Asymptotic models for the planetary and synoptic scales in the atmosphere

Stamen I. Dolaptchiev

Institut für Atmosphäre und Umwelt

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

with Rupert Klein (FU Berlin)

Outline

1. Planetary scale atmospheric motions and reduced models
2. The Planetary Regime
   - Single scale model
   - Two scale model: interactions with the synoptic scale
3. Anisotropic Planetary Regime
4. Balances on the planetary and synoptic scales in numerical experiments

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Planetary Scale Motions

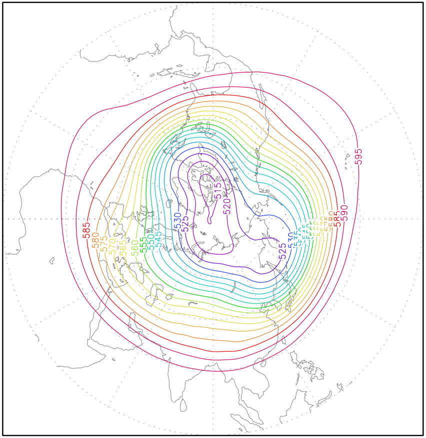

Figure: Time mean geopotential height of the 500 hPa surface for the northern hemisphere, DJF. Based upon ERA40 reanalysis data.

Planetary Scale Motions

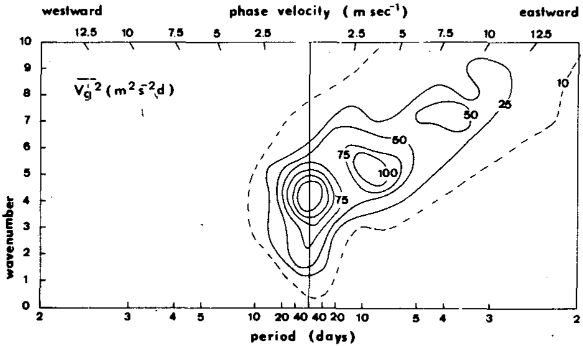

Figure: Power spectrum density of the meridional geostrophic wind at 500 hPa and $50°$N (Fraedrich & Böttger, 1978).

Planetary Scale Motions

- atmospheric phenomena:

  thermally and orographically induced quasi-stationary Rossby waves
  teleconnection patterns (e.g. NAO, PNA)
  zonal mean flows (subtropical and polar jets)

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- planetary spatial scales: $\mathcal{O}(6000\,\text{km})$
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- interactions with the synoptic eddies
  synoptic spatial and temporal scales: 1000 km and 1 day.

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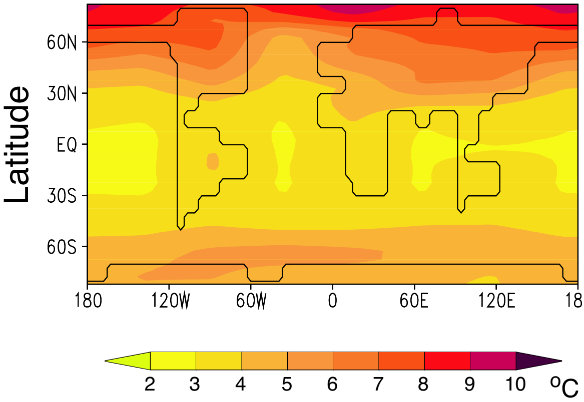

Figure: Annual-mean surface air temperature change in the year 2100 for standard scenario of future CO_2 emissions (Courtesy of S. Rahmstorf).

- SDM, Atmosphere: 2-D(ϕ, λ)-mL
- resolution: $51°$longitude, $10°$latitude

Earth System Models of Intermediate Complexity

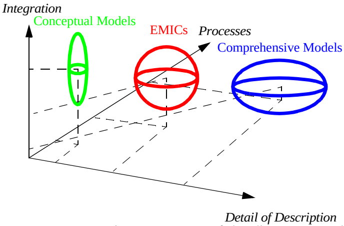

Table 1. References to EMICs

Model	Short list of references
1: Bern 2.5D	Stocker et al. (1992), Marchal et al. (1998)
2: CLIMBER-2	Petoukhov et al. (2000), Ganopolski et al. (2000)
3: EcBilt	Opsteegh et al. (1998)
4: EcBilt-CLIO	Goosse et al. (2000)
5: IAP RAS	Petoukhov et al. (1998), Handorf et al. (1999), Mokhov et al. (2000)
6: MPM	Wang and Mysak (2000), Mysak and Wang (2000)
7: MIT	Prinn et al. (1999), Kamenkovich et al. (2000)
8: MoBidiC	Crucifix et al. (2000a)(2000b)
9: PUMA	Fraedrich et al. (1998), Maier-Reimer et al. (1993)
10: Uvic	Weaver et al. (2000)
11: IMAGE 2	Alcamo (1994), Alcamo et al. (1996)

