MULTISCALE MODELLING OF STRATOCUMULUS CLOUDS

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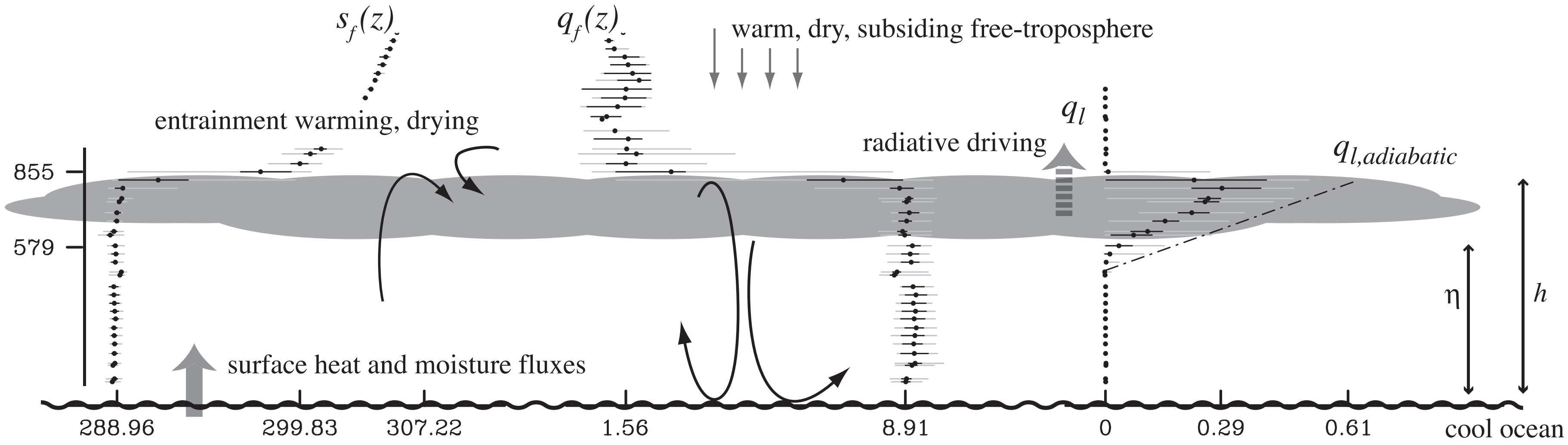
Joint work with ....



To investigate how dynamic processes effect cloud evolution in intermediate timescales, i.e., waves or perturbations to the interface and their interaction with the evolving flow; in part to help understand how dynamic processes may contribute to cloud organization

We present a derivation of the underlying equations from a more formal point of view,with an aim of developing a consistent view of the interplay between thermodynamic and dynamic aspects of stratocumulus layers

To explore the mathematical properties of the solutions and their relevance to observed processes.

Unified Approach to Meteorological Modelling Based on Multiple Scale Asymptotics Techniques developed by Klein & Associates

1. Three-dimensional compressible flow equations
2. Identification of
   - uniformly valid system scales
   - non-dimensional parameters
   - distinguished limits
     $\varepsilon \longrightarrow 0$: $\mathrm{Fr} = \frac{u_{\mathrm{ref}}}{\sqrt{g h_{\mathrm{sc}}}} \sim \varepsilon^2$, $\mathrm{M} = \frac{u_{\mathrm{ref}}}{c_{\mathrm{ref}}} \sim \varepsilon^2$,
     $\mathrm{Ro}_{h_{\mathrm{sc}}} = \frac{u_{\mathrm{ref}}}{2\Omega h_{\mathrm{sc}}} \sim \varepsilon^{-1}$; $(h_{\mathrm{sc}} = p_{\mathrm{ref}}/\rho_{\mathrm{ref}} g)$
3. Dimensionless Equations
4. Specialization of a Multiple scales Ansatz

The moist thermodynamics introduce a number of other dimensionless parameters that must also be tied to the distinguished limit ε e.g.

