Mass flux fluctuation in a cloud resolving simulation with diurnal forcing

Jahanshah Davoudi

Norman McFarlane, Thomas Birner

Physics department, University of Toronto

References

- Brenda G. Cohen and George C. Craig, 2004: Quart. J. Roy. Meteor. Soc. **130**, 933-944.
- Brenda G. Cohen and George C. Craig, 2006: J. Atmos. Sci. **63** , 1996-2004.
- Brenda G. Cohen and George C. Craig, 2006: J. Atmos. Sci. **63** , 2005-2015.
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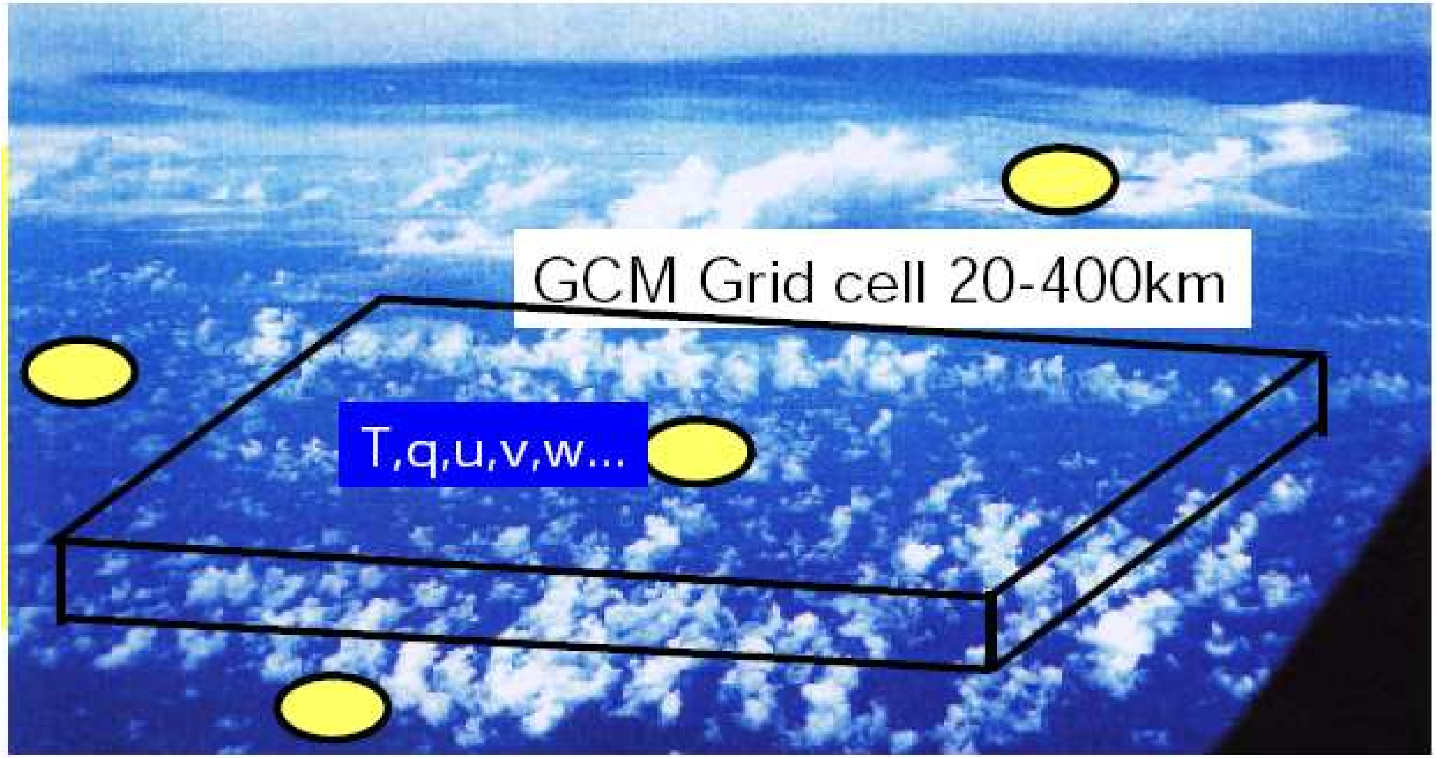
Convective parameterization



Many clouds and especially the processes within them are subgrid-scale size both horizontally and vertically and thus must be parameterized.

This means a mathematical model is constructed that attempts to assess their effects in terms of large scale model resolved quantities.