Table 2. Interactive components of the climate system being implemented into EMICs (for explanation see text)

Model	Atmosphere	Ocean	Biosphere	Sea ice	Inland ice
1	EMBM, 1-D(φ)	2-D(φ, z), 3 basins	B_o, B_T	T	
2	SDM, 2-D(φ, λ)-mL	2-D(φ, z) 3 basins	B_o, B_T, B_V	TD	3-D, polythermal
3	QG, 3-D, T21, L3	3-D, 5.6°×5.6°, L12		T	
4	QG, 3-D, T21-L3	3-D, 3°×3°	B_T, B_V	TD	
5	SDM, 3-D 4.5°×6°, L8	SDM, 2-D(φ, λ) 4.5°×6°, L3 fixed salinity		T	
6	EMBM, 1-D(φ), land/ocean boxes	2-D(φ, z), 3 basins		TD	2-D(φ, z), isothermal
7	SDM, 2-D(φ, z)/atmospheric chemistry	3-D, 4°×1.25° to 3.75°, L15	B_T	T	
8	QG, 2-D(φ, z)-L2	2-D(φ, z), 3 basins	B_o, B_T, B_V	TD	2-D(φ, z), isothermal
9	GCM, 3-D, T21, L5	3-D, 5°×5°, L11	B_o	TD	
10	DEMBM, 2-D(φ, λ)	3-D, 3.6°×1.8°, L 19		TD	3-D, polythermal
11	DEMBM, 2-D(φ, λ)/atmospheric chemistry	2-D(φ, z), 2 basins	B_o, B_T, B_V	T	

Adapted from Claussen et al. (2002)

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Asymptotic regimes

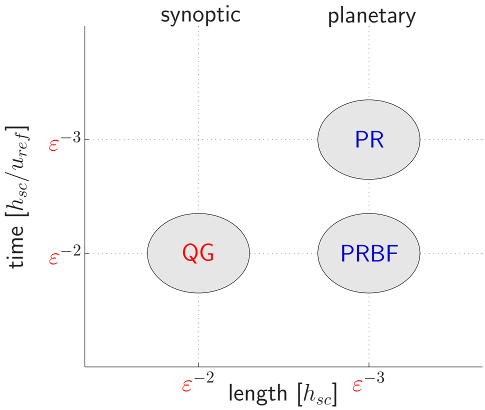

Figure: Scale map for the PR and PRBF, the validity range of the quasi-geostrophic (QG) theory is also shown.

An unified multiple scales asymptotic approach or the derivation of reduced model equations

Klein, 2000; Majda & Klein, 2003; Klein, 2008

1. Universal small parameter: $\varepsilon = \left(\frac{a\Omega^2}{g}\right)^{\frac{1}{3}}$
2. Distinguished limit: expression of the characteristic numbers in terms of ε
3. Multiple scales asymptotic ansatz

$$\boldsymbol{U}(t, \boldsymbol{x}, z; \varepsilon) = \sum_i \varepsilon^i \boldsymbol{U}^{(i)}(\frac{t}{\varepsilon}, t, \varepsilon t, \varepsilon^2 t, \ldots, \frac{\boldsymbol{x}}{\varepsilon}, \boldsymbol{x}, \varepsilon \boldsymbol{x}, \varepsilon^2 \boldsymbol{x}, \ldots, \frac{z}{\varepsilon}, z, \ldots)$$

The Planetary Regime (PR)

We consider horizontal velocities of the order of 10 m/s and weak background potential temperature variations, comparable in magnitude to those adopted in the classical QG theory: $\delta\Theta \sim \varepsilon^2$.

$$\delta\Theta \sim \varepsilon^2 : \qquad \Theta(\underbrace{\varepsilon^3 t}_{t_P}, \lambda_P, \phi_P, z) = 1 + \varepsilon^2 \Theta^{(2)} + \mathcal{O}\big(\varepsilon^3\big) \ ,$$

$$\boldsymbol{u}(t_P, \lambda_P, \phi_P, z) = \boldsymbol{u}^{(0)} + \varepsilon \boldsymbol{u}^{(1)} + \varepsilon^2 \boldsymbol{u}^{(2)} + \mathcal{O}\big(\varepsilon^3\big) \ .$$