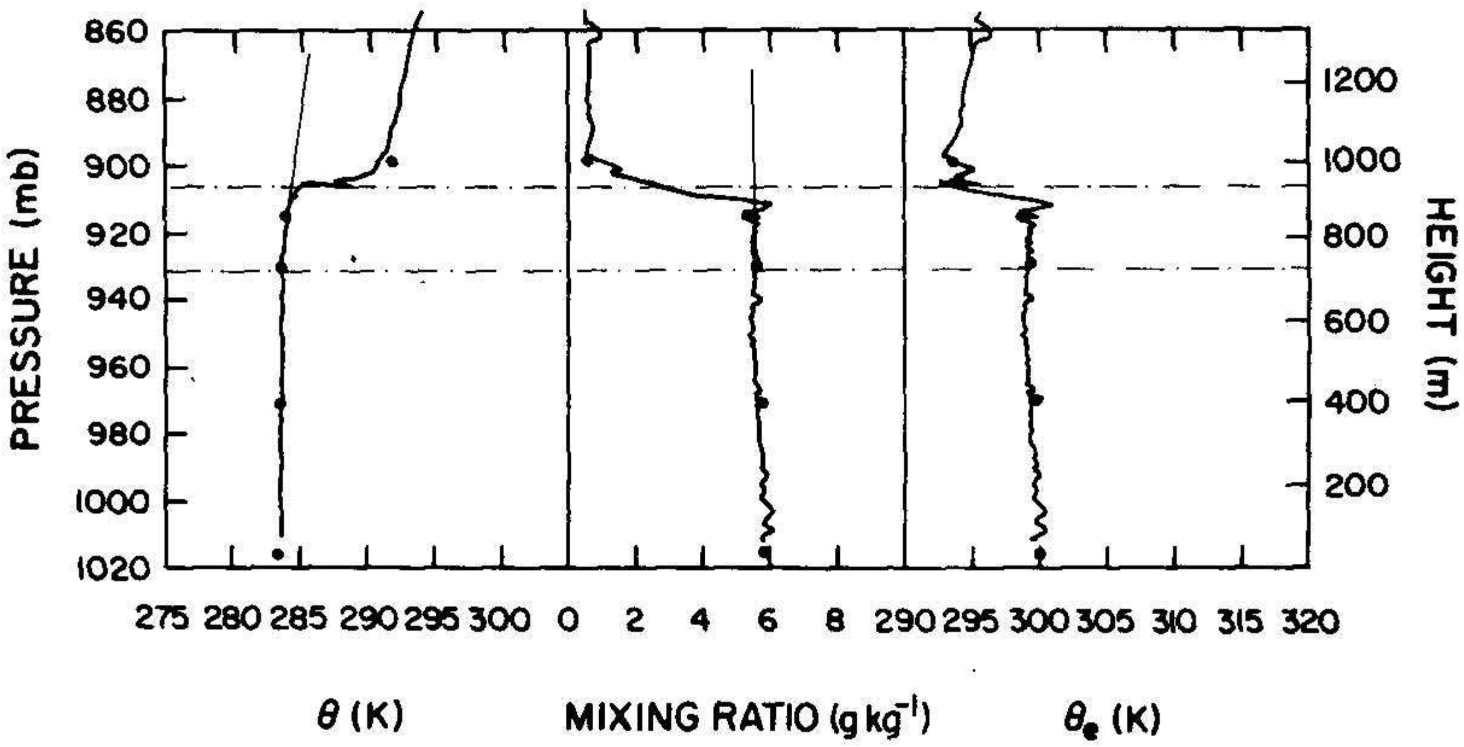
$$\frac{L_v \varrho_{\text{ref}}}{p_{\text{ref}}} = 31.25 \equiv \varepsilon^{-1} L_v^{**}, \qquad \frac{R_v}{R_d} = \frac{461.5}{287.0} \equiv R^{**} \varepsilon^0,$$
$$\frac{R_d}{c_{p_d}} = \frac{\gamma - 1}{\gamma} = \frac{2}{7} \equiv \Gamma^{**} \varepsilon, \qquad \frac{c_l}{c_{p_d}} = \frac{4217}{1007} \equiv c_p^{**} \varepsilon^{-1},$$

Klein and Majda (2006) found that these asymptotic limits allowed for development of mesoscale deep convection models.