Parameterization Basics

Arakawa & Schubert 1974

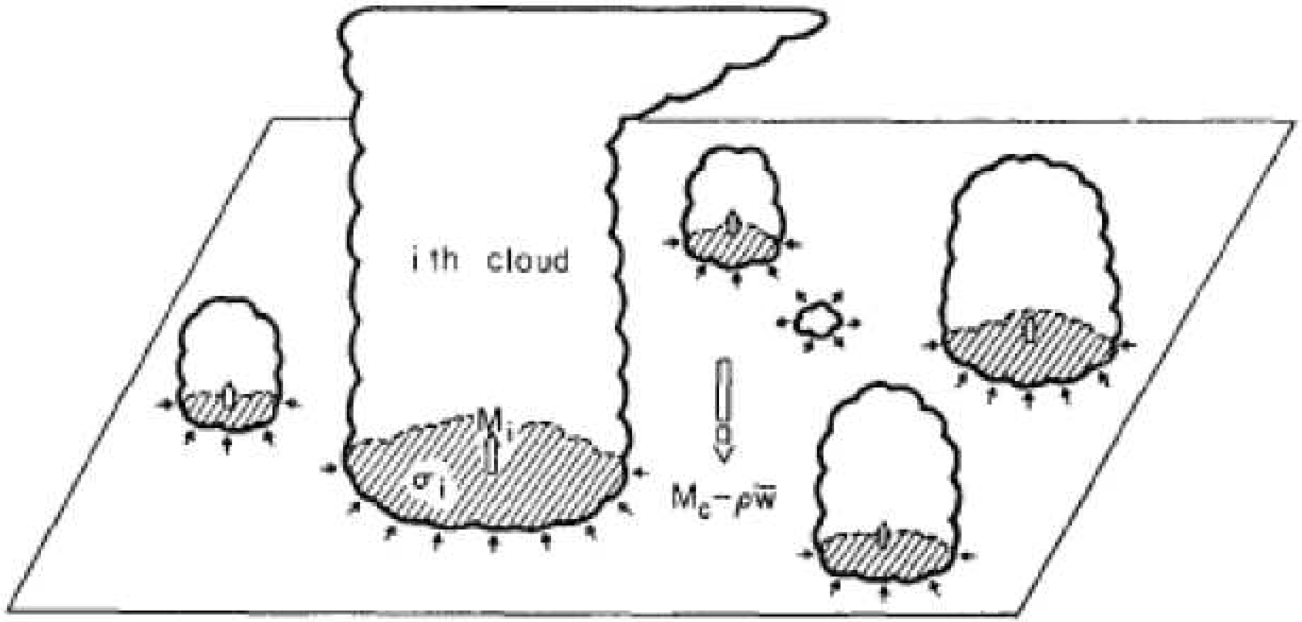
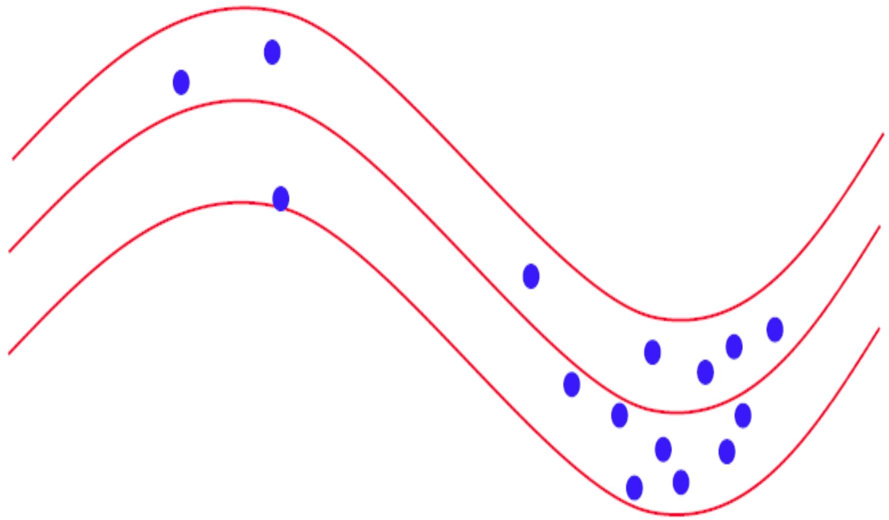


FIG. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

Key Quasi-equilibrium assumption:

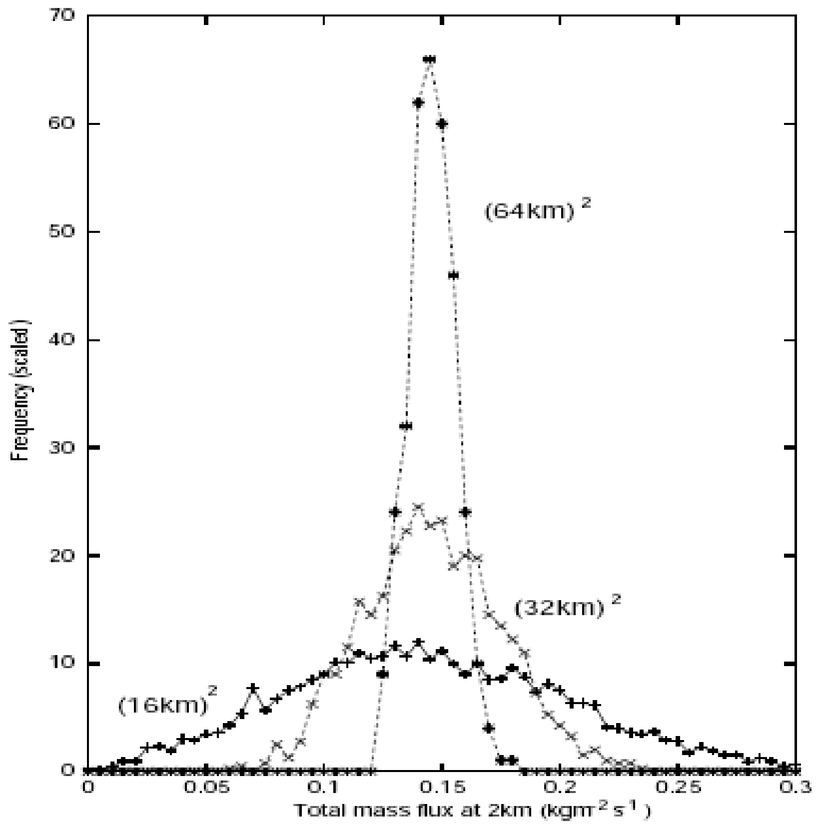
$$\tau_{adj} \ll \tau_{ls}$$

Quasi-equilibrium



- Convective equilibrium requires scale separation
  - Large scale uniform over region containing many clouds
  - Large scale slowly varying so convection has time to respond
- Convective activity within a small grid cell is highly variable(even in statistically stationary state)

Fluctuations in radiative-convective equilibrium



- For convection in equilibrium with a given forcing, the mean mass flux should be well defined.
- At a particular time, this mean value would only be measured in an infinite domain.
- For a region of finite size:
  - What is the magnitude and distribution of variability?
  - What scale must one average over to reduce it to a desired level?

Main assumptions

Assume:

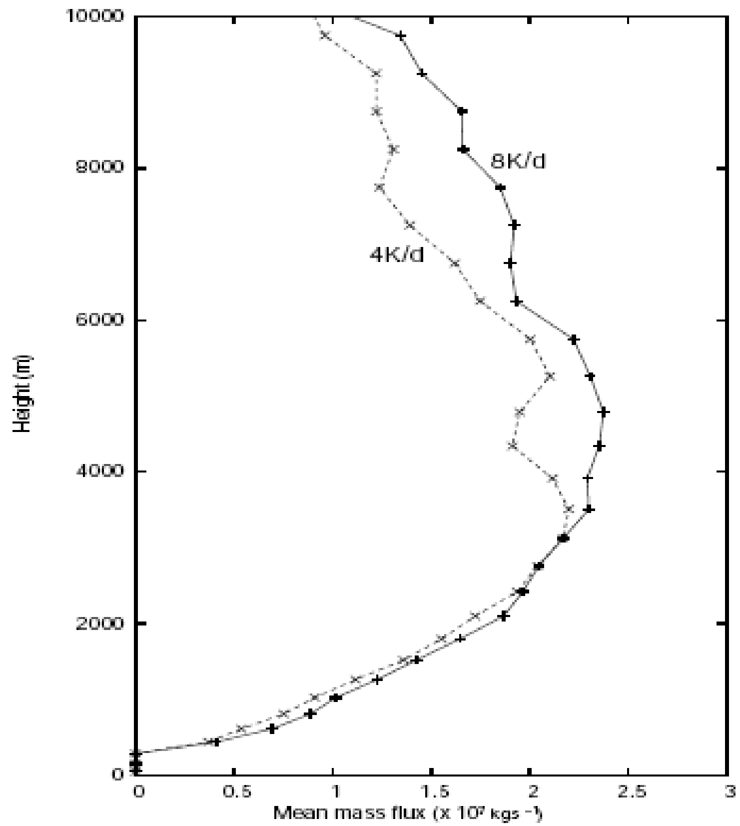
1. Large-scale constraints- mean mass flux within a region $\langle M \rangle$ is given in terms of large scale resolved conditions