The Planetary Regime

Leading order model: **planetary geostrophic equations**

$$\boldsymbol{u}^{(0)} = \frac{1}{f\rho^{(0)}} \boldsymbol{e}_z \times \nabla_P p^{(2)} \,,$$
$$\frac{\partial}{\partial z} \frac{p^{(2)}}{\rho^{(0)}} = \Theta^{(2)} \,,$$
$$\nabla_P \cdot \rho^{(0)} \boldsymbol{u}^{(0)} + \frac{\partial}{\partial z} \rho^{(0)} w^{(3)} = 0 \,,$$
$$\frac{\partial}{\partial t_P} \Theta^{(2)} + \boldsymbol{u}^{(0)} \cdot \nabla_P \Theta^{(2)} + w^{(3)} \frac{\partial}{\partial z} \Theta^{(2)} = 0 \,,$$

- additional boundary condition needed: $p^{(2)}$ at some level or $\overline{p^{(2)}}^{z}$.
- implemented in the atmospheric module of some earth system models of intermediate complexity (EMICs)

The Planetary Regime

Evolution equation for the barotropic component of the pressure $\overline{p^{(2)}}^z$

$$\frac{\partial}{\partial t_P}\left(\frac{\partial}{\partial \tilde{y}_P}\frac{1}{f}\frac{\partial}{\partial y_P}\overline{p^{(2)}}^z - \frac{\beta}{f^2}\frac{\partial}{\partial y_P}\overline{p^{(2)}}^z - f\overline{p^{(2)}}^z\right) - \frac{\partial}{\partial \tilde{y}_P}N + \frac{\beta}{f}N = 0\,,$$

$$N = \frac{\partial}{\partial \tilde{y}_P}\overline{\rho^{(0)}v^{(0)}u^{(0)}}^{\lambda_P,z} - \overline{\rho^{(0)}v^{(0)}u^{(0)}}^{\lambda_P,z}\frac{\tan\phi_P}{a} + \overline{\frac{\partial}{\partial z}p^{(2)}\frac{\partial}{\partial x_P}\frac{p^{(2)}}{\rho^{(0)}}}^{\lambda_P,z}\,.$$

- the barotropic pressure $\overline{p^{(2)}}^z$ is zonally symmetric
- EMICs use a diagnostic closure!

The Planetary and the Synoptic Scales

- Interactions between the planetary and the synoptic scales
- Quasi-geostrophic theory on a sphere: $\delta f \sim f_0 \sim \mathcal{O}(1)$, $N \neq const$
- two scales ansatz resolving additionally to the planetary scales the synoptic spatial scales (internal Rossby deformation radius) and the corresponding advective time scale
- coordinates scaling:

$$\Theta(t_P, \underbrace{\varepsilon^2 t}_{t_S}, \lambda_P, \phi_P, \underbrace{\varepsilon^{-1}\lambda_P}_{\lambda_S}, \underbrace{\varepsilon^{-1}\phi_P}_{\phi_S}, z)$$

$$= 1 + \varepsilon^2 \Theta^{(2)}(t_P, \lambda_P, \phi_P, z) + \varepsilon^3 \Theta^{(3)}(t_P, t_S, \lambda_P, \phi_P, \lambda_S, \phi_S, z) + \mathcal{O}(\varepsilon^4)$$

The Planetary and the Synoptic Scales

geostrophic balance:

$$\boldsymbol{u}^{(0)} = \underbrace{\frac{1}{f}\boldsymbol{e}_r \times \nabla_S \pi^{(3)}}_{:= \boldsymbol{u}_S{}^{(0)}} + \underbrace{\frac{1}{f}\boldsymbol{e}_r \times \nabla_P \pi^{(2)}}_{:= \boldsymbol{u}_P{}^{(0)}}, \qquad \pi^{(i)} = \frac{p^{(i)}}{\rho^{(0)}},$$

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$$\boldsymbol{u}_S{}^{(0)} = \boldsymbol{u}_S{}^{(0)}(t_S, t_P, \lambda_P, \phi_P, \lambda_S, \phi_S, z)$$

$$\boldsymbol{u}_P{}^{(0)} = \boldsymbol{u}_P{}^{(0)}(t_P, \lambda_P, \phi_P, z)$$

