghee Kim

17:30–19:30 Dinner

#### Tuesday

7:00–9:00 Breakfast

9:00–10:00 *The Cremona group and its finitely generated subgroups*, Serge Cantat

10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall

10:30–11:30 *Automorphisms of the Cremona group*, Julie Déserti

11:30–13:15 Lunch

13:15–13:30 Group photo

13:30–14:30 *Special sets of points in product of projective spaces*, Igor Dolgachev

14:30–15:00 Coffee Break, 2nd floor lounge, Corbett Hall

15:00–16:00 *Superattracting dynamics*, Mattias Jonsson

17:30–19:30 Dinner

## Wednesday

- 7:00–9:00** Breakfast  
**9:00–10:00** *Geometric currents and dynamical applications*, Romain Dujardin  
**10:00–10:30** Coffee Break, 2nd floor lounge, Corbett Hall  
**10:30–11:30** *Lee-Yang zeros and 2D rational dynamics*, Mikhail Lyubich  
**11:30–13:30** Lunch  
Free Afternoon  
**17:30–19:30** Dinner

## Thursday

- 7:00–9:00** Breakfast  
**9:00–10:00** *Geometric currents and dynamical applications*, Romain Dujardin  
**10:00–10:30** Coffee Break, 2nd floor lounge, Corbett Hall  
**10:30–11:30** *Lee-Yang zeros and 2D rational dynamics*, Mikhail Lyubich  
**11:30–13:30** Lunch  
**13:30–14:30** *Riemann surface laminations*, John Erik Fornæss  
**14:30–15:00** Coffee Break, 2nd floor lounge, Corbett Hall  
**15:00–16:00** *A Poincaré-Bendixson theorem for homogeneous vector fields and meromorphic connections*, Marco Abate  
**17:30–19:30** Dinner

## Friday

- 7:00–9:00** Breakfast  
**9:00–11:30** Short talks (TBA) and informal discussion  
**11:30–13:30** Lunch

**Checkout by 12 noon.**

\*\* 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

# Multivariable Complex Dynamics

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### ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Marco Abate** (Universita di Pisa)

Title: *A Poincaré-Bendixson theorem for homogeneous vector fields and meromorphic connections*

Abstract: In one complex dimension, a holomorphic germ tangent to the identity is locally topologically conjugated to the time-1 map of a homogeneous vector field. In particular, the study of the real flow of (complex) homogeneous vector fields in (complex) dimension one provides a large amount of informations on the local dynamics of functions tangent to the identity. This suggests that the study of the real flow of complex homogeneous vector fields might also help to understand the local dynamics of holomorphic map tangent to the identity in complex dimension two, at least in generic cases. In this talk I shall describe how it is possible to reduce the study of the real 1-dimensional flow of a complex 2-dimensional homogenous vector field to the study of the geodesic flow of a meromorphic connection on the complex projective line, and how to use this reduction and a new Poincaré-Bendixson theorem for meromorphic connections to describe the recurrent behavior of such flows. Together with a study of the local dynamics of the geodesic flow nearby the poles of the connection this yields a complete description of the topological dynamics of a large class of time-1 maps of homogeneous vector fields in dimension two. (Joint work with F. Tovena)

Speaker: **François Berteloot** (Université de Toulouse)

Title: *Lyapunov exponents and bifurcation currents*

Abstract: For any holomorphic family of rational maps, the Lyapunov function is a good potential for the bifurcation current. This will be used to describe some aspects of the bifurcation locus.

Speaker: **Serge Cantat** (Université de Rennes)

Title: *The Cremona group and its finitely generated subgroups* (two talks)

Abstract: I'll compare the group of birational transformations of the complex plane to other classical groups, like linear groups and groups of diffeomorphisms ; to do that I'll focus on finitely generated subgroups of the Cremona Group.

Speaker: **Julie Déserti** (Université Paris Diderot)

Title: *Automorphisms of the Cremona group*

Abstract: We will explain why an automorphism of the affine group of the complex line is the composition of an interior one and the action of an automorphism of the field  $\mathbb{C}$ ; then we will give an analogous result for the Cremona group and keypoints of the proof.

Speaker: **Igor Dolgachev** (University of Michigan)

Title: *The Cremona group: old and new*

Abstract: I will give a brief introduction into the theory of Cremona transformations.

Title: *Special sets of points in product of projective spaces*

Abstract: A finite set  $S$  of points in the product of projective spaces is called special if the group of pseudo-automorphisms of the blow-up variety is infinite and contains a normal subgroup of finite index of a certain infinite Coxeter group naturally attached to it. A most notorious example of a special set is the set of intersection points of two plane cubic curves. I will explain some of the known examples of special sets of points and discuss some hypothetical constructions of new examples leading to the realization of some simple groups in algebraic geometry.

Speaker: **Romain Dujardin** (Universit Paris Diderot)

Title: *Geometric currents and dynamical applications*

Abstract: This is a set of two lectures aiming at introducing the use of geometric currents in two dimensional holomorphic dynamics. In the first talk, I'll explain the notions of laminar and woven currents, and why and where they appear in holomorphic dynamics. I'll also talk about the possibility of a geometric interpretation of the intersection of such currents.

The second talk will be devoted to the use of these techniques in dynamics, mostly in the iteration of plane birational maps and, more generally, mappings with small topological degree. If time permits, I'll also mention the case of holomorphic endomorphisms of the projective plane.

Speaker: **John Erik Fornæss** (University of Michigan)

Title: *Riemann surface laminations*

Abstract: I'll report on some work in progress with Erlend Fornæss Wold and Nessim Sibony. Basically the problem is to find some way too understand laminations in complex dimension 3 and higher.

Speaker: **Mattias Jonsson** (University of Michigan)

Title: *Superattracting dynamics*

Abstract: I will discuss two types of dynamics in two complex dimensions: local dynamics near a super-attracting fixed point and polynomial dynamics near infinity. The two situations are similar but exhibit interesting differences.

Speaker: **Kyounghee Kim** (Florida State University)

Title: *Linearization and globalization*

Abstract: We will discuss a discrete family of automorphisms. These maps have a curve of fixed points. There are two cases. In the first case, the curve is attracting/repelling, and its basin has full volume. In the other case, the curve is the *center* of a rotation domain, which is *large* because it contains both a curve of fixed points, as well as isolated fixed points. We will discuss how to linearize these mappings globally.

Speaker: **Mikhail Lyubich** (Stonybrook University)

Title: *Lee-Yang zeros and 2D rational dynamics*

Abstract: In a classical work of 1952, Lee and Yang proved that zeros of certain polynomials (the partition functions of Ising models) always lie on the unit circle. Distribution of these zeros is physically important as it controls phase transitions in the model.

We study this distribution for a special “Diamond Hierarchical Lattice”. In this case, it can be described in terms of the dynamics of an explicit rational map in two variables. We prove partial hyperbolicity of this map on an invariant cylinder, and derive from it that the Lee-Yang zeros are organized asymptotically in a transverse measure for the central foliation. From the global complex point of view, these measures get interpreted as slices of the Green (1,1)-current on the projective space.

In the first lecture of our mini-course we will discuss a general background on the Ising models, the Lee-Yang zeros, their general relation to the (pluri)-potential theory, and their transformation under renormalization in hierarchical lattices. In the second lecture we will outline a proof of the above stated results using a mixture of methods from real and complex dynamics.

This mini-course will be based on a joint work with Pavel Bleher and Roland Roeder.