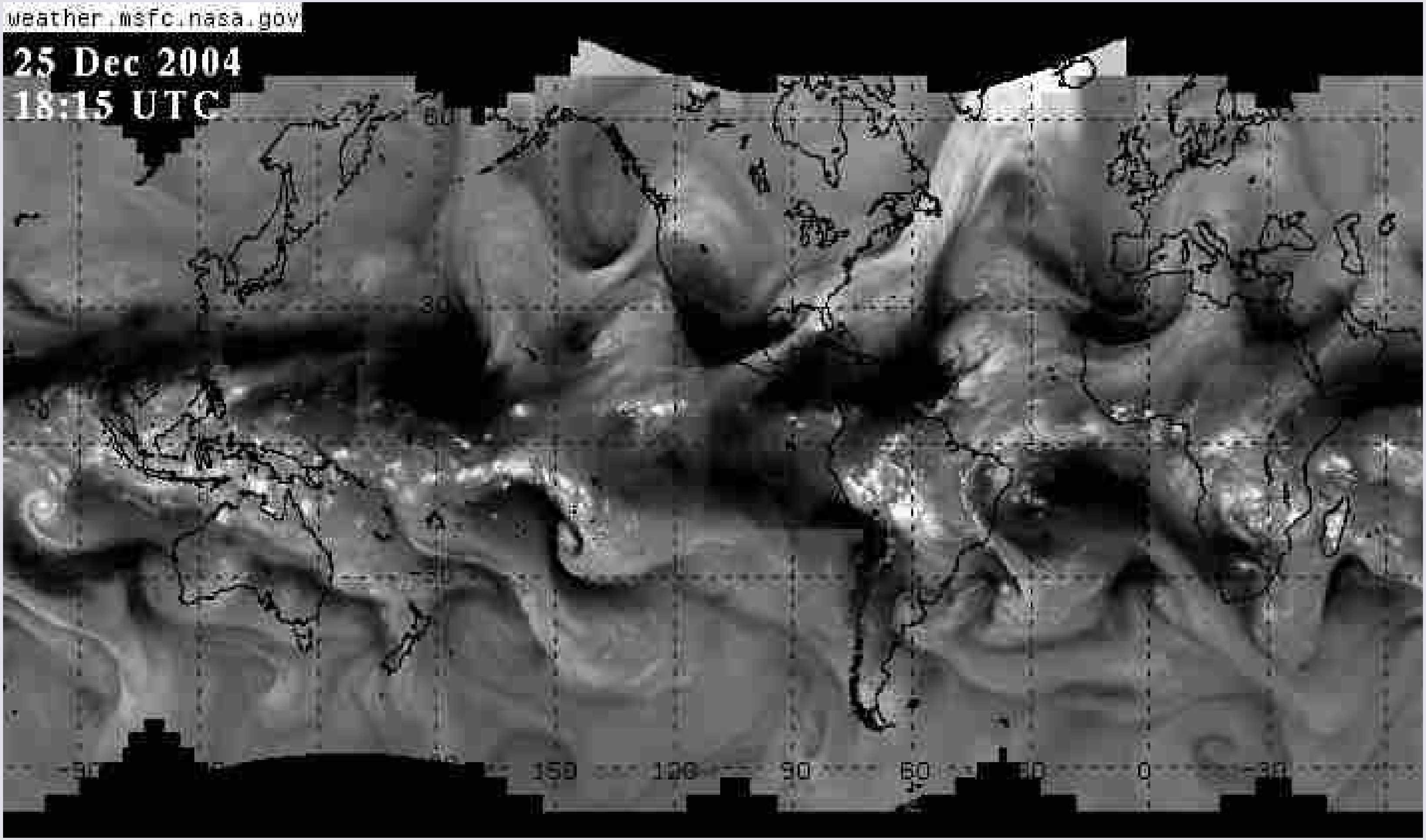
Synoptic/Planetary Multi-Scale Theory for the Tropics

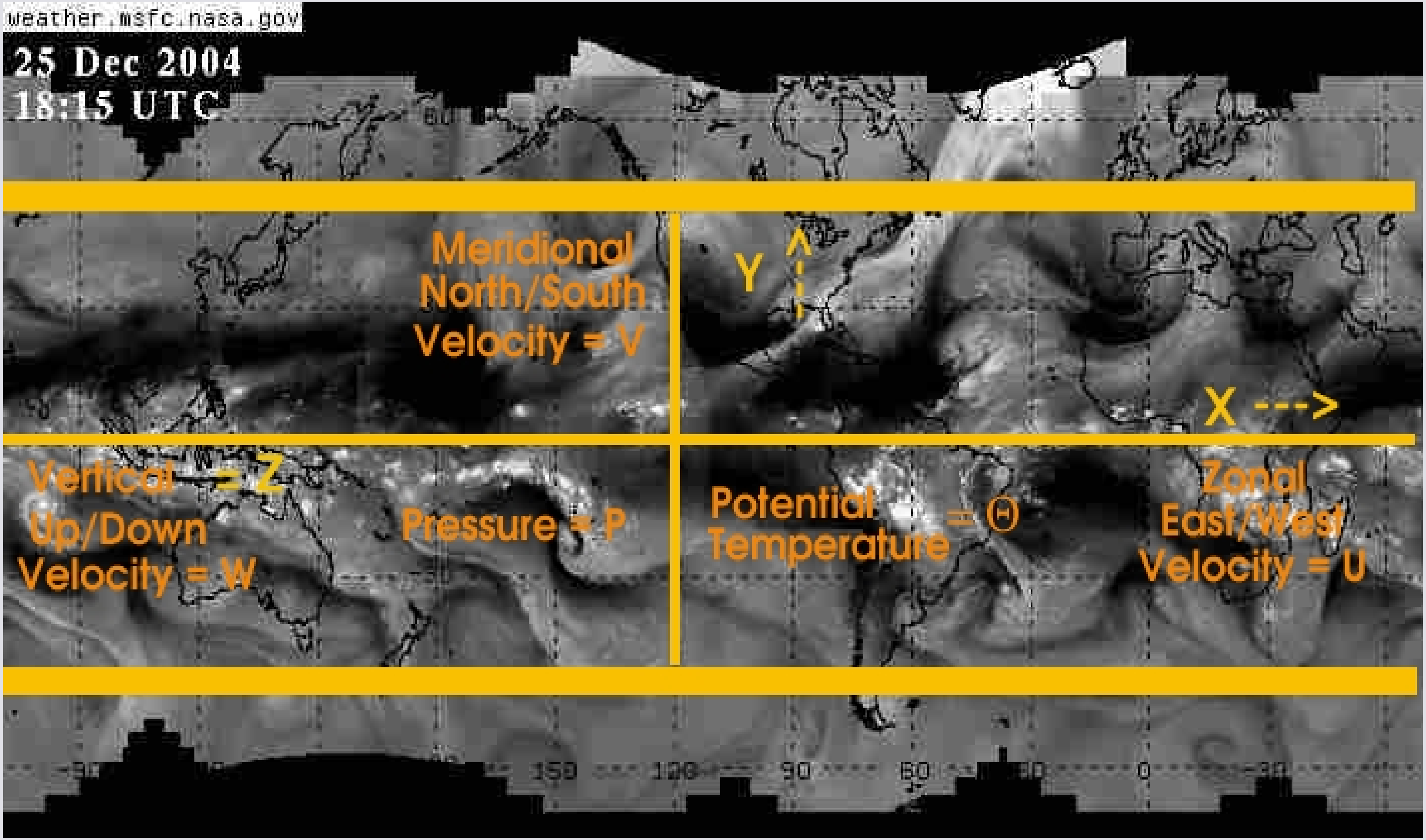
Joseph A. Biello
Department of Mathematical Sciences
University of California, Davis

- joint work with Andy Majda
- New multiscale, wave/mean flow theory of the Madden-Julian oscillation
- New nonlinear wave/wave interaction theory for tropical/ midlatitude interactions

Water Vapor


weather.msfc.nasa.gov
25 Dec 2004
18:15 UTC
60
30
150
120
90
60
0
-30

Coordinate System



The Equatorial Primitive Equations

Incompressible, Hydrostatic, Coriolis
force on β-plane

$$\begin{aligned} u_t + \vec{u}\cdot\nabla\, u - \beta\, y\, v &= -p_x + S_u \\ v_t + \vec{u}\cdot\nabla\, v + \beta\, y\, u &= -p_y + S_v \\ \theta_t + \vec{u}\cdot\nabla\,\theta + N^2 w &= S_\theta \\ p_z &= \theta \\ u_x + v_y + w_z &= 0 \end{aligned}$$

- $\vec{u} = (u, v, w) = (\mathrm{East}, \mathrm{North}, \mathrm{Up})$
- $\theta =$ potential temperature perturbation
- $p =$ pressure perturbation
- $N = \sqrt{\frac{g}{\theta_0}\frac{d\theta_0}{dz}} =$ buoyancy frequency
- $\beta\, y$ is vertical component of Coriolis force near equator
- Rigid lid $\Rightarrow w = 0$ at $z = 0,\ 16\,\mathrm{km}$

- Heat and momentum sources and sinks $= S_\theta,\ S_u,\ S_v$

The Equatorial Primitive Equations

Incompressible, Hydrostatic, Coriolis force on β-plane

$$
\begin{aligned}
u_t + \vec{u}\cdot\nabla\, u - \beta\, y\, v &= -p_x + S_u \\
v_t + \vec{u}\cdot\nabla\, v + \beta\, y\, u &= -p_y + S_v \\
\theta_t + \vec{u}\cdot\nabla\,\theta + N^2 w &= S_\theta \\
p_z &= \theta \\
u_x + v_y + w_z &= 0
\end{aligned}
$$

- $\vec{u} = (u, v, w) = (\text{East, North, Up})$
- θ = potential temperature perturbation
- p = pressure perturbation
- $N = \sqrt{\frac{g}{\theta_0}\frac{d\theta_0}{dz}}$ = buoyancy frequency
- $\beta\, y$ is vertical component of Coriolis force near equator
- Rigid lid $\Rightarrow$ $w = 0$ at $z = 0,\ 16\,\mathrm{km}$