The Planetary and the Synoptic Scales

Planetary scale dynamics:

$$\left(\frac{\partial}{\partial t_P} + \boldsymbol{u}_P{}^{(0)} \cdot \nabla_P + w_P^{(3)} \frac{\partial}{\partial z}\right) PV^{(2)} = 0\,, \qquad PV^{(2)} = \frac{f}{\rho^{(0)}} \frac{\partial \Theta^{(2)}}{\partial z}\,.$$

Synoptic scale dynamics:

$$\left(\frac{\partial}{\partial t_S} + \left(\boldsymbol{u}_S^{(0)} + \boldsymbol{u}_P^{(0)}\right) \cdot \nabla_S\right) PV^{(3)} + \beta v_S^{(0)} + \frac{f}{\rho^{(0)}} \boldsymbol{u}_S^{(0)} \cdot \frac{\partial}{\partial z} \frac{\nabla_P \rho^{(0)} \Theta^{(2)}}{\partial \Theta^{(2)}/\partial z} = 0\,,$$

$$PV^{(3)} = \frac{1}{f} \Delta_S \pi^{(3)} + \frac{f}{\rho^{(0)}} \frac{\partial}{\partial z} \left(\frac{\rho^{(0)} \partial \pi^{(3)}/\partial z}{\partial \Theta^{(2)}/\partial z}\right)$$

- advection of synoptic scale PV by the planetary scale velocity field
- advection of PV resulting from the planetary scale gradient of $\Theta^{(2)}$ by the synoptic scale velocities

The Planetary and the Synoptic Scales

Planetary-synoptic interactions in the evolution equation for the planetary scale structure of $\overline{p^{(2)}}^{z}$

$$\frac{\partial}{\partial t_P}\left(\frac{\partial}{\partial \tilde{y}_P}\frac{1}{f}\frac{\partial}{\partial y_P}\overline{p^{(2)}}^{z} - \frac{\beta}{f^2}\frac{\partial}{\partial y_P}\overline{p^{(2)}}^{z} - f\overline{p^{(2)}}^{z}\right) - \frac{\partial}{\partial \tilde{y}_P}N + \frac{\beta}{f}N = 0\,,$$

$$N = \frac{\partial}{\partial \tilde{y}_P}\overline{\rho^{(0)}\left(v_P^{(0)}u_P^{(0)} + v_S^{(0)}u_S^{(0)}\right)}^{S,\lambda_P,z} - \overline{\rho^{(0)}\left(v_P^{(0)}u_P^{(0)} + v_S^{(0)}u_S^{(0)}\right)}^{S,\lambda_P,z}\frac{\tan\phi_P}{a}$$

$$+\overline{\frac{\partial}{\partial z}p^{(2)}\frac{\partial}{\partial x_P}\frac{p^{(2)}}{\rho^{(0)}}}^{\lambda,z}\,.$$

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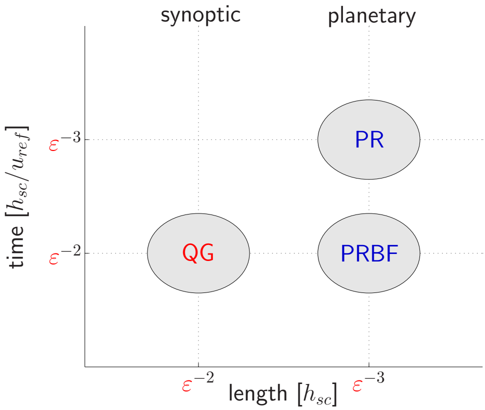

Figure: Scale map for the PR and PRBF, the validity range of the quasi-geostrophic (QG) theory is also shown.