Equation of State

$$\varrho \theta_e = p^{[1-\Gamma^{**}\varepsilon(1+\varepsilon^{-1}c_p^{**}q_t)^{-1}]}(1+q_t)(1+R^{**}q_v)^{[-1+\Gamma\varepsilon(1+\varepsilon^{-1}c_p^{**}q_t)^{-1}]}$$
$$\left(\frac{q_v}{q_{vs}}\right)^{-R^{**}\Gamma^{**}\varepsilon q_v(1+c_p^{**}\varepsilon^{-1}q_t)^{-1}} \exp\left(\frac{L_v^{**}\Gamma^{**}(1+R^{**}q_v)\varrho q_v}{(1+c_p^{**}\varepsilon^{-1}q_t)(1+q_t)p}\right)$$

- Describe the thermodynamics in terms of equivalent potential temperature θ_e and total water mixing ratio q_t since θ_e has the additional advantage of being weakly conserved also in the presence of precipitation.
- Assume the leading order equations feel the effects of radiation as a source term in the θ_e equation while precipitation acts principally as a source (sink) for q_t.



Albrecht *et al*, JAS 1995

- Resolve a shallow layer of fluid of depth of 500–600m (i.e $\varepsilon^{\frac{3}{2}} h_{\mathrm{sc}}$).
- Horizontal length scales of approximately 500–600m and 70–100 km (i.e $\varepsilon^{-1} h_{\mathrm{sc}}$).
- We consider the time scales associated with the horizontal advection i.e. $\varepsilon^{-\frac{3}{2}} t_{\mathrm{ref}}$ (5 hrs) and convective time scale t_{ref} (20 min) associated with 500–600m scale and speeds of 0.2m/s ($\varepsilon^{\frac{3}{2}} u_{\mathrm{ref}}$).

Thus the new co-ordinate system: $\boldsymbol{X}_{||} = \varepsilon^{-1}\boldsymbol{x}_{||}$, $\boldsymbol{\xi}_{||} = \varepsilon^{-\frac{3}{2}}\boldsymbol{x}_{||}$, $\eta = \varepsilon^{-\frac{3}{2}} z$, $T = \varepsilon t$ and $\tau = t$.

1. Handling of the pressure gradient term
   (a) Pressure above the boundary layer assuming drier troposphere than boundary layer
   (b) Integrate the vertical momentum balance using the equation of state and assume continuity in pressure at the top of the boundary layer.
2. Depth averaged equation subject to free surface kinematic boundary conditions on $\eta = H$ and vanishing velocities at lower boundary condition $\eta = 0$
3. Fast Scale Averaged Equations using spatio-temporal (τ, ξ) sublinear growth conditions

$$H^{(1)} \sim 200\text{m}; \quad \theta_e^{(6)} \sim 0.5\text{--}1.5\text{K}; \quad q_t^{(6)} \sim 1\text{--}2\text{g/kg}; \quad \boldsymbol{v}_{||}^{(1)} \sim 3\text{m/s}$$

$$\frac{\partial H^{(1)}}{\partial T} + \boldsymbol{v}_{||}^{(0)} . \nabla_X H^{(1)} + H^{(0)} \nabla_X \cdot \boldsymbol{v}_{||}^{(1)} + H^{(1)} \nabla_X \cdot \boldsymbol{v}_{||}^{(0)} = \overline{E}^{(6)}$$

$$\frac{\partial \theta_e^{(6)}}{\partial T} + \boldsymbol{v}_{||}^{(0)} \nabla_X \theta_e^{(6)} = \frac{(\overline{w \varrho \theta_e})_s^{(11)}}{H^{(0)}} + \frac{[\overline{(\varrho \theta_e)_H E}]^{(11)}}{H^{(0)}} + \frac{(\overline{H \left\langle \varrho \mathcal{S}_{\theta_e} \right\rangle})^{(8)}}{H^{(0)}}$$

$$\frac{\partial q_t^{(6)}}{\partial T} + \boldsymbol{v}_{||}^{(0)} \nabla_X q_t^{(6)} = \frac{(\overline{w \varrho q_t})_s^{(11)}}{H^{(0)}} + \frac{[\overline{(\varrho q_t)_H E}]^{(11)}}{H^{(0)}} + \frac{(\overline{H \left\langle \varrho \mathcal{S}_{q_t} \right\rangle})^{(8)}}{H^{(0)}}$$