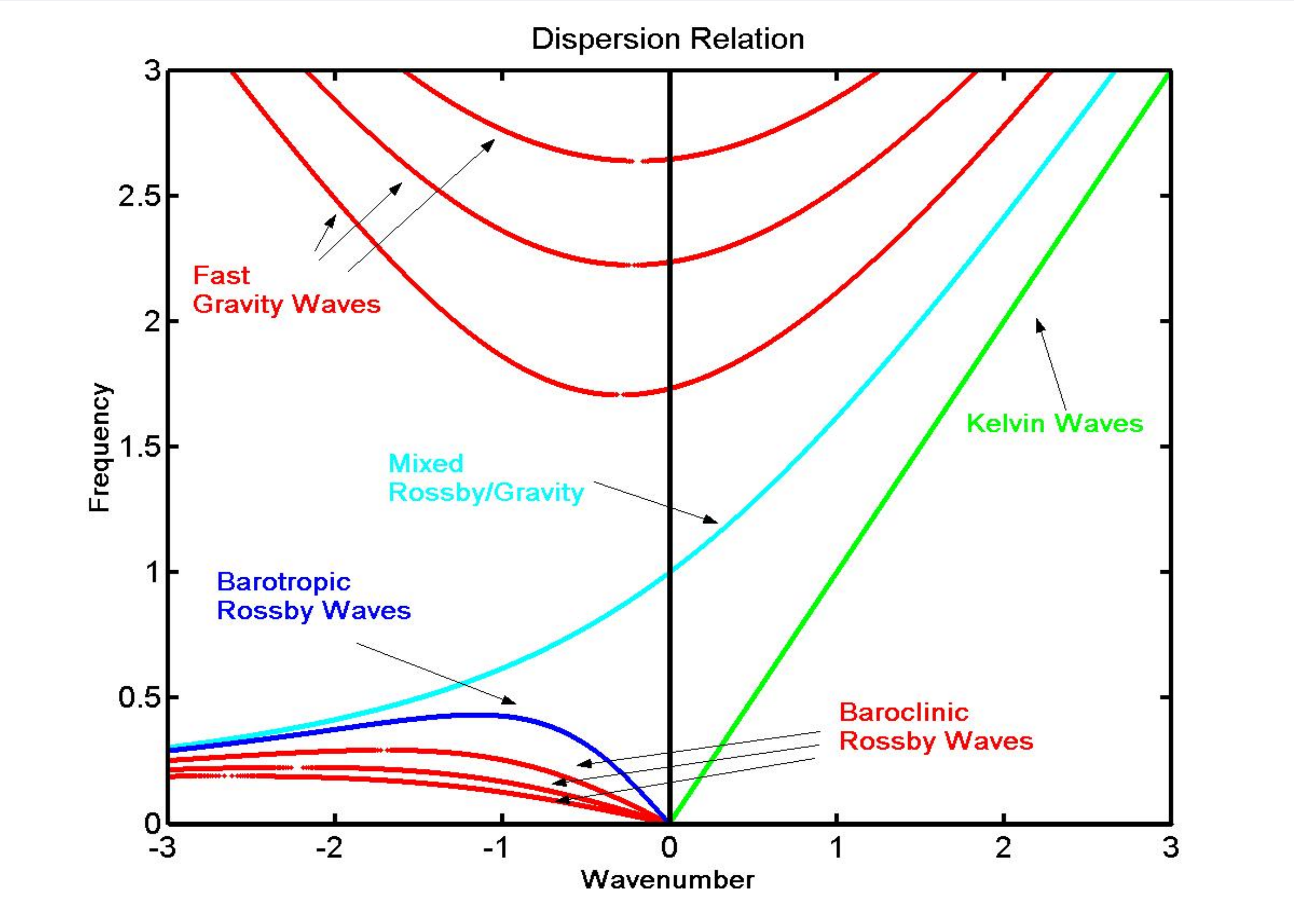
- Heat and momentum sources and sinks $= S_\theta,\ S_u,\ S_v$

Schematically, these are equivalent to

$$\boldsymbol{\Psi}_t + \mathbf{L}\,\boldsymbol{\Psi} + \mathbf{N}\,(\boldsymbol{\Psi},\ \boldsymbol{\Psi}) = \mathbf{S}(x, t)$$

Linear Theory of Equatorial Waves

$\mathbf{L}$ is skew self adjoint $\Longrightarrow$ dispersive waves



Multiple Scales Approach

- Long waves are nearly dispersionless
- Fast waves are not forced
- Forcing $\sim O(\epsilon) \sim 10^\circ\,\mathrm{K/day}$
- Multiple spatial scales, $O(1),\ O(\delta^{-1})$

$$\mathbf{S}(x,t) \longrightarrow \epsilon\,\mathbf{S}(x,\delta\,x,\delta\,t) \qquad\qquad \mathbf{\Psi}(x,t) \longrightarrow \epsilon\,\mathbf{\Psi}(x,\delta\,x,\delta\,t)$$

$$\boxed{\mathbf{\Psi}_t + \mathbf{L}\,\mathbf{\Psi} + \epsilon\,\mathbf{N}\,(\mathbf{\Psi},\ \mathbf{\Psi}) = \mathbf{S}(x,\delta\,x,\delta\,t)}$$

Multiple Scales Approach

- Long waves are nearly dispersionless
- Fast waves are not forced
- Forcing $\sim O(\epsilon) \sim 10^\circ \,\mathrm{K/day}$
- Multiple spatial scales, $O(1),\ O(\delta^{-1})$

$$\mathbf{S}(x,t) \longrightarrow \epsilon\,\mathbf{S}(x,\delta\,x,\delta\,t) \qquad \boldsymbol{\Psi}(x,t) \longrightarrow \epsilon\,\boldsymbol{\Psi}(x,\delta\,x,\delta\,t)$$

$$\boxed{\boldsymbol{\Psi}_t + \mathbf{L}\,\boldsymbol{\Psi} + \epsilon\,\mathbf{N}\,(\boldsymbol{\Psi},\ \boldsymbol{\Psi}) = \mathbf{S}(x,\delta\,x,\delta\,t)}$$

Time $\sim$ few days, 1500 km/ Planetary length scales

$\delta \sim \epsilon \Longrightarrow$ Madden-Julian Oscillation

Multiple Scales Approach

- Long waves are nearly dispersionless
- Fast waves are not forced
- Forcing $\sim O(\epsilon) \sim 10^{\circ}\,\mathrm{K/day}$
- Multiple spatial scales, $O(1),\ O(\delta^{-1})$

$$\mathbf{S}(x,t) \longrightarrow \epsilon\,\mathbf{S}(x,\delta\,x,\delta\,t) \qquad \boldsymbol{\Psi}(x,t) \longrightarrow \epsilon\,\boldsymbol{\Psi}(x,\delta\,x,\delta\,t)$$

$$\boxed{\boldsymbol{\Psi}_t + \mathbf{L}\,\boldsymbol{\Psi} + \epsilon\,\mathbf{N}\,(\boldsymbol{\Psi},\ \boldsymbol{\Psi}) = \mathbf{S}(x,\delta\,x,\delta\,t)}$$

Time $\sim$ few days, 1500 km/ Planetary length scales

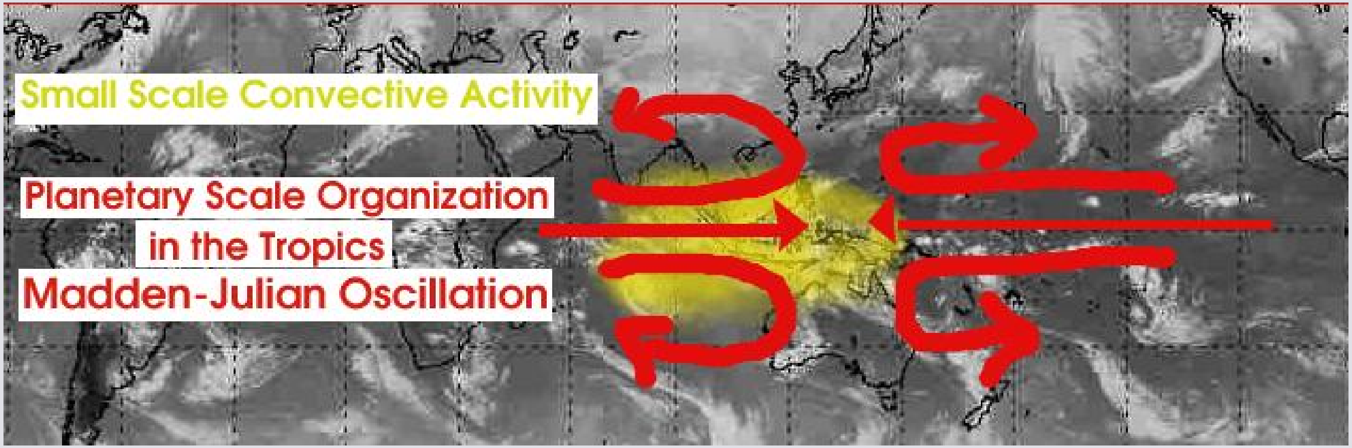
$\delta \sim \epsilon \Longrightarrow$ Madden-Julian Oscillation