Motions with planetary zonal & synoptic meridional scales

- β-plane approximation for a sphere
- anisotropic scaling

$$\Theta(\underbrace{\varepsilon^2 t}_{t_S}, \underbrace{\varepsilon^3 t}_{t_P}, \lambda_P, \underbrace{\varepsilon^{-1}\lambda_P}_{\lambda_S}, \underbrace{\varepsilon^{-1}\phi_P}_{\phi_S}, z) = 1 + \varepsilon^2 \Theta^{(2)}(\lambda_P, z, t_P)$$
$$+ \varepsilon^3 \Theta^{(3)}(t_P, t_S, \lambda_P, \lambda_S, \phi_S, z) + \mathcal{O}(\varepsilon^4)$$
$$\boldsymbol{u}(t_P, t_S, \lambda_P, \lambda_S, \phi_S, z) = \boldsymbol{u}^{(0)} + \varepsilon \boldsymbol{u}^{(1)} + \varepsilon^2 \boldsymbol{u}^{(2)} + \mathcal{O}(\varepsilon^3) \ .$$

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- solvability condition $\Longrightarrow \Theta^{(2)}(z)$
- Leading order model: classical QG on a sphere

Motions with planetary zonal & synoptic meridional scales

Case: dynamics on a plane: $\lambda_P, \lambda_S, \phi_S \to X, x, y$

Motions with planetary zonal & synoptic meridional scales

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Next order model: **planetary scale structure**, **next order QG corrections**

$$\frac{d}{dt_S} PV^{(4)} + \boldsymbol{u}^{(1)} \cdot \nabla_S PV^{(3)} = S_{qg} - \frac{d}{dt_P} PV^{(3)} - \frac{d}{dt_S} \frac{\partial}{\partial X} v^{(0)} ,$$
$$PV^{(4)} = \tilde{\Delta}_S \Phi^{(4)} + \frac{1}{f_0} \frac{\partial}{\partial X} \frac{\partial}{\partial x} \pi^{(3)} - \frac{f_0}{2} y^2$$

$$\frac{d}{dt_{S,P}} = \left(\frac{\partial}{\partial t_{S,P}} + \boldsymbol{u}^{(0)} \cdot \nabla_{S,P} \right) , \tilde{\Delta} = \frac{1}{f_0} \Delta_S + f_0 \frac{\partial^2}{\partial z^2} , S_{qg}(\pi^{(3)}) .$$

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- solvability condition for the planetary-scale dynamics
- **Case**: synoptic scales only $\Longrightarrow$ QG^{+1} of Muraki et al. (1999)

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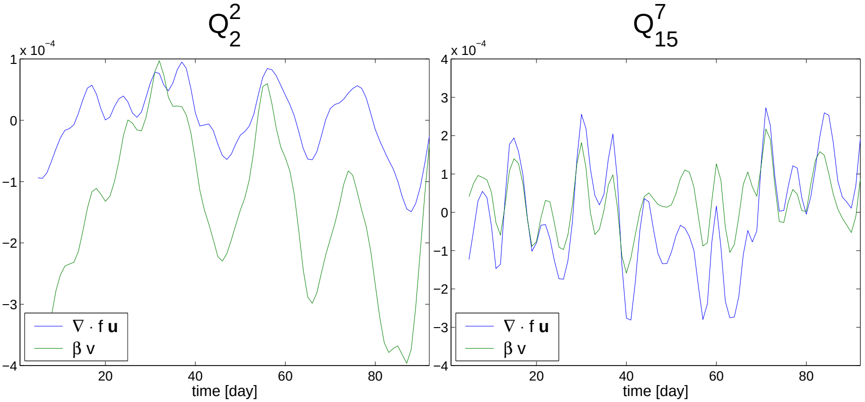
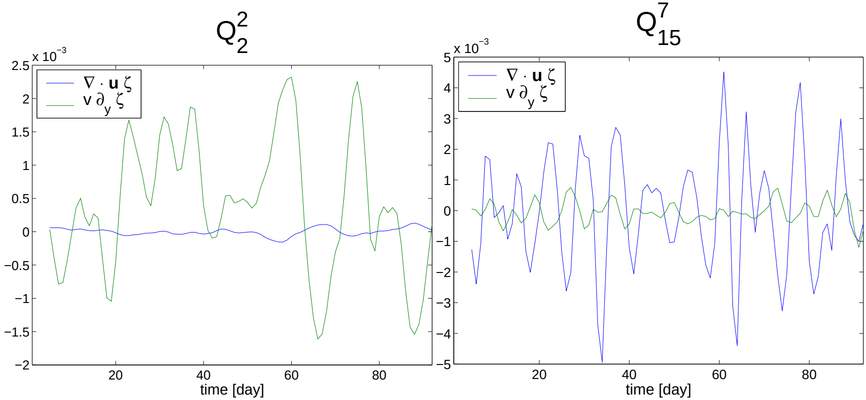
The Planetary Regime in numerical experiments



$$\overline{f \nabla_P \cdot \boldsymbol{u}^{(0)}}^S + \beta \overline{v^{(0)}}^S = 0$$