$$
\frac{\partial \boldsymbol{v}_{_{||}}^{(1)}}{\partial T} + \boldsymbol{v}_{_{||}}^{(0)} . \nabla_X \boldsymbol{v}_{_{||}}^{(1)} + (\widehat{\boldsymbol{\Omega}} \times \boldsymbol{v}^{(1)})_{_{||}} - \frac{(\overline{w \varrho \boldsymbol{v}_{_{||}}})_s^{(6)}}{H^{(0)}} =
$$

$$
\frac{1}{H^{(0)}} \Big\{ \theta_e^{(3)} \left[H^{(0)} \nabla_X H^{(3)} + H^{(1)} \nabla_X H^{(2)} + H^{(2)} \nabla_X H^{(1)} \right]
- \left(-\theta_e^{(4)} + q_t^{(4)} + \tilde{R}^{**} q_v^{(4)} \right) \left[H^{(1)} \nabla_X H^{(1)} + H^{(0)} \nabla_X H^{(2)} \right]
- \left(-\theta_e^{(5)} + q_t^{(5)} + \tilde{R}^{**} q_v^{(5)} \right) H^{(0)} \nabla_X H^{(1)} + \nabla_X \Phi \Big\}
$$

where

$$
\Phi = \frac{{H^{(0)}}^2}{2} \left(-{\theta_e}^{(6)} + {q_t}^{(6)} \right) + \tilde{R}^{**} \left(\frac{\beta_0}{2} \eta_c^2 + \frac{\beta_1}{3} \eta_c^3 + \frac{1}{2} q_t^{(6)} \eta_c^2 \right)
$$

- Let $\varrho\mathcal{S}_{q_t} = R_e - R_p$ where R_p is the rate of production of precipitation and R_e is the rate of evaporation of precipitation.
- Take $R_p = C_o(\varrho q_l)^{\alpha_p}$ but neglect evaporative cooling of the subcloud layer and evaporation in the drizzle part of the cloud.
- $$\left\langle \mathcal{S}_{q_t}{}^{(8)} \right\rangle H^{(0)} = \mathcal{D}^{**} \left\langle q_l^{(6)} \right\rangle^{\alpha_p} H^{(0)}$$
  where $\mathcal{D}^{**}$ is a constant of order 1 representing the precipitation conversion rate.

- Assume that the long wave radiative cooling is assumed to be the primary forcing mechanisms and hence the short wave radiation neglected.
- Use the exponential formulation to compute long wave radiation
- The depth averaged source terms are given by

$$\left\langle {\mathcal{S}_{\theta_e}}^{(8)} \right\rangle H^{(0)} = \Delta F^{(1)} \left\langle q_l^{(6)} \right\rangle H^{(0)}$$

where $\Delta F^{(1)}$ is a measure of difference of the radiative flux at the cloud top and cloud bottom.

Assume $[\overline{(\varrho\theta_e)_H E}]^{(11)} = [E\Delta(\varrho\theta_e)_H]^{(11)} = \alpha(\overline{H \left\langle \varrho\mathcal{S}_{\theta_e} \right\rangle})^{(8)}$ where α is an order one parameter and can be interpreted as non dimensional entrainment rate efficiency.

$\alpha > 1 \Rightarrow$ shear driven entrainment overwhelms that due to radiative cooling

$\alpha = 1 \Rightarrow$ balance between entrainment warming and radiative cooling

1. *The temperature inversion is strong*

$$E^{(8)} = \alpha \frac{\left\langle S_{\theta_e}^{(8)} \right\rangle H^{(0)}}{\Delta(\varrho\theta_e)_H^{(3)}} = \alpha \frac{\Delta F^{(1)} \left\langle q_l^{(6)} \right\rangle H^{(0)}}{\Delta(\varrho\theta_e)_H^{(3)}} \sim 0.5\mathrm{cm/s}$$