---

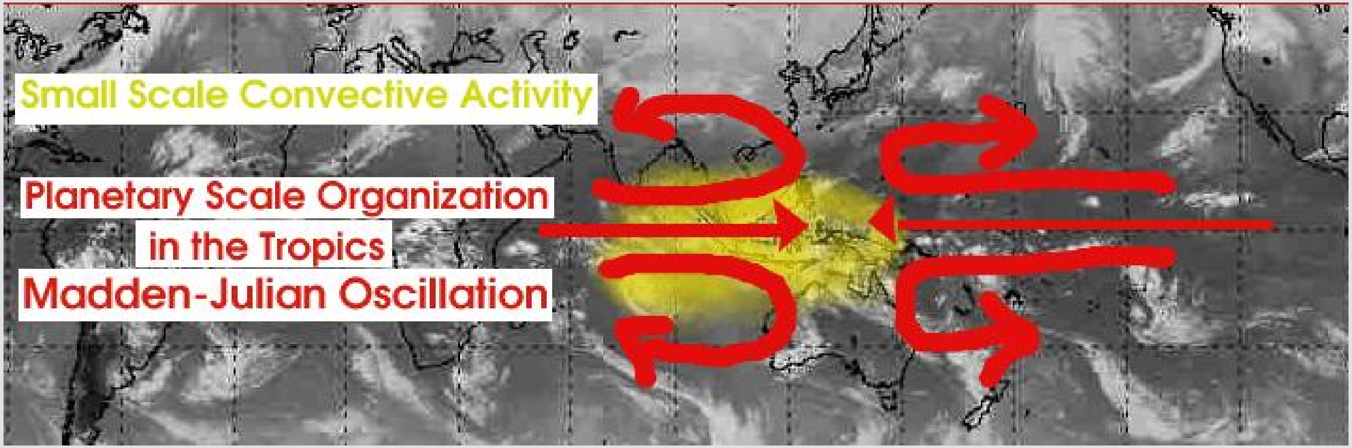
Time $\sim$ few weeks, Planetary length scales

$\delta^{\mathbf{2}} \sim \epsilon \Longrightarrow$ Nonlinear Wave/wave interaction, tropics/midlatitudes

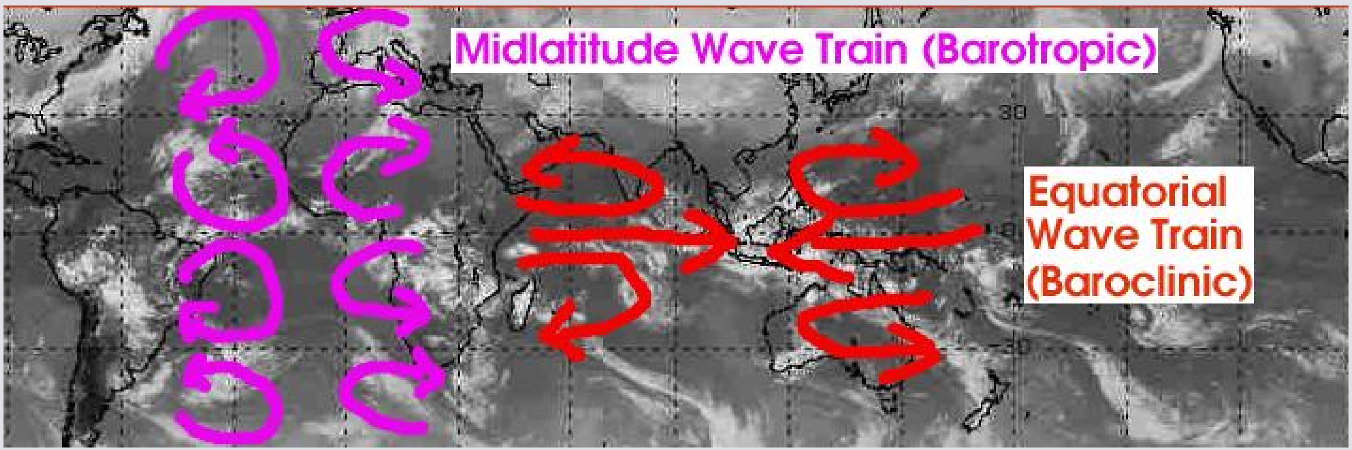
MJO Asymptotics: Few Days, 1500/40000 km



MJO Asymptotics: Few Days, 1500/40000 km



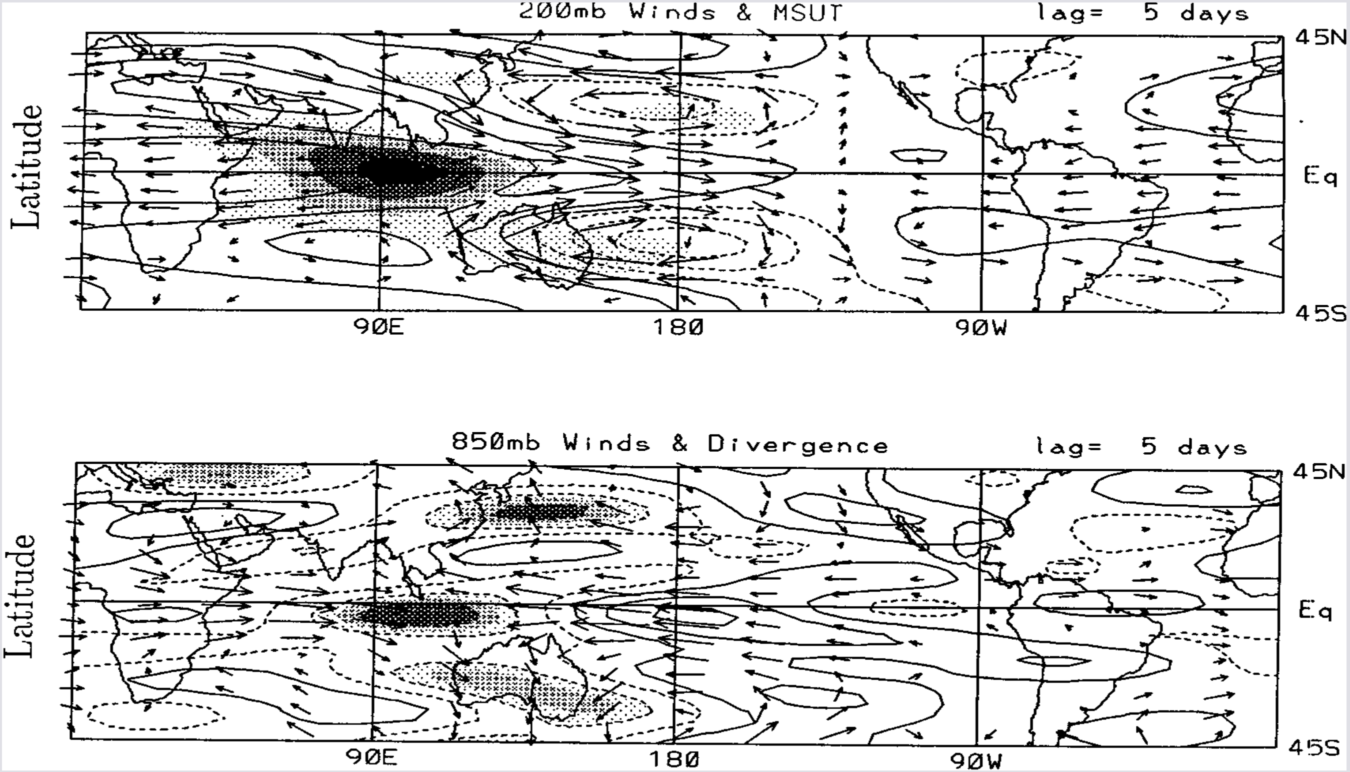
Tropics/Midlatitude: Few Weeks, 40000 km



The IPESD Multiscale Model for the MJO

- FRAMEWORK: IPESD multiscale models (Majda/Klein 2003)
- STRUCTURE: MJO structure given by specified heating profiles (PNAS 2004, JAS 2005, DAO 2006, BM+ Moncrieff JAS 2007)
  - planetary scale direct heating
  - upscale fluxes of momentum and heat from synoptic scales
- DYNAMICS: Khouider/Majda multi-cloud model. (Khouider/Majda JAS 2005, 2006, 2007)
  - active moisture through cloud model
  - nonlinear feedback from planetary to synoptic scales
  - organized embedded structures in a traveling envelope (Majda, Stechmann, Khouider PNAS)

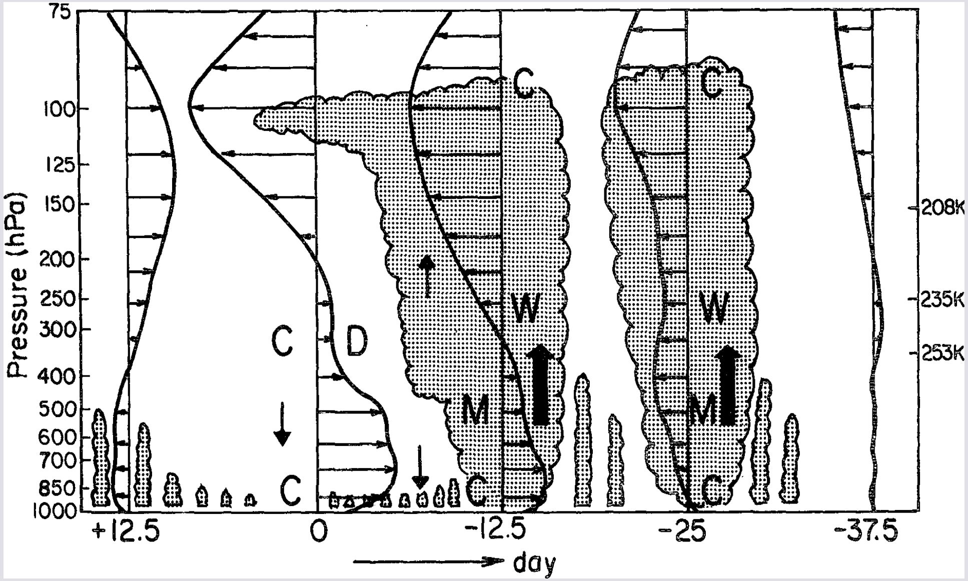
MJO: Large scale wind pattern



From Hendon & Salby *J. Atmos. Sci.*, **51**, p 2230, fig. 3.

- Top: Top of Troposphere, Winds and precipitation.
- Bottom: Bottom of Troposphere, winds and divergence. ***

MJO: Vertical Shear and Convection



- Schematic showing correlation of convection and vertical shear
- Time goes from right to left (reverse of previous slide) and can be interpreted as left = west, right = east.

Lin & Johnson *J. Atmos. Sci.*, **53**, p 701, fig. 16.