The Planetary Regime in numerical experiments



$$\overline{\nabla_S \cdot \boldsymbol{u}^{(0)} \zeta^{(0)}}^S = 0$$

Conclusions and outlook

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- behavior of the model in the tropics
  Majda & Klein 2003; Majda, 2007:
  $X_M = \varepsilon^2 x, X_S = \varepsilon^{\frac{5}{2}} x, X_P = \varepsilon^{\frac{7}{2}} x$
  $\Longrightarrow MEWTG, IPESD$

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- numerical implementation of the Planetary Regime, single scale/ two scale
- MTV strategy for the synoptic scale dynamics

Outline

5 An unified multiple scales asymptotic approach

An unified multiple scales asymptotic approach or the derivation of reduced model equations

1. Universal parameters:

for the rotating earth:
$a \sim 6\ 10^3$ km,
$\Omega \sim 7\ 10^{-5}$ s^{-1},
$g \sim 10$ m s^{-2}

for a variety of atmospheric flow regimes:
$p_{ref} \sim 1$ bar,
$T_{ref} \sim 290$ K,
$u_{ref} \sim 10$ m s^{-1}

An unified multiple scales asymptotic approach or the derivation of reduced model equations

2. Characteristic numbers

three independent nondimensional numbers

$$\pi_1 = \frac{c_{ref}}{\Omega a} \approx \frac{1}{2}\,, \quad \pi_2 = \frac{u_{ref}}{c_{ref}} \approx 0.03\,, \quad \pi_3 = \frac{a\Omega^2}{g} \approx 0.006\,,$$

where $c_{ref} = \sqrt{\gamma R T_{ref}}$.

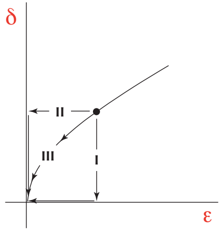
difficulties with multiple small parameter expansions

$$F = \frac{a + b\varepsilon}{1 + \delta}$$

$$\lim_{\varepsilon \to 0} \lim_{\delta \to 0} F = \lim_{\delta \to 0} \lim_{\varepsilon \to 0} F = a$$

$$G = \frac{a\varepsilon + b\delta}{\varepsilon + \delta}$$

$$\lim_{\varepsilon \to 0} \lim_{\delta \to 0} G = a \qquad \lim_{\delta \to 0} \lim_{\varepsilon \to 0} G = b$$



An unified multiple scales asymptotic approach or the derivation of reduced model equations

3. Distinguished limit

Expression of the characteristic numbers in terms of ε:

$$\pi_1 = \frac{c_{ref}}{\Omega a} = \pi_1^* \,,$$
$$\pi_2 = \frac{u_{ref}}{c_{ref}} = \varepsilon^2 \pi_2^* \,,$$
$$\pi_3 = \frac{a\Omega^2}{g} = \varepsilon^3 \pi_3^* \,,$$
$$\text{as } \varepsilon \to 0$$

where π_1^*, π_2^* and π_3^* are $\mathcal{O}(1)$ and $\varepsilon \sim \frac{1}{8} \dots \frac{1}{6}$.

An unified multiple scales asymptotic approach or the derivation of reduced model equations

3. Classical dimensionless parameter

Mach, Froude and Rossby number are expressed in terms of ε

$$M = \frac{u_{ref}}{\sqrt{\gamma R T_{ref}}} = \pi_2 \sim \varepsilon^2 \,,$$

$$Fr = \frac{u_{ref}}{\sqrt{g h_{sc}}} = \sqrt{\gamma}\pi_2 \sim \varepsilon^2 \,,$$

$$Ro_{h_{sc}} = \frac{u_{ref}}{2\Omega h_{sc}} = \frac{a}{2h_{sc}}\pi_2 \sim \frac{1}{\varepsilon} \,,$$

where $h_{sc} = p_{ref}/(g\rho_{ref})$.
The Rossby number is “small” if we use the internal Rossby deformation radius $L_{syn} \sim 1000$km $\sim \varepsilon^{-2} h_{sc}$

$$Ro_{L_{syn}} = \frac{u_{ref}}{2\Omega L_{syn}} \sim \varepsilon^2 Ro_{h_{sc}} \sim \varepsilon \,.$$

An unified multiple scales asymptotic approach or the derivation of reduced model equations