2. *The temperature inversion is moderate*

$$E^{(7)} = \alpha \frac{\left\langle S_{\theta_e}^{(8)} \right\rangle H^{(0)}}{\Delta(\varrho\theta_e)_H^{(4)}} = \alpha \frac{\Delta F^{(1)} \left\langle q_l^{(6)} \right\rangle H^{(0)}}{\Delta(\varrho\theta_e)_H^{(4)}} \sim 1\mathrm{cm/s}$$

3. *The temperature inversion is weak*

$$E^{(6)} = \alpha \frac{\left\langle S_{\theta_e}^{(8)} \right\rangle H^{(0)}}{\Delta(\varrho\theta_e)_H^{(5)}} = \alpha \frac{\Delta F^{(1)} \left\langle q_l^{(6)} \right\rangle H^{(0)}}{\Delta(\varrho\theta_e)_H^{(5)}} \sim 3\mathrm{cm/s}$$

- Assume that the cloud base appears where the saturation mixing ratio matches the total mixing ratio in the sub-cloud layer
- The saturated water vapour mixing ratio, q_{vs} is obtained from

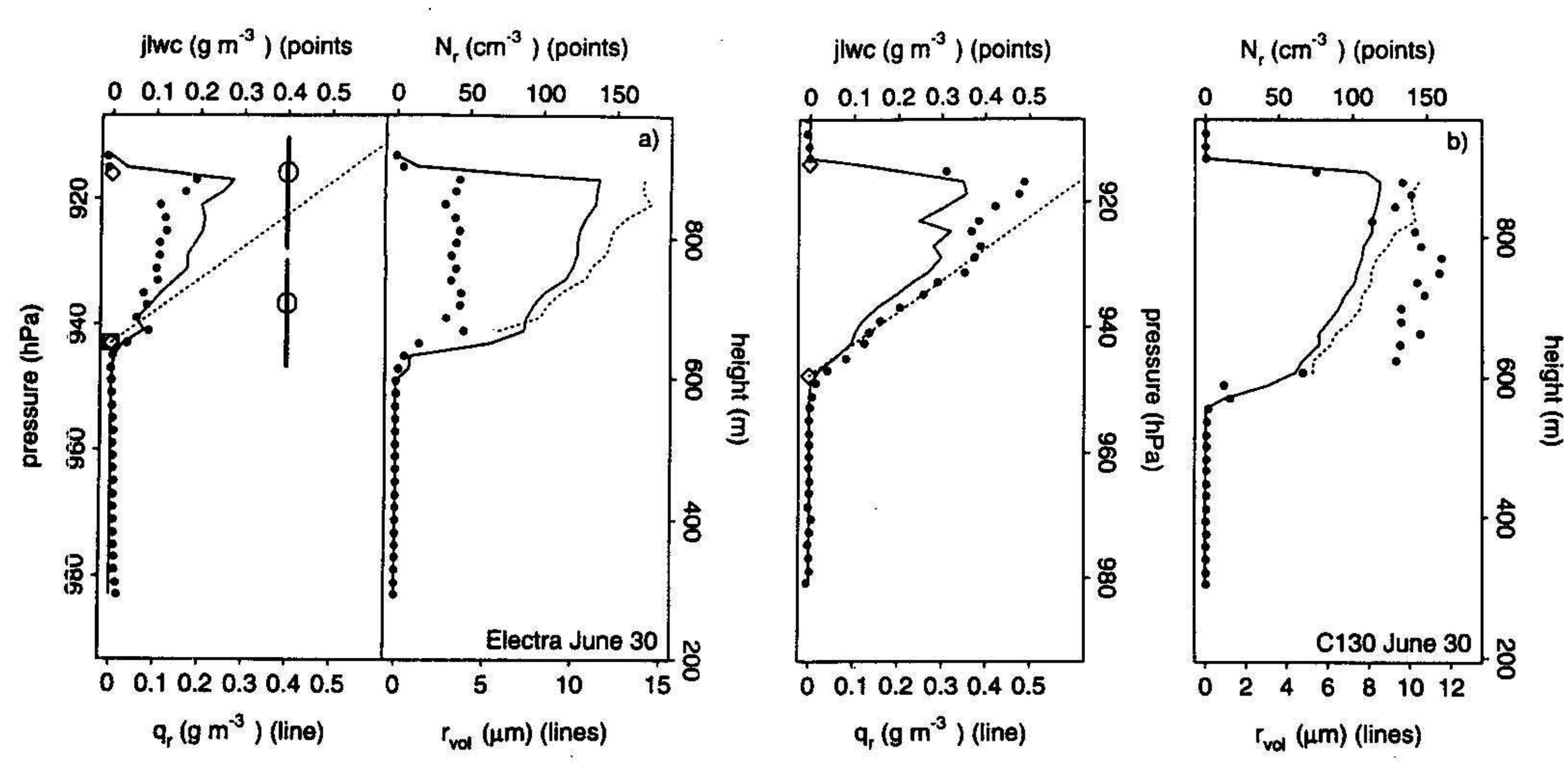
$$q_{vs} = \frac{\delta^4 e_s^{**} \exp\left(\frac{A^{**}}{\delta^3}\left[1 - \frac{\varrho}{p}\frac{(1+R^{**}q_v)}{(1+q_t)}\right]\right)}{R^{**}p - \delta^4 R^{**} e_s^{**} \exp\left(\frac{A^{**}}{\delta^3}\left[1 - \frac{\varrho}{p}\frac{(1+R^{**}q_v)}{(1+q_t)}\right]\right)}.$$