2. Scale separation- environment sufficiently uniform in time and space to average over a large number of clouds

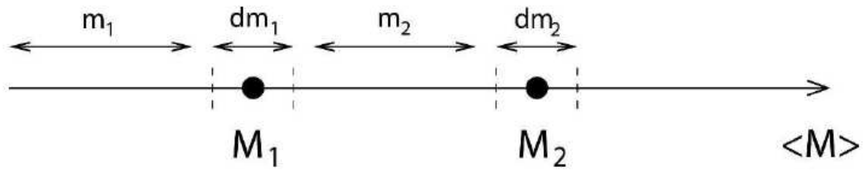
3. Weak interactions- clouds feel only mean effects of total cloud field(no organization)

Find the distribution function subject to these constraints

Other constraints



- $\langle m \rangle$ is not necessarily a function of large scale forcing
- Observations suggest that $\langle m \rangle$ is independent of large scale forcing
- Response to the change in forcing is to change the number of clouds.
- $\langle m \rangle$ might be only sensitive to the initial perturbation triggering it and the dynamical entrainment processes.



- mass flux of individual clouds are statistically un-correlated :

$$P_M(n) = Prob\{N\,[(0,M]) = n\} = \frac{(\lambda M)^n e^{-\lambda M}}{n!} \quad n = 0, 1, \cdots$$

given $\lambda = 1/(\langle m \rangle) = \frac{\langle N \rangle}{\langle M \rangle}$ is fixed.

- Poisson point process implies:

$$P(m) = \frac{1}{\langle m \rangle} e^{-\frac{m}{\langle m \rangle}}$$

- The total Mass flux for a given N Poisson distributed plumes is a Compound point process:

$$M = \sum_{i=0}^{N} m_i$$

Predicted distribution

So the Generating function of M is calculated exactly:

$$G(t) = \langle e^{tM} \rangle_{m,N} = \langle e^{t \sum_i^N m_i} \rangle_{m,N}$$
$$\langle e^{tM} \rangle_{m,N} = \langle g^N(t) \rangle_N$$
$$= e^{-\Lambda} e^{\Lambda g(t)}$$

where

$$g(t) = \langle e^{tm} \rangle_m \qquad \Lambda = \langle N \rangle$$

Therefore the probability distribution of the total mass flux is exactly given by:

$$P(M) = P(M) = \left(\frac{\langle N \rangle}{\langle m \rangle} \right)^{1/2} e^{-\langle N \rangle} M^{-1/2} e^{-M/\langle m \rangle} I_1 \left(2 \left(\frac{\langle N \rangle}{\langle m \rangle} M \right)^{1/2} \right)$$

All the moments of M are analytically tractable and are functions of $\langle N \rangle$ and $\langle m \rangle$.

$$\frac{\langle (\delta M)^2 \rangle}{\langle M \rangle^2} = \frac{2}{\langle N \rangle}$$
$$\frac{\langle (\delta M)^3 \rangle}{\langle M \rangle^3} = \frac{6}{\langle N \rangle^2}$$

Estimates

- In a region with area A and grid size $\Delta x \gg L$ where L the mean cloud spacing is:

$$L = (A/\langle N \rangle)^{1/2} = (\langle m \rangle A/\langle M \rangle)^{1/2}$$

- Assume latent heat release balance radiative cooling S,
rate of Latent heating $\simeq$ Convective mass flux $\times$ Typical water vapor mass mixing ratio q

$$l_v q \frac{\langle M \rangle}{A} = S$$

- Estimate:
$S = 250Wm^{-2}$, $q = 10gkg^{-1}$ and $l_v = 2.5 \times 10^6 Jkg^{-1}$ gives

$$\langle M \rangle / A = 10^{-2} kgs^{-1} m^{-2}$$

- $\langle m \rangle = w\rho\sigma$ with $w \simeq 10ms^{-1}$ and $\sigma \simeq 1km^2$ gives