4. Multiple scales asymptotic ansatz

$$\boldsymbol{U}(t, \boldsymbol{x}, z; \varepsilon) = \sum_i \varepsilon^i \boldsymbol{U}^{(i)}(\frac{t}{\varepsilon}, t, \varepsilon t, \varepsilon^2 t, \ldots, \frac{\boldsymbol{x}}{\varepsilon}, \boldsymbol{x}, \varepsilon \boldsymbol{x}, \varepsilon^2 \boldsymbol{x}, \ldots, \frac{z}{\varepsilon}, z, \ldots)$$

Example: quasi-geostrophic theory

Characteristic length and time scales, the rescaled coordinates

$$L_{syn} \sim 1000\,\text{km} \sim \varepsilon^{-2} h_{sc} \Rightarrow x_S = \frac{x}{L_{syn}} = \frac{\varepsilon^2 x}{h_{sc}} = \varepsilon^2 x'$$

$$T_{syn} \sim 1\,\text{day} \sim \varepsilon^{-2} \frac{h_{sc}}{u_{ref}} \Rightarrow t_S = \frac{t}{T_{syn}} = \frac{\varepsilon^2 t}{h_{sc}/u_{ref}} = \varepsilon^2 t'$$

The asymptotic ansatz for the velocity takes the form

$$\boldsymbol{u}(t, \boldsymbol{x}, z; \varepsilon) = \sum_i \varepsilon^i \boldsymbol{u}^{(i)}(x_S, y_S, t_S, z)$$

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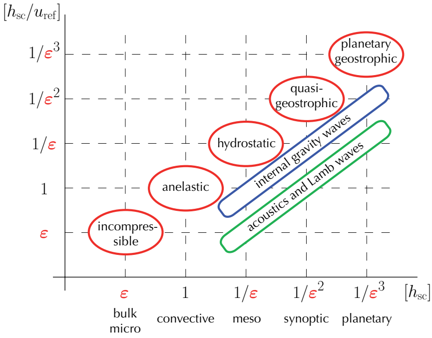

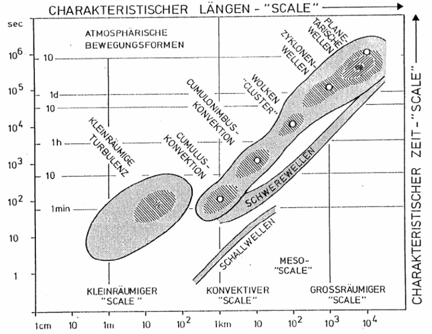

CLASSICAL RESULTS:

Coordinate scalings	Simplified model obtained
$\mathcal{U}^{(i)} = \mathcal{U}^{(i)}(t, \mathbf{x}, z)$	Anelastic & pseudo-incompressible models
$\mathcal{U}^{(i)} = \mathcal{U}^{(i)}(t, \varepsilon\mathbf{x}, z)$	Linear large scale internal gravity waves
$\mathcal{U}^{(i)} = \mathcal{U}^{(i)}(\frac{t}{\varepsilon}, \mathbf{x}, \frac{z}{\varepsilon})$	Linear small scale internal gravity waves
$\mathcal{U}^{(i)} = \mathcal{U}^{(i)}(\varepsilon^2 t, \varepsilon^2\mathbf{x}, z)$	Mid-latitude Quasi-Geostrophic model
$\mathcal{U}^{(i)} = \mathcal{U}^{(i)}(\varepsilon^2 t, \varepsilon^2\mathbf{x}, z)$	Equatorial Weak Temperature Gradient models
$\mathcal{U}^{(i)} = \mathcal{U}^{(i)}(\varepsilon^2 t, \varepsilon^{-1}\xi(\varepsilon^2\mathbf{x}), z)$	Semi-geostrophic model
$\mathcal{U}^{(i)} = \mathcal{U}^{(i)}(\varepsilon^{\frac{5}{2}} t, \varepsilon^{\frac{7}{2}} x, \varepsilon^{\frac{5}{2}} y, z)$	Equatorial Kelvin, Yanai & Rossby Waves

NEW MODELS

- Intraseasonal Planetary Equatorial Dynamics
  Majda & Klein (2003); Majda & Biello (2004)
- Deep Mesoscale Convection
  Klein & Majda (2006)
- Hurricanes
  Mikusky (2007); Mikusky, Klein & Owinoh (in prep.)

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