- Congestus clouds - weak winds/easterlies
- Westward tilted anvil - westerly onset
- Strong westerlies trail convection

MJO: Some previous work

- Planetary scale response to moving heat source, Chao (1987) based on Gill model of tropical heating.
- Linearized evaporation wind feedback, Emmanuel (1987), Neelin et al. (1987).
- Boundary layer friction causing convective instability, Wang & Rui (1990).
- Stochastic linearized convection, Salby et al. (1994).
- Radiation instability, Raymond (2001).
- Phenomenological model of upscale momentum transport from mesoscales $O(300\,\mathrm{km})$ to planetary scale (with a scale gap on $O(1000\,\mathrm{km})$), Moncrieff (2004). Based on Grabowski (2001) super- parametrization .

IPESD: Systematic multi-scale asymptotics

- Majda & Klein, *J. Atmos. Sci*, **60** (2003) : from primitive equations

$$\delta\,\mathbf{\Psi}_T + \mathbf{L}\,\mathbf{\Psi} + \delta\,\mathbf{M}\,\mathbf{\Psi}_X + \epsilon\,\mathbf{N}\,(\mathbf{\Psi},\,\mathbf{\Psi}) = \mathbf{S}'(x,X,T) + \delta\,\overline{\mathbf{S}}(X,T)$$

- Large (planetary) scale means plus fluctuations on small (synoptic) scales:

$$\mathbf{\Psi} = \overline{\mathbf{\Psi}}(X,T) + \psi'(x,X,T) \quad \text{where} \quad X = \delta\,x,\; T = \delta\,t$$

IPESD: Systematic multi-scale asymptotics

- Majda & Klein, *J. Atmos. Sci*, **60** (2003) : from primitive equations

$$\delta\,\mathbf{\Psi}_T + \mathbf{L}\,\mathbf{\Psi} + \delta\,\mathbf{M}\,\mathbf{\Psi}_X + \epsilon\,\mathbf{N}\,(\mathbf{\Psi},\ \mathbf{\Psi}) = \mathbf{S}'(x, X, T) + \delta\,\overline{\mathbf{S}}(X, T)$$

- Large (planetary) scale means plus fluctuations on small (synoptic) scales:

$$\mathbf{\Psi} = \overline{\mathbf{\Psi}}(X, T) + \psi'(x, X, T) \quad \text{where} \quad X = \delta\,x,\ T = \delta\,t$$

- Distinguished limit: $\boxed{\epsilon = \delta}$

IPESD: Systematic multi-scale asymptotics

- Majda & Klein, *J. Atmos. Sci*, **60** (2003) : from primitive equations

$$\delta\,\mathbf{\Psi}_T + \mathbf{L}\,\mathbf{\Psi} + \delta\,\mathbf{M}\,\mathbf{\Psi}_X + \epsilon\,\mathbf{N}\,(\mathbf{\Psi},\ \mathbf{\Psi}) = \ \mathbf{S}'(x, X, T) + \delta\,\overline{\mathbf{S}}(X, T)$$

- Large (planetary) scale means plus fluctuations on small (synoptic) scales:

$$\mathbf{\Psi} = \ \overline{\mathbf{\Psi}}(X, T) + \psi'(x, X, T) \quad \text{where} \quad X = \delta\,x,\ T = \delta\,t$$

- Distinguished limit: $\boxed{\epsilon = \delta}$

- $O(1)$: Balanced dynamics for fluctuations plus constraint

$$\begin{aligned} \mathbf{L}\,\psi' &= \mathbf{S}' \\ \mathbf{L}\,\overline{\mathbf{\Psi}} &= 0 \end{aligned} \Longrightarrow \begin{aligned} \psi' &= \mathbf{L}^{-1}\,\mathbf{S}' \\ \overline{\mathbf{S}'} &= 0 \quad \text{solvability} \end{aligned} \Longrightarrow \overline{\psi'} = 0$$

IPESD: Systematic multi-scale asymptotics

- Majda & Klein, *J. Atmos. Sci*, **60** (2003) : from primitive equations

$$\delta\,\mathbf{\Psi}_T + \mathbf{L}\,\mathbf{\Psi} + \delta\,\mathbf{M}\,\mathbf{\Psi}_X + \epsilon\,\mathbf{N}\,(\mathbf{\Psi},\,\mathbf{\Psi}) = \mathbf{S}'(x, X, T) + \delta\,\overline{\mathbf{S}}(X, T)$$

- Large (planetary) scale means plus fluctuations on small (synoptic) scales:

$$\mathbf{\Psi} = \overline{\mathbf{\Psi}}(X, T) + \psi'(x, X, T) \quad \text{where} \quad X = \delta\,x,\; T = \delta\,t$$

- Distinguished limit: $\boxed{\epsilon = \delta}$

- $O(1)$: Balanced dynamics for fluctuations plus constraint

$$\begin{aligned} \mathbf{L}\,\psi' &= \mathbf{S}' \\ \mathbf{L}\,\overline{\mathbf{\Psi}} &= 0 \end{aligned} \;\Longrightarrow\; \begin{aligned} \psi' &= \mathbf{L}^{-1}\,\mathbf{S}' \\ \overline{\mathbf{S}'} &= 0 \quad \text{solvability} \end{aligned} \;\Longrightarrow\; \overline{\psi'} = 0$$

- $O(\delta)$: Planetary dynamics forced by upscale fluxes

$$\overline{\mathbf{\Psi}}_T + \mathbf{M}\,\overline{\mathbf{\Psi}}_X = \overline{\mathbf{S}} - \overline{\mathbf{N}\,(\psi',\,\psi')} \quad \text{since} \quad \mathbf{N}\,(\overline{\mathbf{\Psi}},\,\overline{\mathbf{\Psi}}) = O(\delta)$$

FRAMEWORK: IPESD Theory (Majda/Klein 2003)

Synoptic Scale (Balanced) Dynamics:

$$
\begin{aligned}
u'_\tau - y\,v' + p'_x &= S'_u \\
v'_\tau + y\,u' + p'_y &= S'_v \\
\theta'_\tau + w' &= S'_\theta(\theta', \overline{\Theta}) \\
p'_z &= \theta' \\
u'_x + v'_y + w'_z &= 0 \\
\overline{S'_\theta} &= 0
\end{aligned}
$$

Planetary Scale Quasi-Linear Dynamics:

$$
\begin{aligned}
\overline{U}_t - y\overline{V} + \overline{P}_X &= F^U \; - d_0\,\overline{U} \\
y\overline{U} + \overline{P}_y &= 0 \\
\overline{\Theta}_t + \overline{W} &= F^\theta \; - d_\theta\,\overline{\Theta} + \overline{S}_\theta(\theta', \overline{\Theta}) \\
\overline{P}_z &= \overline{\Theta} \\
\overline{U}_X + \overline{V}_y + \overline{W}_z &= 0
\end{aligned}
$$

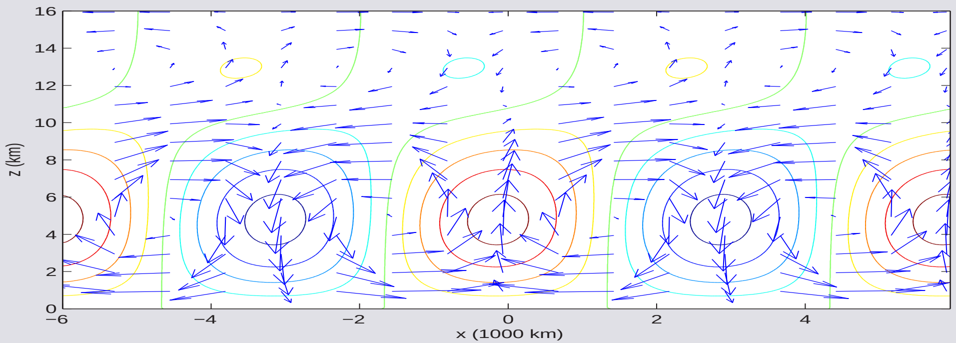
The fluxes from the synoptic scales are given by

$$
\begin{aligned}
F^U &= -\overline{(v'\,u')_y} - \overline{(w'\,u')_z} \\
F^\theta &= -\overline{(v'\,\theta')_y} - \overline{(w'\,\theta')_z}
\end{aligned}
$$

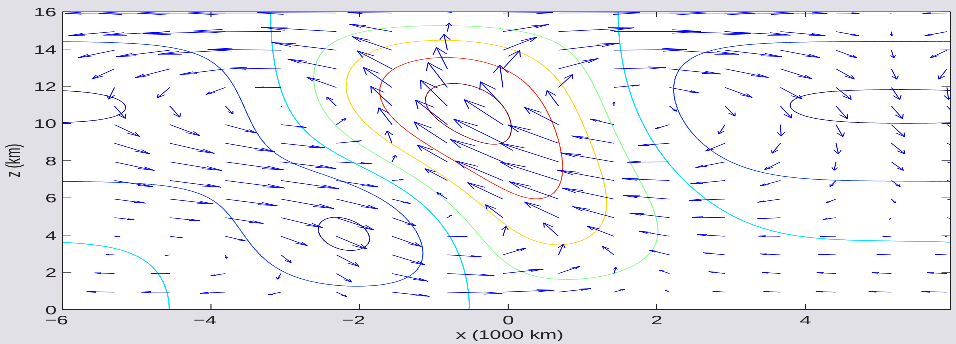
Each forcing effect, i.e. upscale vertical and meridional momentum and temperature transport and planetary scale mean heating can be considered separately and superposed

MJO Model: Convection organized on small scales

- Heating rate traces cloudiness (latent heat release).
- Fluctuations on 1500 km spatial scales
- Clouds/heating localized near equator above Western Pacific.