$$\langle m \rangle \simeq 10^7 kgs^{-1}$$

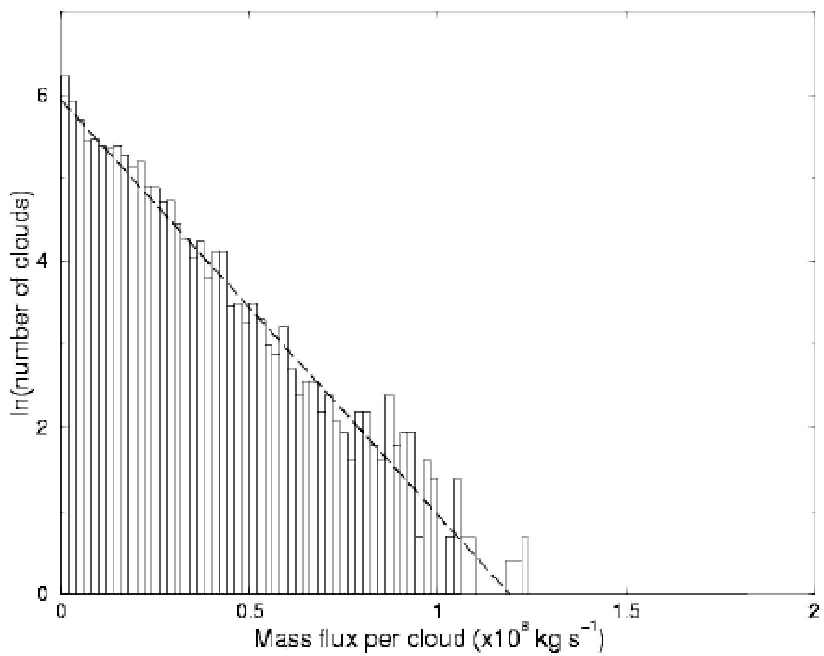
hence

$$L \simeq 30km$$

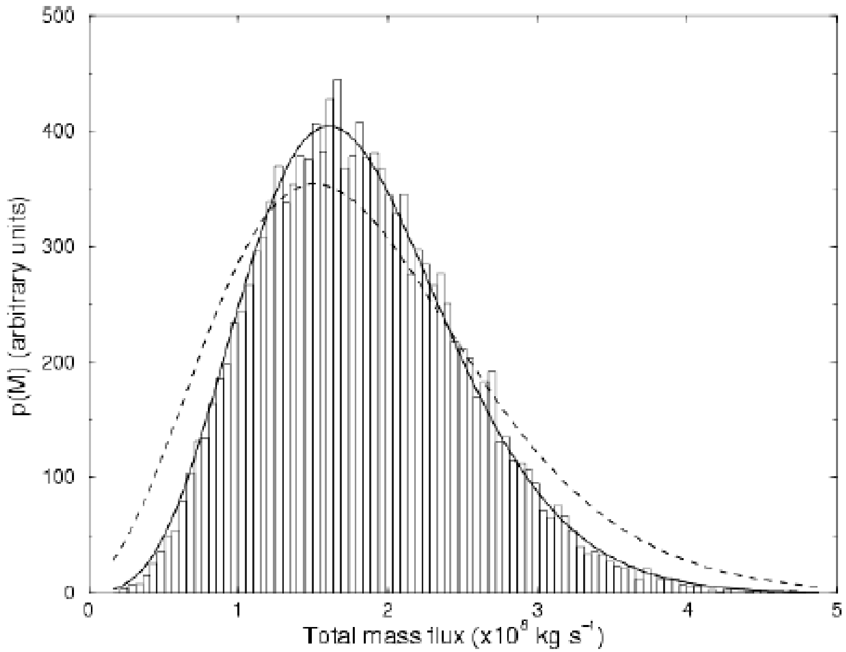
- $\frac{\delta M}{M} = \sqrt{2}\frac{L}{\Delta x}$

CRM distributions of cloud mass flux

Mass flux per cloud



Total mass flux distribution



Craig and Cohen JAS (2006)

Resolution: 2km$\times$ 2km$\times$ 50 levels

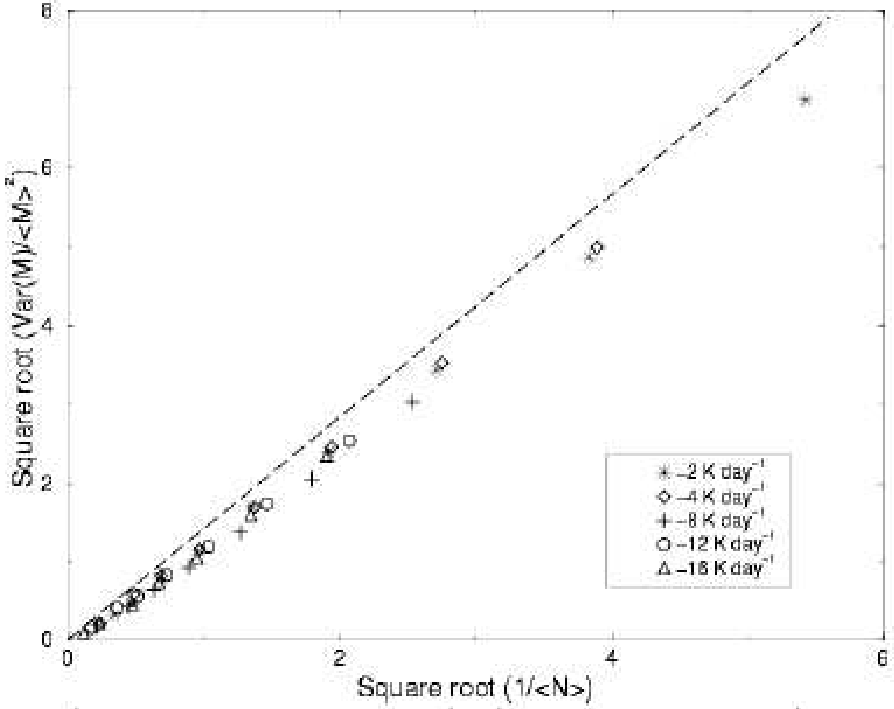
Domain: 128 km$\times$128 km$\times$ 21 km

Boundary conditions: doubly periodic, fixed SST of 300 K

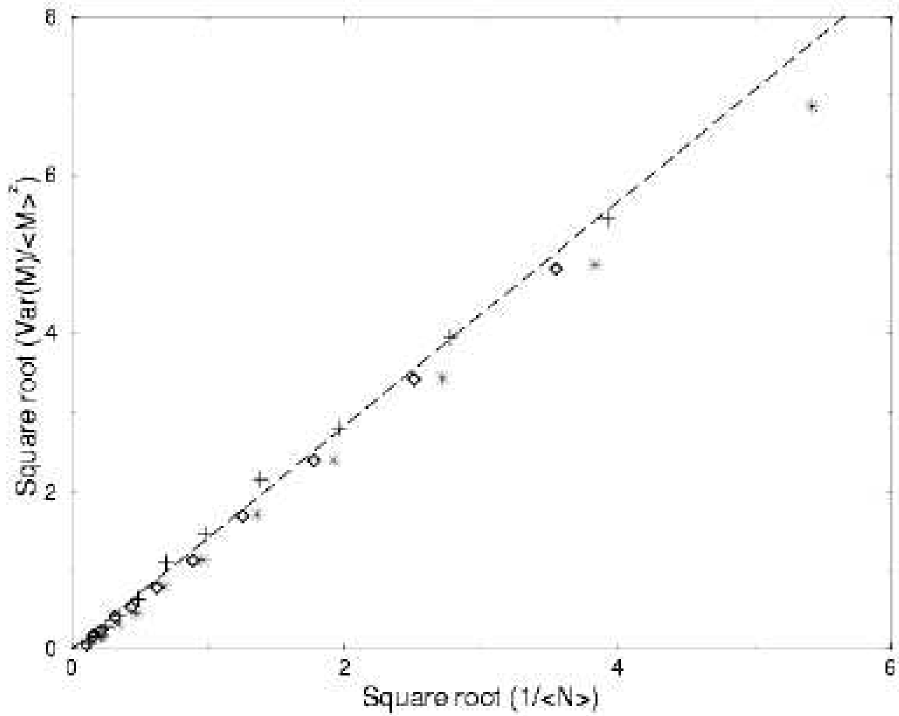
Forcings: fixed tropospheric cooling of 2,4,8,12,16 K day^{-1}

CRM mass flux variance

Without organization



Shear (organization)



Left panel: normalized standard deviation of are-integrated convective mass flux versus characteristic cloud spacing.