- $$q_{vs}^{(6)} = q_s^{(6)} + \beta_1 \eta \qquad \Rightarrow \qquad \eta_c = \frac{1}{\beta_1}\left(q_t^{(6)} - q_s^{(6)}\right)$$

  where

$$q_s^{(6)} = \frac{A^{**}}{2} q_{vs}^{(4)} \left[2\left(\theta_e^{(5)} - q_v^{(5)}\right) + A^{**}\left(\theta_e^{(4)} - q_v^{(4)}\right)^2\right]$$

$$\beta_1 = -A^{**}\Gamma^{**} q_{vs}^{(4)} \qquad \text{with} \qquad q_{vs}^{(4)} = \frac{e_s^{**}}{R^{**}} \exp\left(A^{**}\theta_e^{(3)}\right)$$



(Austin et al, JAS 1995)

The liquid water mixing ratio is given by

$$q_l = \begin{cases} q_t - q_{vs} & \text{if } q_t > q_{vs}, \\ 0 & \text{otherwise} \end{cases}$$

Thus

$$\left\langle q_l^{(6)} \right\rangle H^{(0)} = \frac{\beta_1}{2}(H^{(0)} - \eta_c)^2$$

This result is consistent with Albrecht et al (1990) observations of shallow stratocumulus clouds.

Momentum Fluxes

$$(\overline{\varrho \boldsymbol{v}_{||} w})_s = -C_D \varrho_s \left| \boldsymbol{v}_{||} \right| \boldsymbol{v}_{||}$$

Assume $C_D = 10^{-3} \sim \delta^5 C_D^{**}$,

$$(\overline{\varrho u w})_s^{(5)} = -C_D^{**} \left| \boldsymbol{v}_{||} \right|^{(0)} \boldsymbol{v}_{||}^{(0)}$$

and

$$(\overline{\varrho u w})_s^{(6)} = -C_D^{**} \left(\left| \boldsymbol{v}_{||} \right|^{(0)} \boldsymbol{v}_{||}^{(1)} + \left| \boldsymbol{v}_{||} \right|^{(1)} \boldsymbol{v}_{||}^{(0)} \right)$$

where $\left| \boldsymbol{v}_{||} \right|^{(0)} = \sqrt{{u^{(0)}}^2 + {v^{(0)}}^2}$ and $\left| \boldsymbol{v}_{||} \right|^{(1)} = \frac{u^{(0)} v^{(1)} + u^{(1)} v^{(0)}}{\sqrt{{u^{(0)}}^2 + {v^{(0)}}^2}}$.