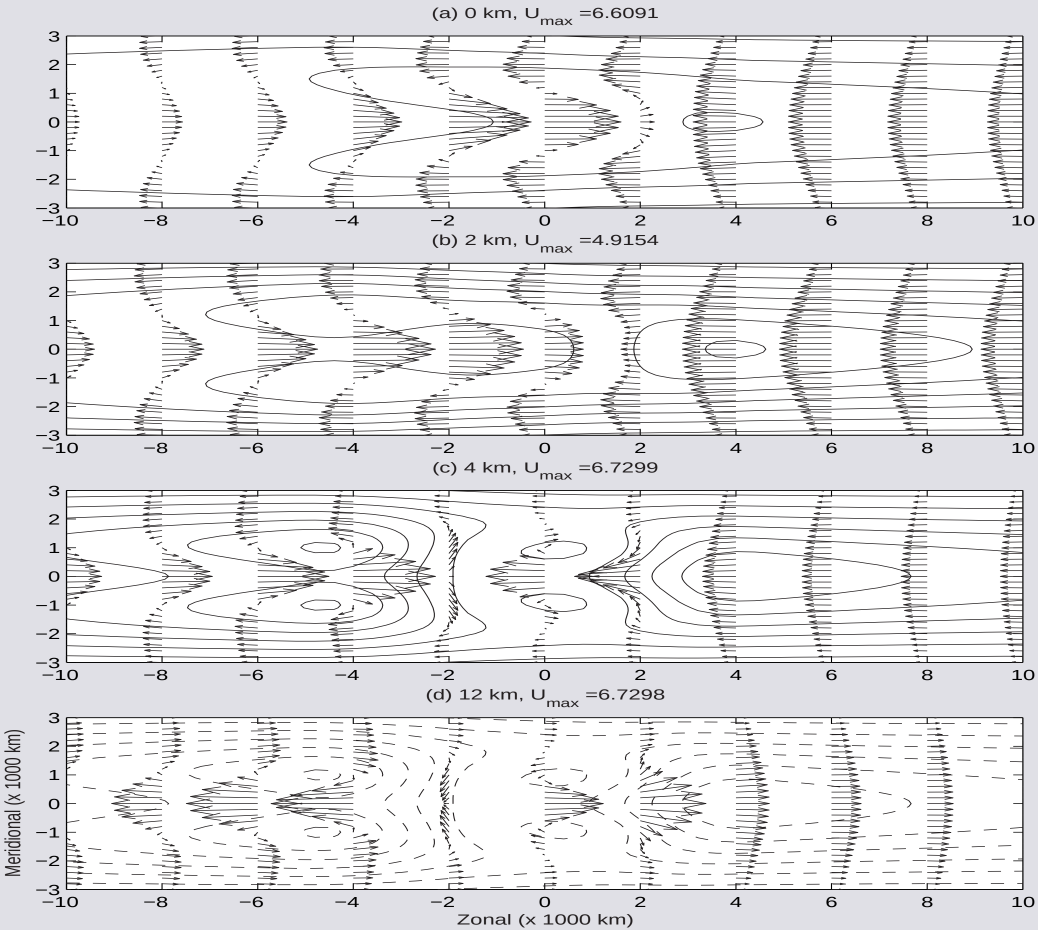
- East: Lower troposphere ***congestus clouds***
- West: High, westward tilted anvil ***superclusters***
- Flow vectors and heating contours
- Upscale flux, $\overline{\mathbf{N}(\psi', \psi')} \neq 0$
  $\Rightarrow$ Vertical/Longitudinal Tilt



Equatorial MJO model: Flow in the Horizontal Plane

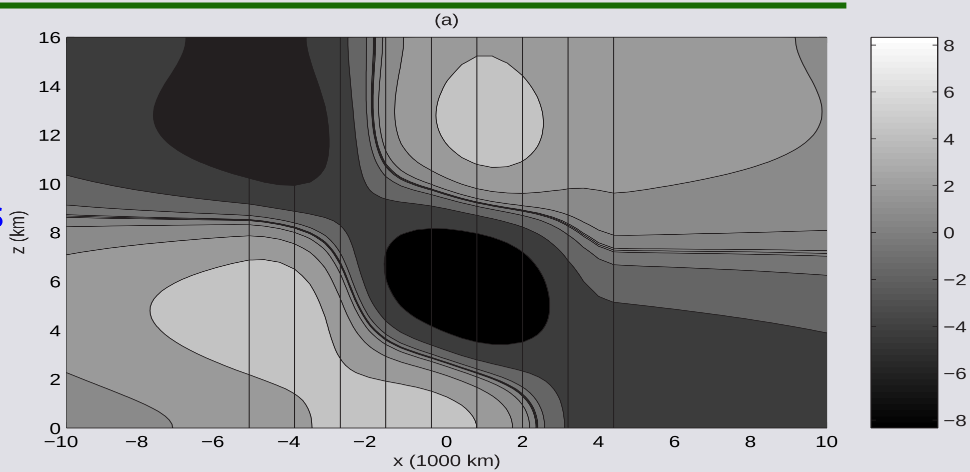
- Congestus heating in the east and westward tilted superclusters in the west of a moving warm pool.
- Planetary mean heating is weaker, but has same structure of synoptic scale fluctuations.
- Pressure and flow at $z = 0, 2, 4, 12$ km.

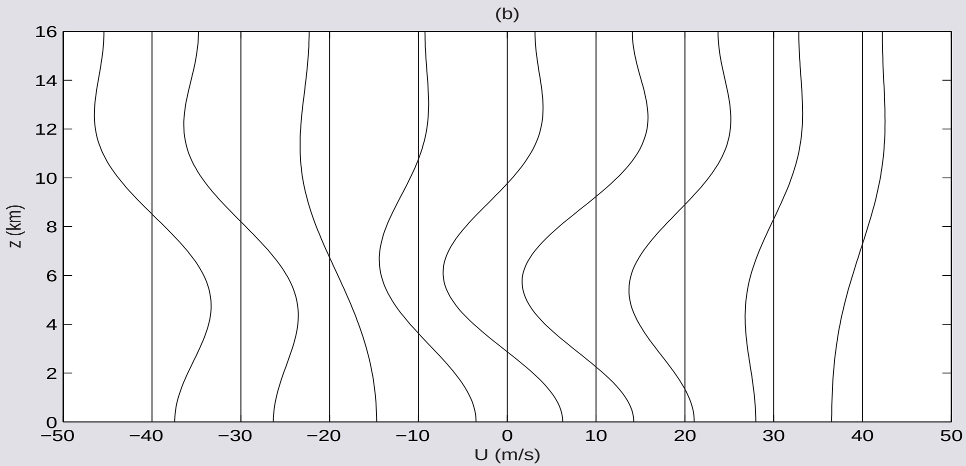
***



Equatorial MJO model: Winds above the equator

- Lower troposphere congestus heating in the east
- Westward tilted anvil superclusters in the west
- (a) Zonal velocity:
  westerly = light,
  easterly = dark
  versus height and longitude above equator
- (b) Height vs Velocity.





Summary: Systematic Multi-scale Theory for the MJO

- asymptotic framework in Majda & Klein, *J. Atmos. Sci (2003)* amounts to a closure of the nonlinear equatorial primitive equations on large scales

Summary: Systematic Multi-scale Theory for the MJO

- asymptotic framework in Majda & Klein, *J. Atmos. Sci (2003)* amounts to a closure of the nonlinear equatorial primitive equations on large scales
- Balanced motion on 1500 km scales and Planetary scale response

Summary: Systematic Multi-scale Theory for the MJO

- asymptotic framework in Majda & Klein, *J. Atmos. Sci (2003)* amounts to a closure of the nonlinear equatorial primitive equations on large scales
- Balanced motion on 1500 km scales and Planetary scale response
- Planetary scales forced by: upscale momentum and temperature fluxes plus weaker planetary mean heating.

Summary: Systematic Multi-scale Theory for the MJO

- asymptotic framework in Majda & Klein, *J. Atmos. Sci (2003)* amounts to a closure of the nonlinear equatorial primitive equations on large scales
- Balanced motion on 1500 km scales and Planetary scale response
- Planetary scales forced by: upscale momentum and temperature fluxes plus weaker planetary mean heating.
  - Isolate the features of the planetary scale organization arising from either upscale fluxes or planetary mean heating

Summary: Systematic Multi-scale Theory for the MJO

- asymptotic framework in Majda & Klein, *J. Atmos. Sci (2003)* amounts to a closure of the nonlinear equatorial primitive equations on large scales
- Balanced motion on 1500 km scales and Planetary scale response
- Planetary scales forced by: upscale momentum and temperature fluxes plus weaker planetary mean heating.
  - Isolate the features of the planetary scale organization arising from either upscale fluxes or planetary mean heating
- Heating fluctuations determined from realistic cloud structure

Summary: Systematic Multi-scale Theory for the MJO

- asymptotic framework in Majda & Klein, *J. Atmos. Sci (2003)* amounts to a closure of the nonlinear equatorial primitive equations on large scales
- Balanced motion on 1500 km scales and Planetary scale response
- Planetary scales forced by: upscale momentum and temperature fluxes plus weaker planetary mean heating.
  - Isolate the features of the planetary scale organization arising from either upscale fluxes or planetary mean heating
- Heating fluctuations determined from realistic cloud structure
- Planetary flows are predicted with MJO features
  - Westerly wind burst; maximum at $z = 5$ km.
  - Quadrupolar structure
  - Vertical shear with upward/westward tilt
  - Upper troposphere outflow from warm pool

The Nonlinear Interaction of Midlatitude/Equatorial Waves

How do convection in the tropics and midlatitude waves interact with one another on *intraseasonal* time scales and planetary length scales in the presence of mean vertical and meridional shear?

OR

How does the breakup of the ***MJO affect us*** ?