Right panel: Various degrees of convection organization: un-sheared(*), weak shear (□), strong shear(+).

Simulations with a 'cloud resolving' model

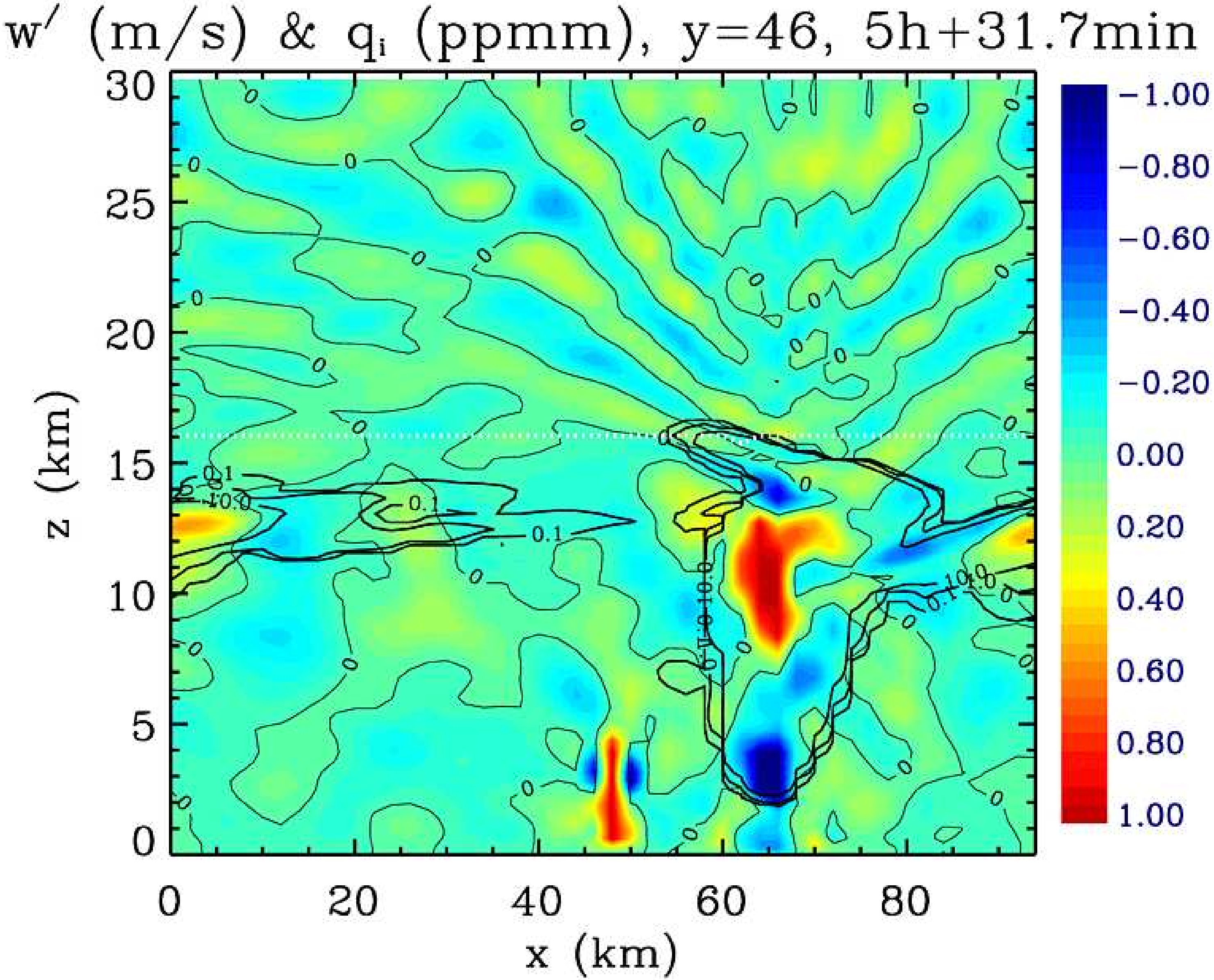
Resolution: $2\text{km} \times 2\text{km} \times 90$ levels

Domain: $96\text{ km} \times 96\text{ km} \times 30\text{ km}$

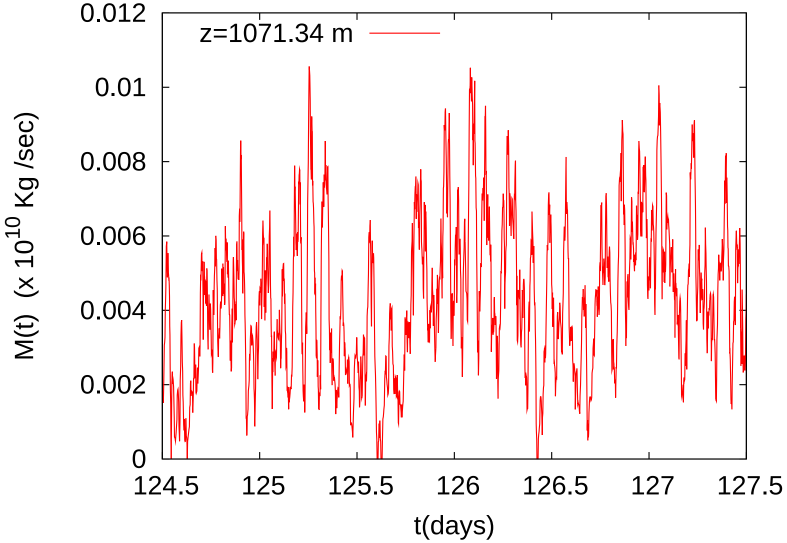
Boundary conditions: doubly periodic, fixed SST of 300 K