Equivalent Potential Temperature Flux

$$(\overline{\varrho w \theta_e})_s = -C_{\theta_e} \varrho_s |\boldsymbol{v}_{||}| (\theta_e - \tilde{\theta}_e)$$

$$(\overline{\varrho w \theta_e})_s^{(11)} = -C_{\theta_e}^{**} \left| \boldsymbol{v}_{||} \right|^{(0)} \left(\theta_e^{(6)} - \tilde{\theta}_e^{(6)} \right)$$

Total Moisture Flux

$$(\overline{\varrho w q_t})_s = -C_{q_t}\varrho_s|\boldsymbol{v}_{||}|(q_t - \tilde{q}_s)$$

$$(\overline{\varrho w q_t})_s^{(10)} = -C_{q_t}^{**}\,|\boldsymbol{v}_{||}|^{(0)}\left(q_t^{(5)} - \tilde{q}_s^{(5)}\right)$$

$$-C_{q_t}^{**}\,|\boldsymbol{v}_{||}|^{(0)}\left(q_t^{(5)} - \tilde{q}_s^{(5)}\right) + [\overline{(\varrho q_t)_H E}]^{(10)} + (\overline{H\left\langle \varrho \mathcal{S}_{q_t}\right\rangle})^{(7)} = 0$$

$$(\overline{\varrho w q_t})_s^{(11)} = -C_{q_t}^{**}\left[|\boldsymbol{v}_{||}|^{(0)}\left(q_t^{(6)} - \tilde{q}_s^{(6)}\right) + |\boldsymbol{v}_{||}|^{(1)}\left(q_t^{(5)} - \tilde{q}_s^{(5)}\right)\right]$$

$$\frac{\partial \theta_e^{(6)}}{\partial T} + \boldsymbol{v}_{\shortparallel}^{(0)} \nabla_X \theta_e^{(6)} - \frac{\beta_1}{2H^{(0)}}(1-\alpha)\Delta F^{(1)} \left(H^{(0)} - \eta_c \right)^2 + \frac{C_{\theta_e}^{**}}{H^{(0)}} |\boldsymbol{v}_{\shortparallel}|^{(0)} \left(\theta_e^{(6)} - \tilde{\theta}_e^{(6)} \right) = 0$$

$\Delta\theta_e$	Δq_t
9.2	0.74
8.4	−0.06
3.8	0.17
1.7	−1.93

(°C); net radiation,

$\Delta\theta_e$	Δq_t
−3.1	−2.43
0.5	−2.93
−4.1	−3.72
−1.8	−4.28
−1.8	−3.06
−1.1	−3.47

(Price et al, QJRMS 1999)

$$\nabla_X H^{(1)} = 0 \qquad \Rightarrow \qquad \nabla_X \cdot \boldsymbol{v}_{||}^{(1)} = 0.$$

$$\theta_e^{(3)} H^{(0)} \nabla_X H^{(2)} = C_D^{**} \left|\boldsymbol{v}_{||}\right|^{(0)} \boldsymbol{v}_{||}^{(0)}$$

$$\frac{\partial \boldsymbol{v}_{||}^{(1)}}{\partial T} + \boldsymbol{v}_{||}^{(0)} . \nabla_X \boldsymbol{v}_{||}^{(1)} + (\widehat{\boldsymbol{\Omega}} \times \boldsymbol{v}^{(1)})_{||} + \frac{1}{H^{(0)}} \nabla_X \Phi - \theta_e^{(3)} \nabla_X H^{(3)}$$
$$+ \frac{C_D^{**}}{H^{(0)}} \left(\left|\boldsymbol{v}_{||}\right|^{(0)} \boldsymbol{v}_{||}^{(1)} + \left|\boldsymbol{v}_{||}\right|^{(1)} \boldsymbol{v}_{||}^{(0)} \right) + \frac{q_t^{(4)}}{\theta_e^{(3)}} \frac{C_D^{**}}{H^{(0)}} \tilde{R}^{**} \left|\boldsymbol{v}_{||}\right|^{(0)} \boldsymbol{v}_{||}^{(0)} = 0$$

$$\frac{\partial q_t^{(6)}}{\partial T} + \boldsymbol{v}_{||}^{(0)} \nabla_X q_t^{(6)} + \frac{\beta_1}{2H^{(0)}} \mathcal{D}^{**} \left(H^{(0)} - \eta_c \right)^{2\alpha_p}$$
$$+ \frac{C_{q_t}^{**}}{H^{(0)}} \left|\boldsymbol{v}_{||}\right|^{(0)} \left(q_t^{(6)} - \tilde{q}_s^{(6)} \right) = 0$$