- Dispersion balanced nonlinear wave/wave interaction theory.
- Nonlinear theory: *J. Atmos. Sci.*, **60**, (2003),
- with Boundary Layer: *GAFD*, **98**, (2004)
- Hamiltonian structure and solitary waves: *Stud. Appl. Math.*, **112**, (2004)

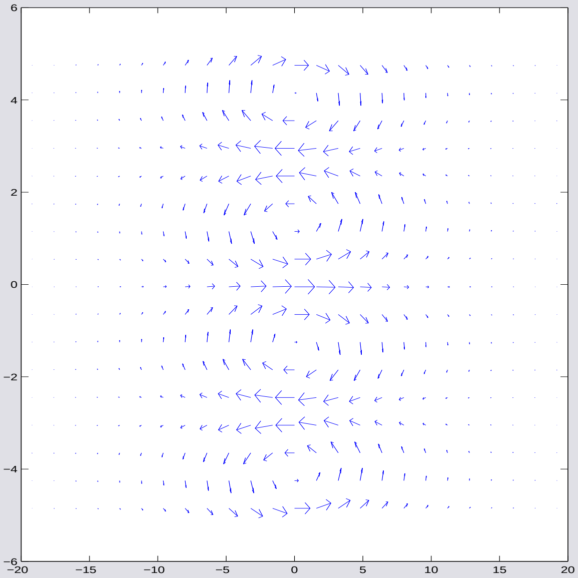
The main question

- How do Baroclinic Waves (equatorial) and Baroptropic Waves (midlatitude) exchange energy nonlinearly ?
  - Teleconnections from Tropics to midlatitudes
  - El Nino and Madden-Julian oscillation interaction with midlatitude waves
  - Time scales ~ 20 days: increase prediction time
- The art of this type of asymptotics:

Identify the principal length and timescales and thereby the essential dynamics

Horizontal Structure

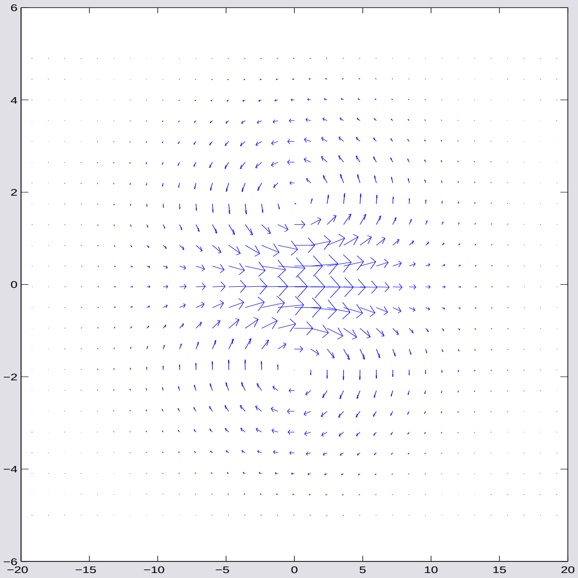
Barotropic Wave (BT)



Vertical Mean Winds
Independent of Height

$$u \propto \cos(l\, y)$$

Baroclinic Wave (BC)

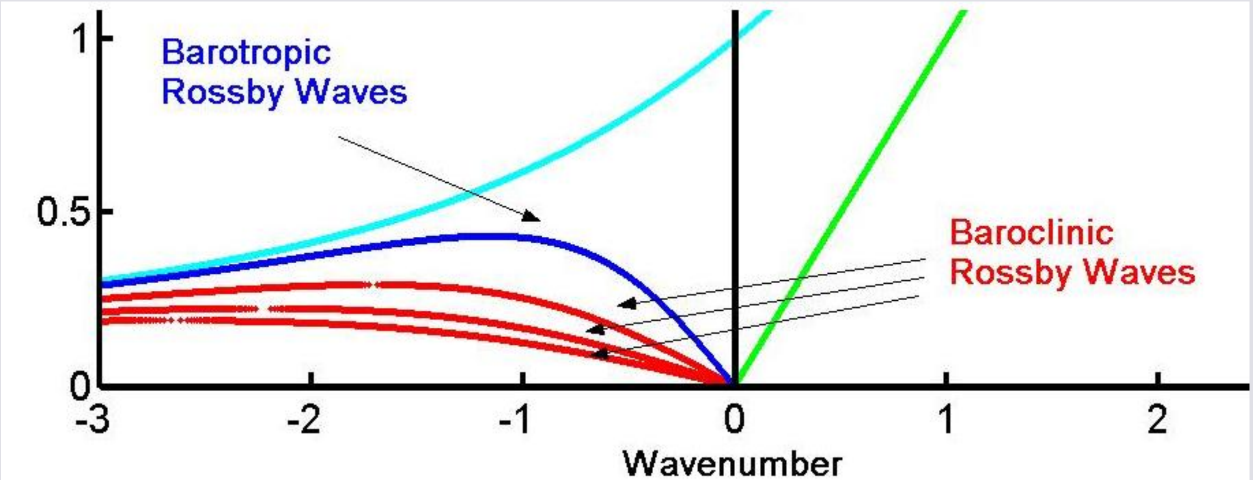


Vertical Mean Shear

$$u \propto D_{m+1}(y)\ \cos\left(\frac{\pi z}{H}\right)$$

$D_m(y)$ parabolic cylinder functions

Equatorial Barotropic and Baroclinic Waves



Barotropic Long Wave Speed

$$c_{BT} \equiv -\frac{\mathbf{1}}{l^2}, \quad \text{N/S wavenumber}$$

Baroclinic Long Wave Speed

$$c_{BC} \equiv -\frac{1}{2m+1}, \quad m \quad \text{integer}$$

Resonance if $l = \sqrt{2m+1}$

$m = 1 \Longrightarrow$ **5400 km, 16 m/s westward**

Previous Work: Linear theory and climate models

- Webster (1971, 1981, 1982), Kasahara & Silva Dias (1986), Hoskins & Jin (1991), Wang & Xie (1996): linearized model for midlatitude barotropic waves forced by tropics.
  - role of vertical and horizontal shear
  - role of nearly dispersionless long wavelength Rossby waves
  - Vertical shear linearly couples equatorial Rossby Waves to midlatitudes
- Lim & Chang (1981, 1986), Hoskins & Yang (2000), Lin et al. (2000)
  - role of nearly resonant forcing
  - role of vertical mean shear
  - midlatitude dynamics drives tropical *intraseasonal response*

The Model: Long Wave, Slow Time Scaling

$L_E \equiv$ N/S length scale $= 1500$ km
$L =$ E/W wave scale
$T_E \equiv$ equatorial timescale $= 8$ hours
$T =$ wave travel time

Small parameter,

$$\delta \equiv \frac{L_E}{L} = \frac{T_E}{T} = \frac{|v|}{|u|}$$

The Model: Long Wave, Slow Time Scaling

$L_E \equiv$ N/S length scale $= 1500$ km
$L =$ E/W wave scale
$T_E \equiv$ equatorial timescale $= 8$ hours
$T =$ wave travel time

Small parameter,

$$\delta \equiv \frac{L_E}{L} = \frac{T_E}{T} = \frac{|v|}{|u|}$$

Long Wave Equatorial Baroclinic/Barotropic Equations

$$\begin{aligned} \frac{\overline{D}}{Dt}u_1 - \vec{v}_1 \cdot \nabla\psi_y - y\,v_1 + (p_1)_x &= 0 \\ \frac{\overline{D}}{Dt}p_1 + div(\vec{v}_1) &= 0 \\ (p_1)_y + y\,u_1 + \delta^2\left(\frac{\overline{D}}{Dt}v_1 + \vec{v}_1 \cdot \nabla\psi_x\right) &= 0 \end{aligned}$$

$$\frac{\overline{D}}{Dt}(\psi_{yy}) + \psi_x - div\left((\vec{v}_1\,u_1)_y\right) + \delta^2\left(\frac{\overline{D}}{Dt}(\psi_{xx}) + div\left((\vec{v}_1\,v_1)_x\right)\right) = 0$$

Amplitude Equations for Long Equatorial Rossby Waves

- Westward traveling waves resonate

- Low Froude number, weakly nonlinear asymptotic expansion

$$\mathbf{v}_H = \epsilon\,[A(X,\tau)\,\mathbf{v}_{BC}(y)\,\cos(z) + B(X,\tau)\,\mathbf{v}_{BT}(y)] + O(\epsilon^2)$$

- τ = slow time
- X = planetary
- $D \approx 1$, dispersion
- moving frame

$$\begin{aligned} A_\tau - DA_{XXX} + BA_X + AB_X = 0 \quad &\text{Baroclinic} \\ B_\tau - B_{XXX} + AA_X = 0 \quad &\text{Barotropic} \end{aligned}$$

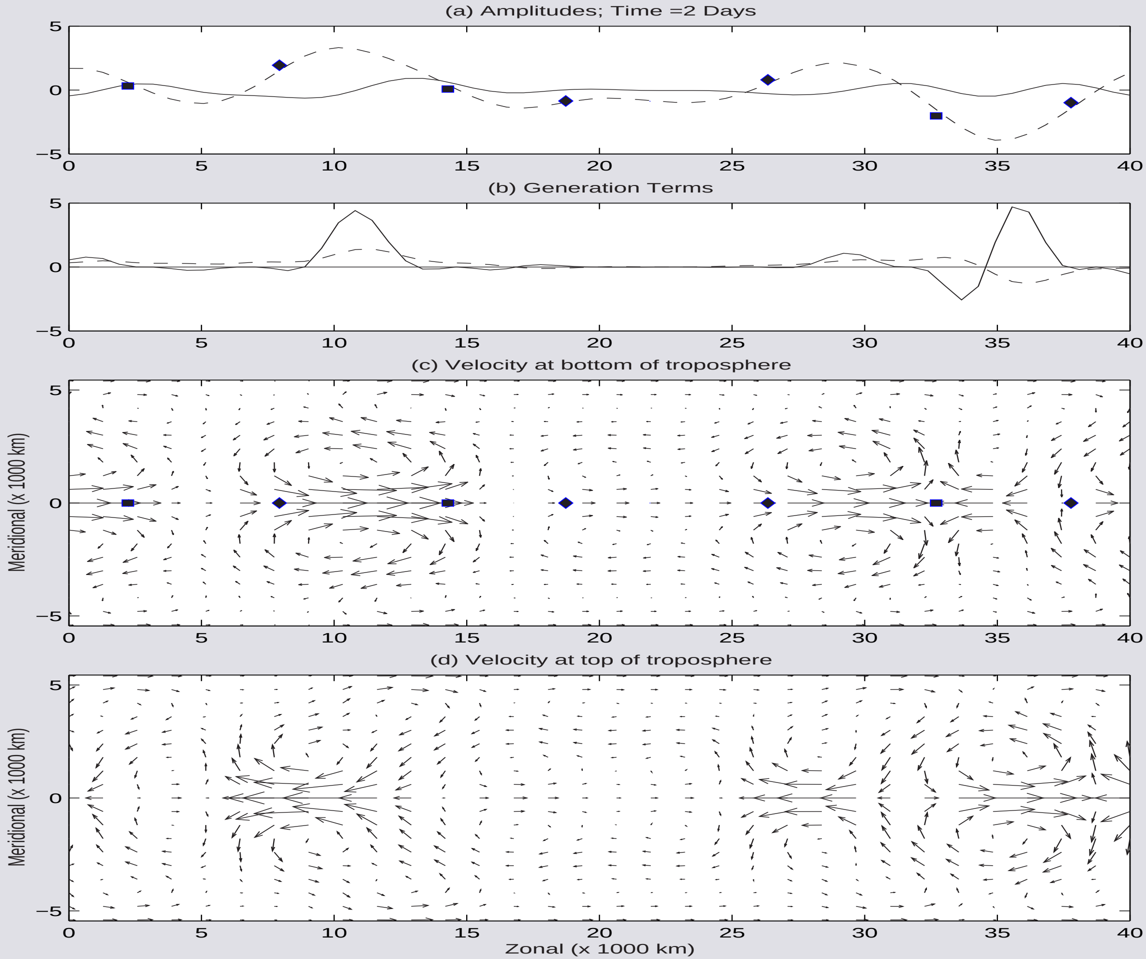
Nonlinear Dynamics: energy transfer from tropics to midlatitudes