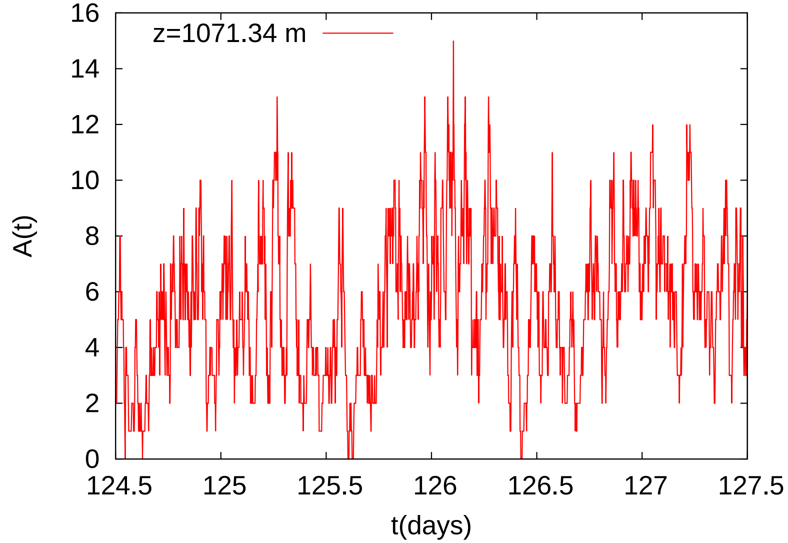
Forcing: An-elastic equations with fully interactive radiation scheme

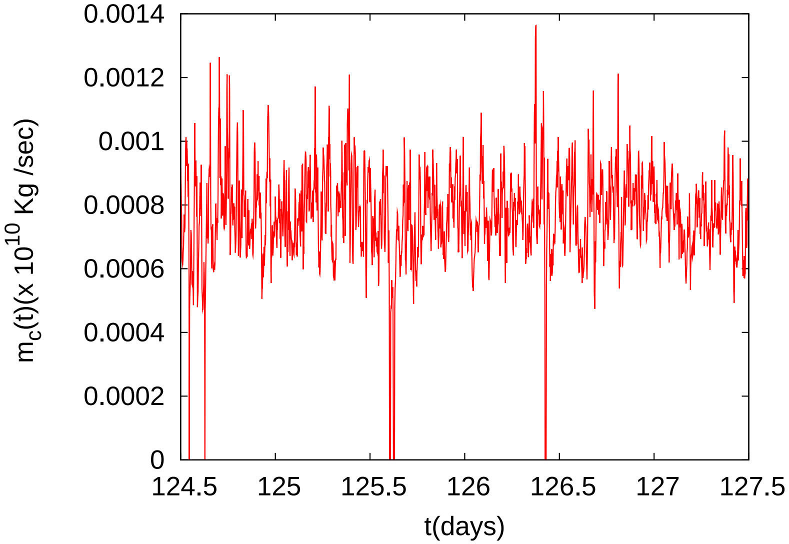
A 2D cut through the convective field



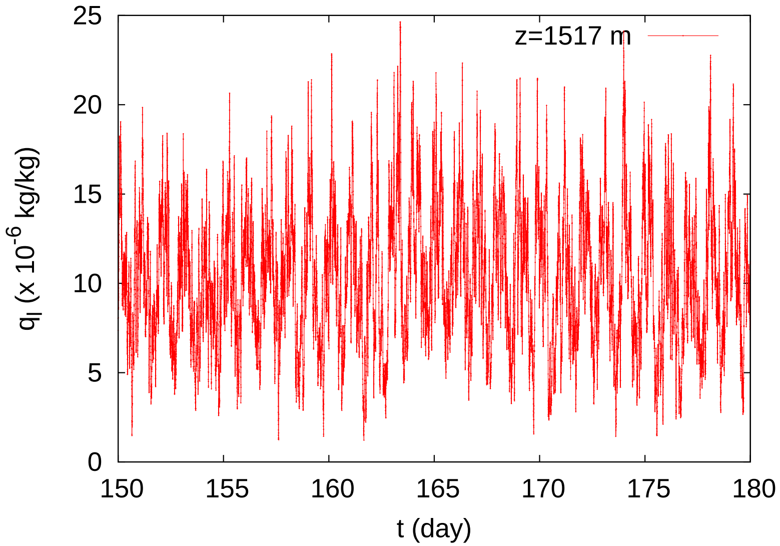
In cloud properties up-drafts M(t) and A(t)