$$\frac{\partial q_t^{(6)}}{\partial T} + \boldsymbol{v}_{\shortparallel}^{(0)} \nabla_X q_t^{(6)} + \frac{\Delta(\varrho q_t)_H^{(4)}}{\Delta(\varrho \theta_e)_H^{(4)}} \frac{\beta_1}{2H^{(0)}} \alpha \Delta F^{(1)} (H^{(0)} - \eta_c)^2 +$$

$$\frac{\beta_1}{2H^{(0)}} \mathcal{D}^{**} \left(H^{(0)} - \eta_c\right)^{2\alpha_p} + \frac{C_{q_t}^{**}}{H^{(0)}} \left|\boldsymbol{v}_{\shortparallel}\right|^{(0)} \left(q_t^{(6)} - \tilde{q}_s^{(6)}\right) = 0$$

$$(-\theta_e^{(4)} + \tilde{R}^{**} q_t^{(4)}) H^{(0)} \nabla_X H^{(1)} = C_D^{**} \left| \boldsymbol{v}_{||} \right|^{(0)} \boldsymbol{v}_{||}^{(0)}$$

$$H^{(0)} \nabla_X \cdot \boldsymbol{v}_{||}^{(1)} + \boldsymbol{v}_{||}^{(0)} \nabla_X H^{(1)} = 0.$$

$$\frac{\partial \boldsymbol{v}_{||}^{(1)}}{\partial T} + \boldsymbol{v}_{||}^{(0)} . \nabla_X \boldsymbol{v}_{||}^{(1)} + (\widehat{\boldsymbol{\Omega}} \times \boldsymbol{v}^{(1)})_{||} - (-\theta_e^{(4)} + \tilde{R}^{**} q_t^{(4)}) \nabla_X H^{(2)}$$
$$+ \frac{1}{H^{(0)}} \nabla_X \Phi + \frac{C_D^{**}}{H^{(0)}} \left(\left| \boldsymbol{v}_{||} \right|^{(0)} \boldsymbol{v}_{||}^{(1)} + \left| \boldsymbol{v}_{||} \right|^{(1)} \boldsymbol{v}_{||}^{(0)} \right)$$
$$+ \frac{H^{(1)}}{H^{(0)}} (-\theta_e^{(4)} + \tilde{R}^{**} q_t^{(4)}) \nabla_X H^{(1)} = 0.$$

$$C_D^{**} \left|\boldsymbol{v}_{_{||}}\right|^{(0)} \boldsymbol{v}_{_{||}}^{(0)} = 0 \qquad \Rightarrow \qquad \boldsymbol{v}_{_{||}}^{(0)} = 0$$

$$\frac{\partial H^{(1)}}{\partial T} + H^{(0)} \nabla_X \cdot \boldsymbol{v}_{_{||}}^{(1)} + \frac{\beta_1}{2} \alpha \Delta F^{(1)} \left(H^{(0)} - \eta_c \right)^2 = 0$$

$$\frac{\partial \boldsymbol{v}_{_{||}}^{(1)}}{\partial T} + (\widehat{\boldsymbol{\Omega}} \times \boldsymbol{v}^{(1)})_{_{||}} + \varphi \nabla_X H^{(1)} + \frac{1}{H^{(0)}} \nabla_X \Phi = 0$$

$$\frac{\partial \theta_e^{(6)}}{\partial T} - \frac{\beta_1}{2H^{(0)}} (1-\alpha) \Delta F^{(1)} \left(H^{(0)} - \eta_c \right)^2 = 0$$

$$\frac{\partial q_t^{(6)}}{\partial T} + \frac{\Delta(\varrho q_t)_H^{(5)}}{\Delta(\varrho \theta_e)_H^{(5)}} \frac{\beta_1}{2H^{(0)}} \alpha \Delta F^{(1)} (H^{(0)} - \eta_c)^2 +$$

$$\frac{\beta_1}{2H^{(0)}} \mathcal{D}^{**} \left(H^{(0)} - \eta_c \right)^{2\alpha_p} = 0$$

Concluding Remarks