- Mean BT flow $= 2.5\,\mathrm{ms}^{-1}$
- Mean BC flow $= 5\,\mathrm{ms}^{-1}$
- Initial energy in mean flow or BT wave only, seeded randomly

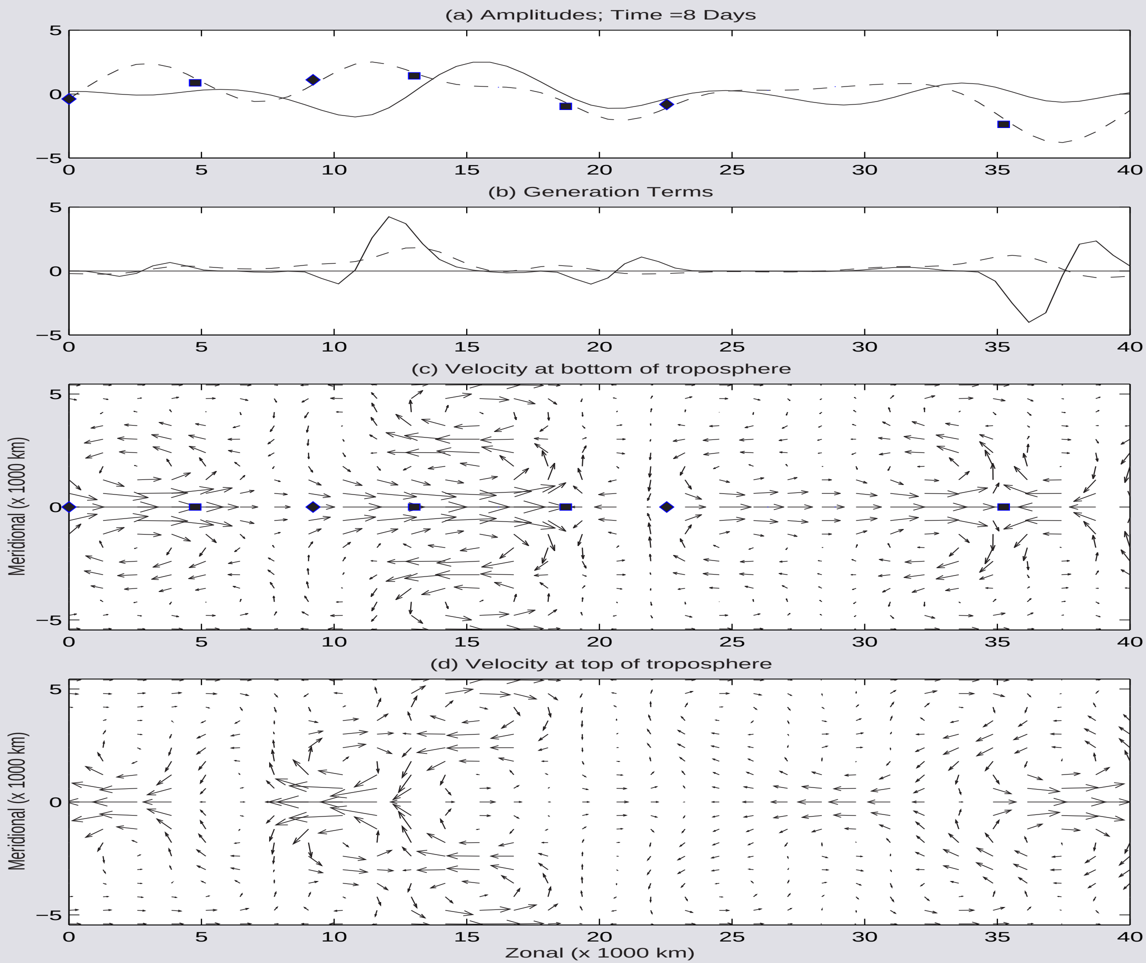
Results

- Development of Coherent BT Rossby Wave Train - midlatitude connection
- Timescale of less than 20 days