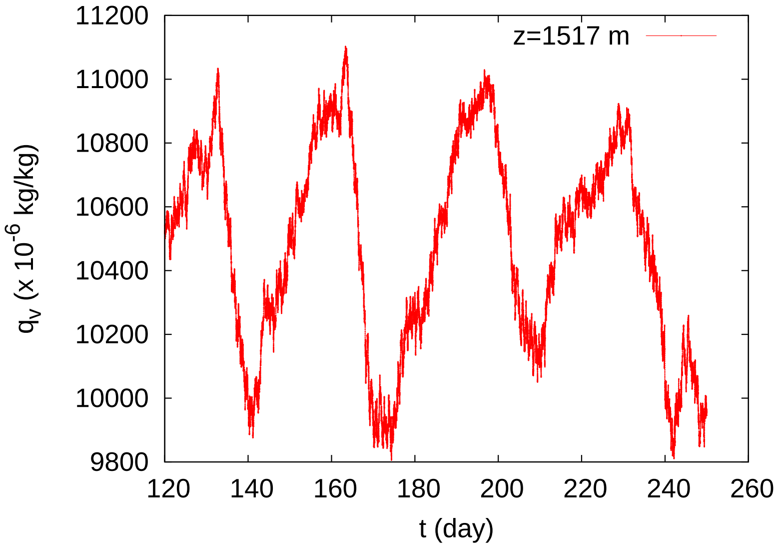


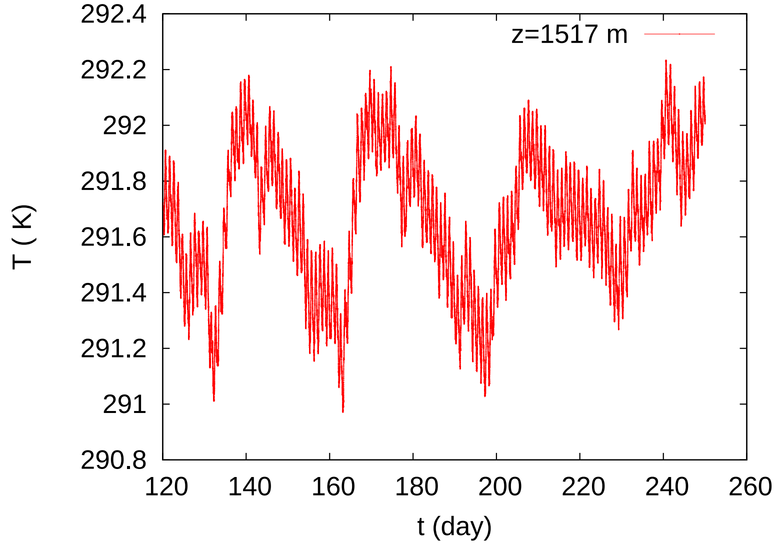




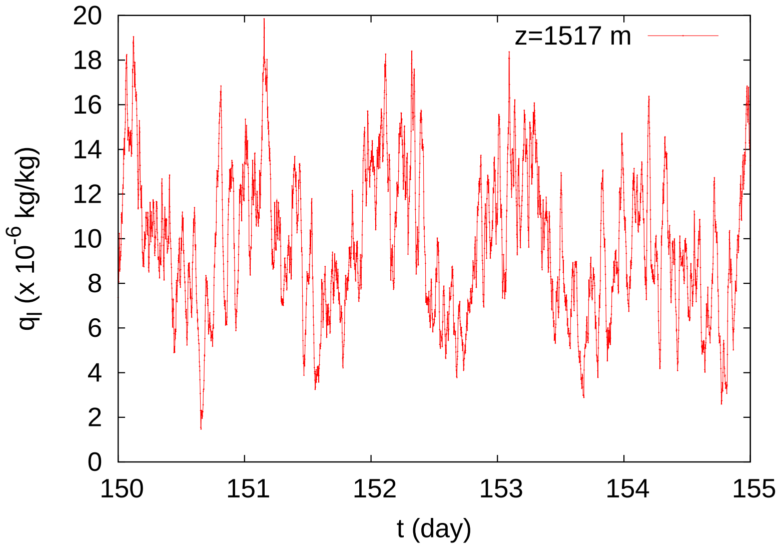
Long time portraits of $q_l(t)$, $q_v(t)$ and T(t)

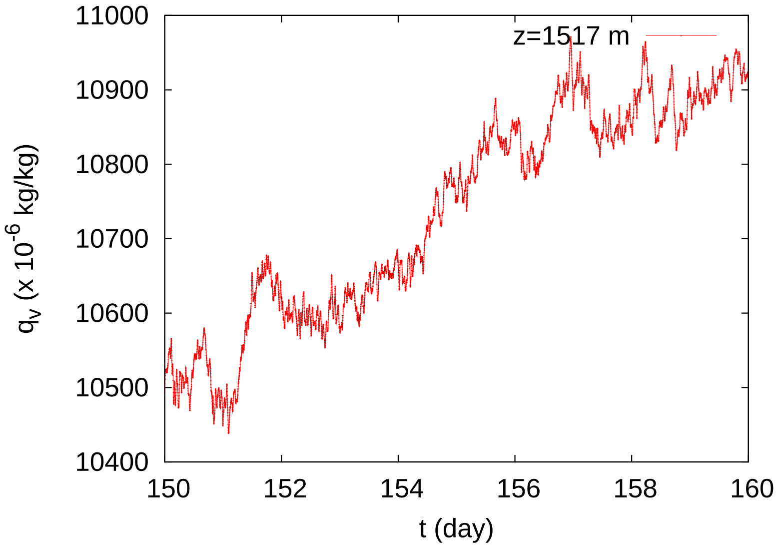


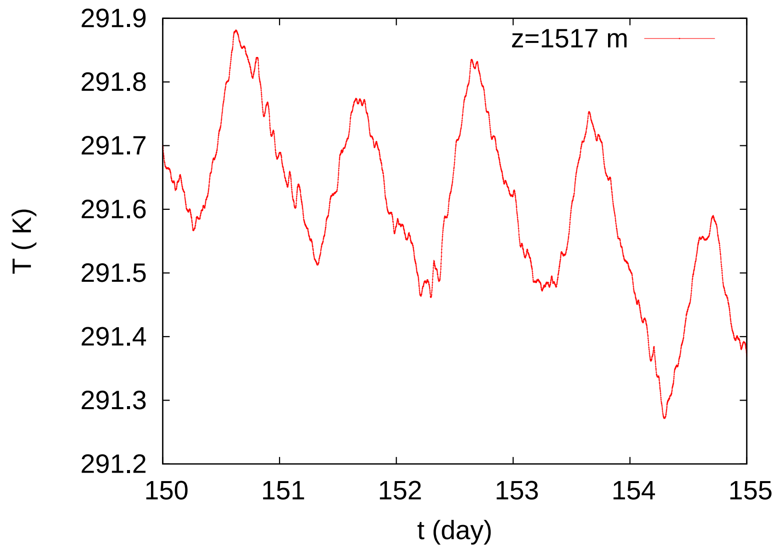




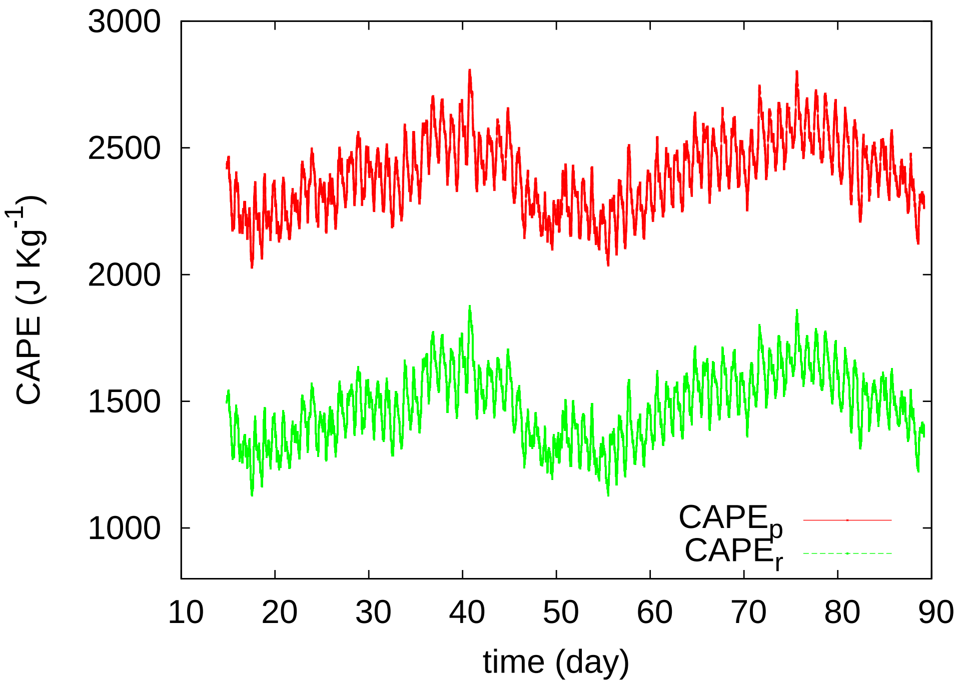
short time portraits of $q_l(t)$, $q_v(t)$ and T(t)