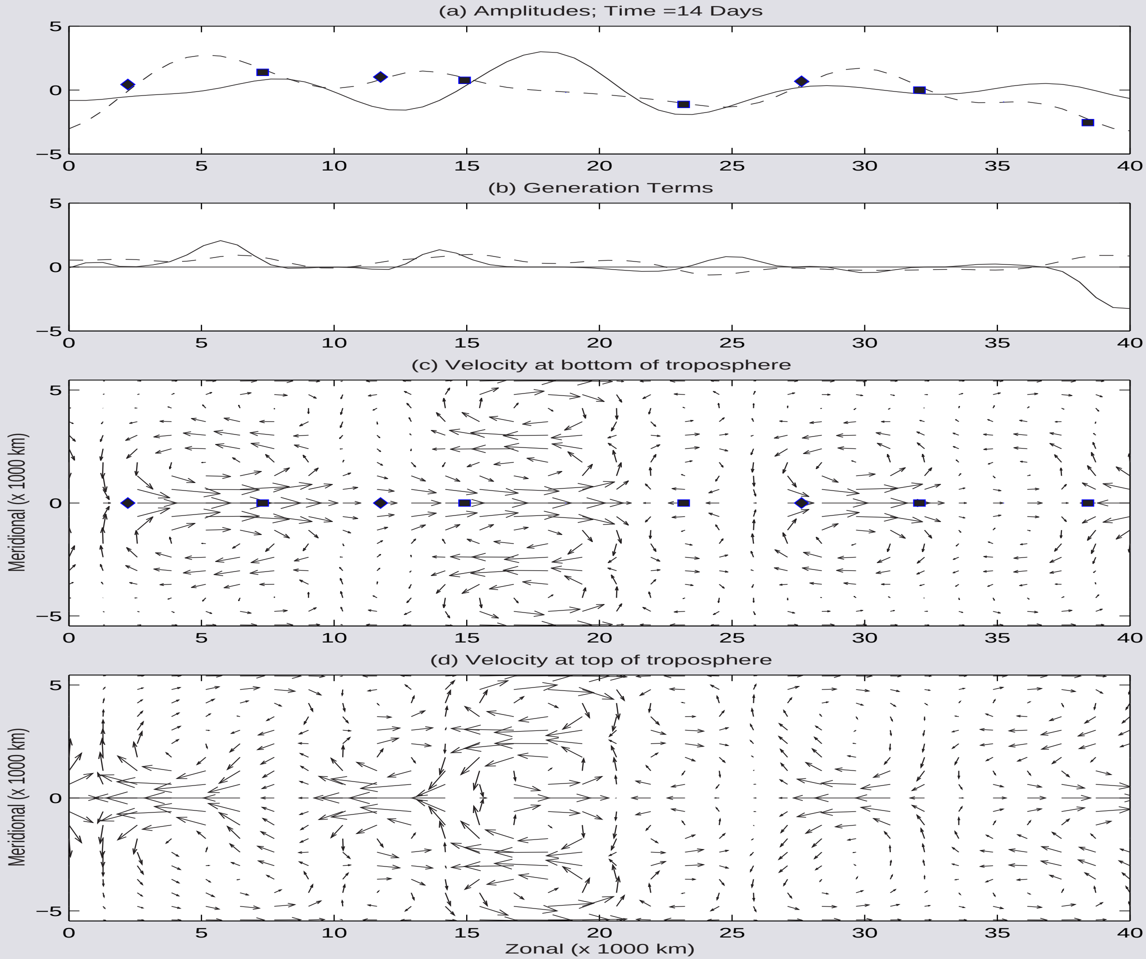
Tropics ⟶ midlatitudes: 2 days



Tropics ⟶ midlatitudes: 8 days



Tropics ⟶ midlatitudes: 14 days



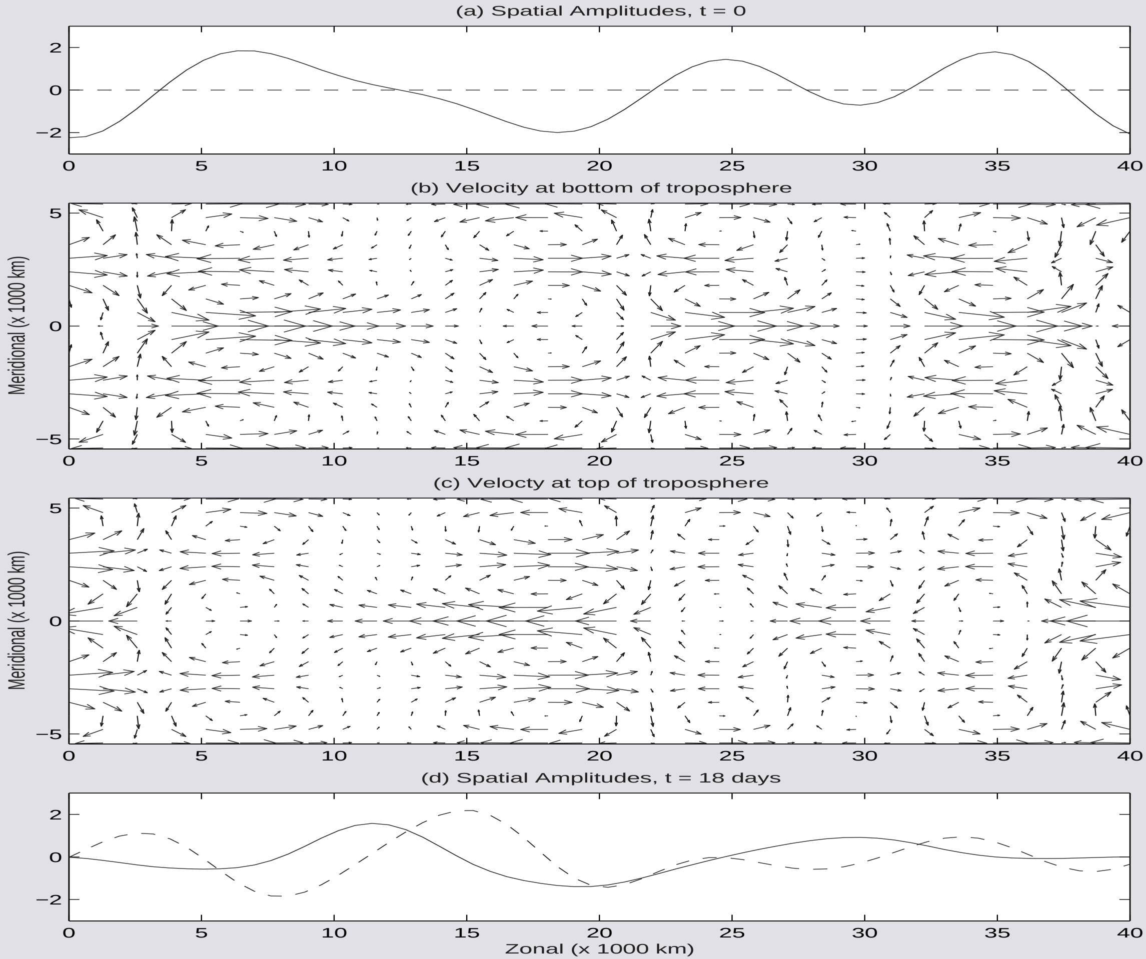
Nonlinear Dynamics: energy transfer from midlatitudes to the tropics

- Numerical integration of amplitude equations for a selection of mean flow profiles
- Mean BT flow $= 0\,\mathrm{ms}^{-1}$
- Mean BC flow $= 5\,\mathrm{ms}^{-1}$
- Initial energy in mean flow or BT wave only, seeded randomly

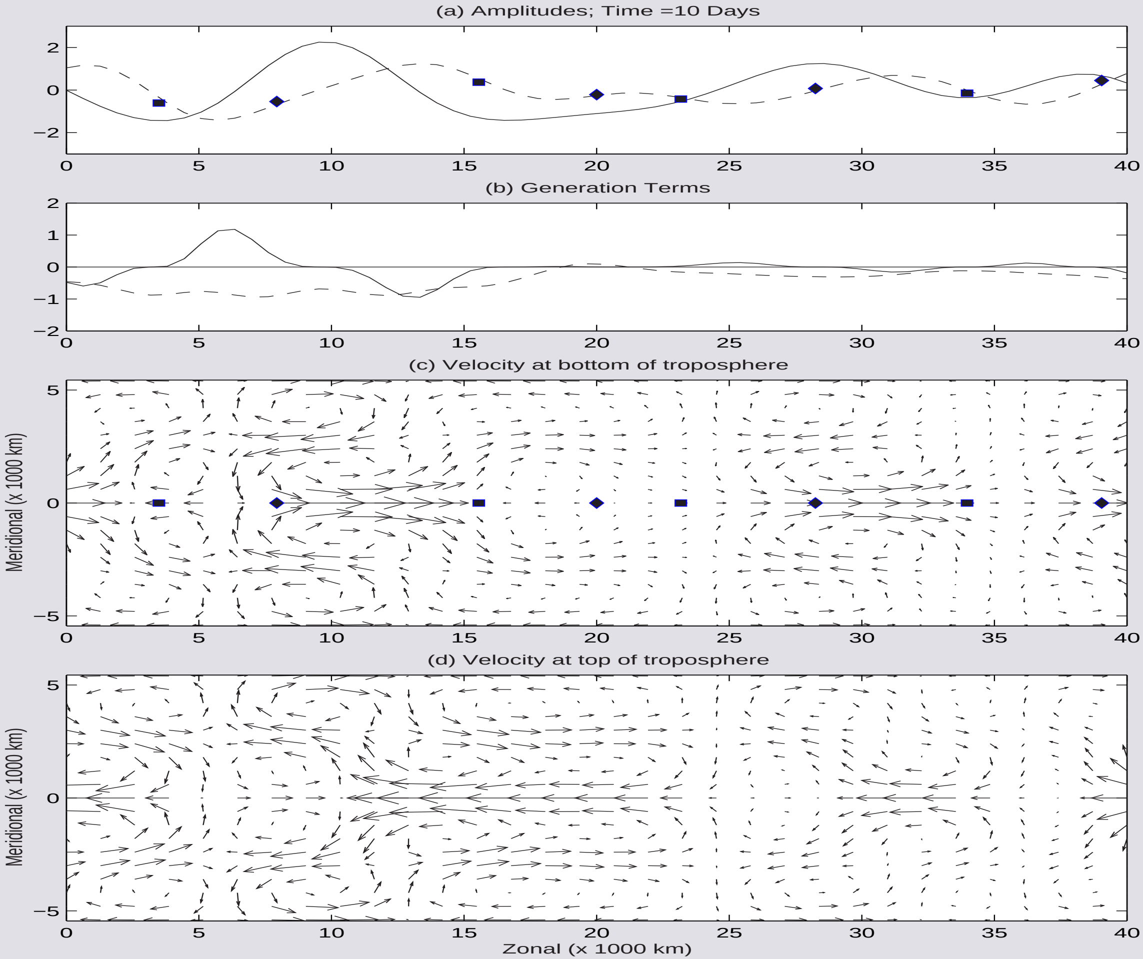
Results

- Strong driving of BC waves within 20 days
- Structure of a Westerly Wind Burst

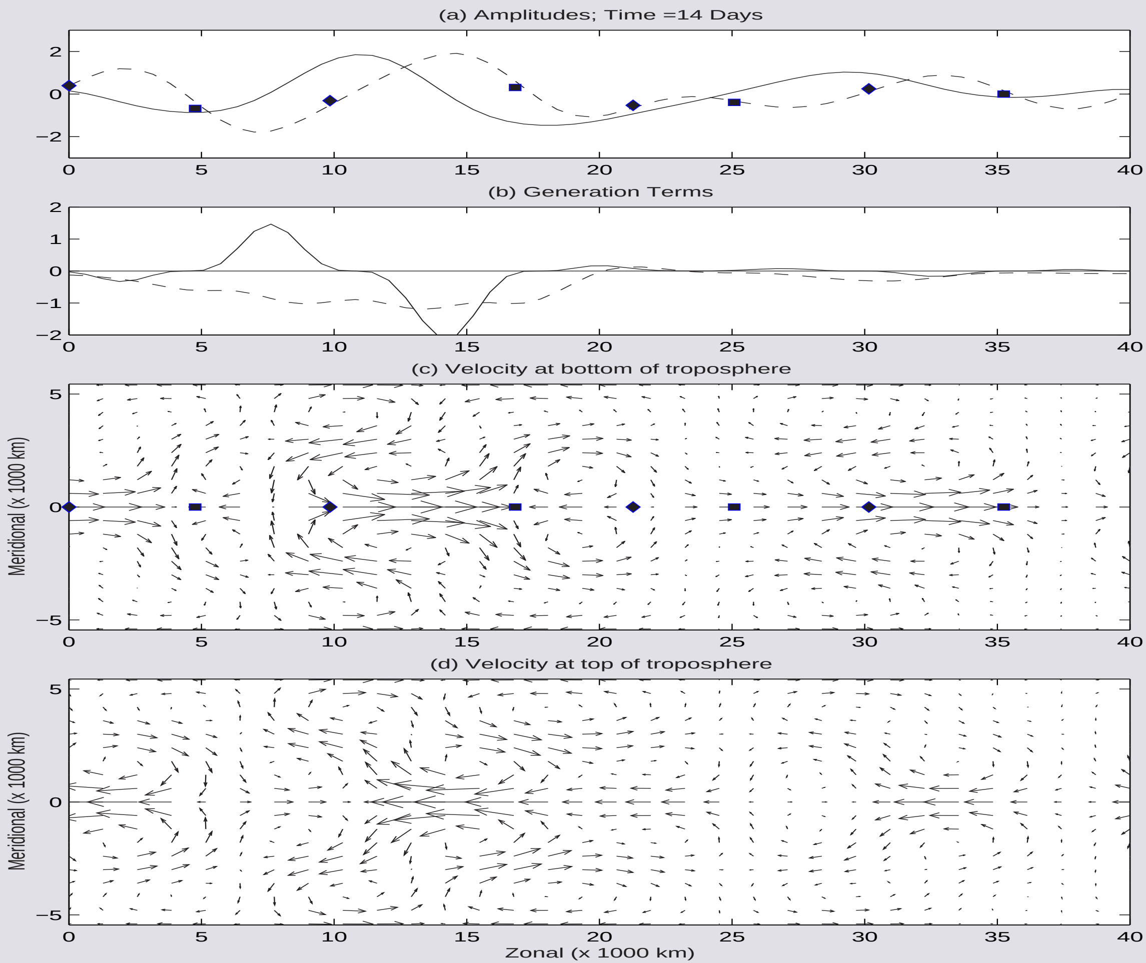
Midlatitudes ⟶ tropics: Initial condition



Midlatitudes ⟶ tropics: 10 days



Midlatitudes ⟶ tropics: 14 days



Summary of Tropics/Midlatitude wave theory

- *New equations* in the context of atmospheric sciences
- *New equations* in the context of applied mathematics
- Dispersion balanced nonlinearity.
- Equatorial waves drive *Midlatitude Rossby Waves* relevant for the *breakup of the MJO*.