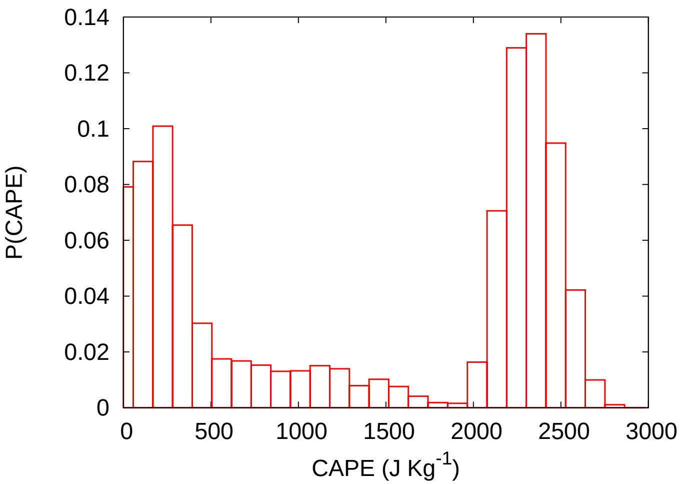


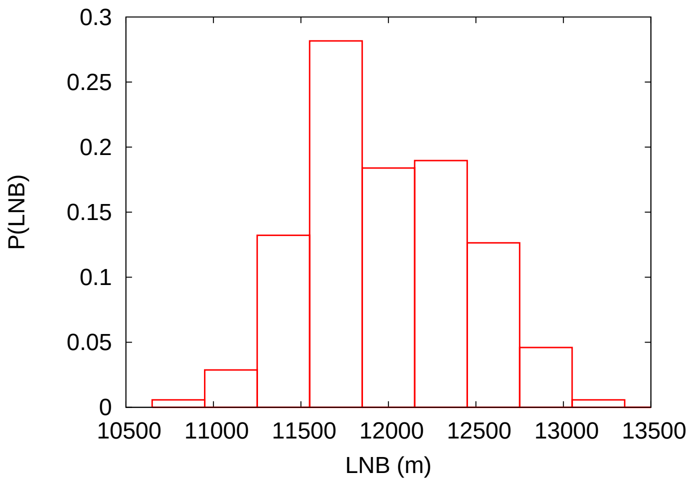




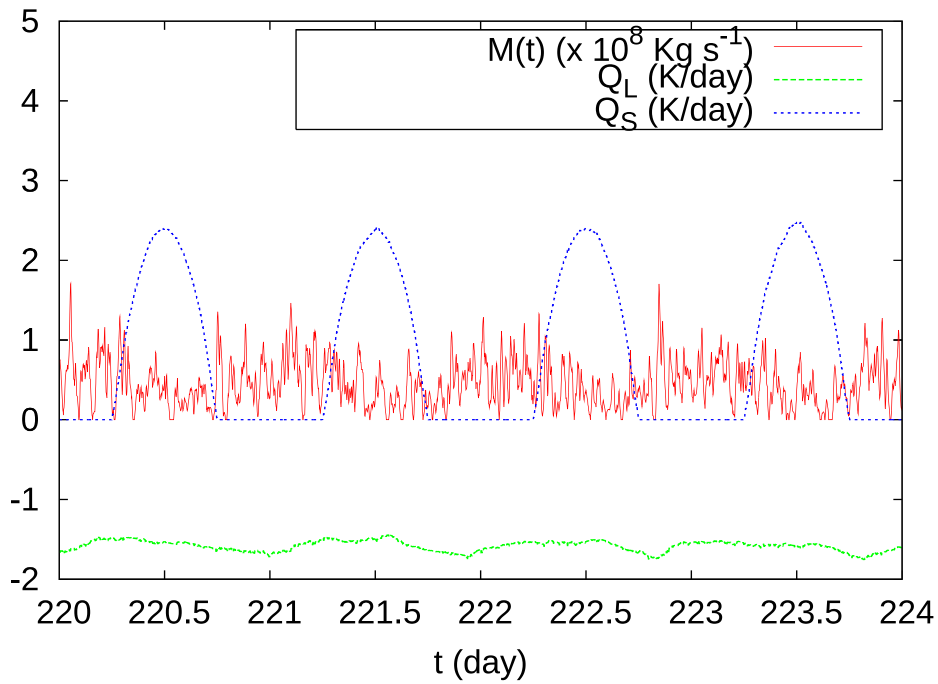
Distribution of CAPE and LNB

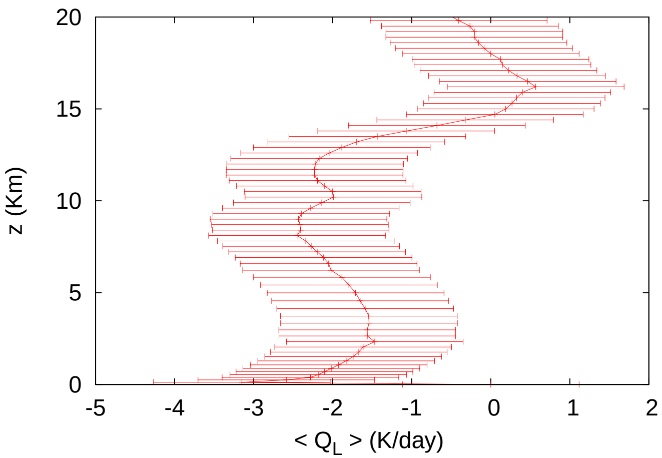


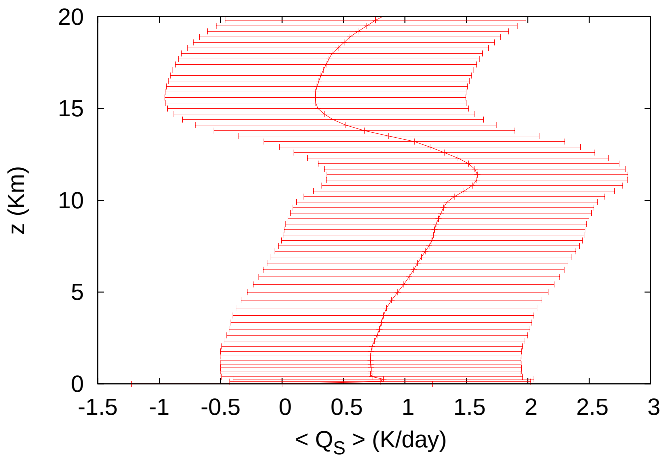




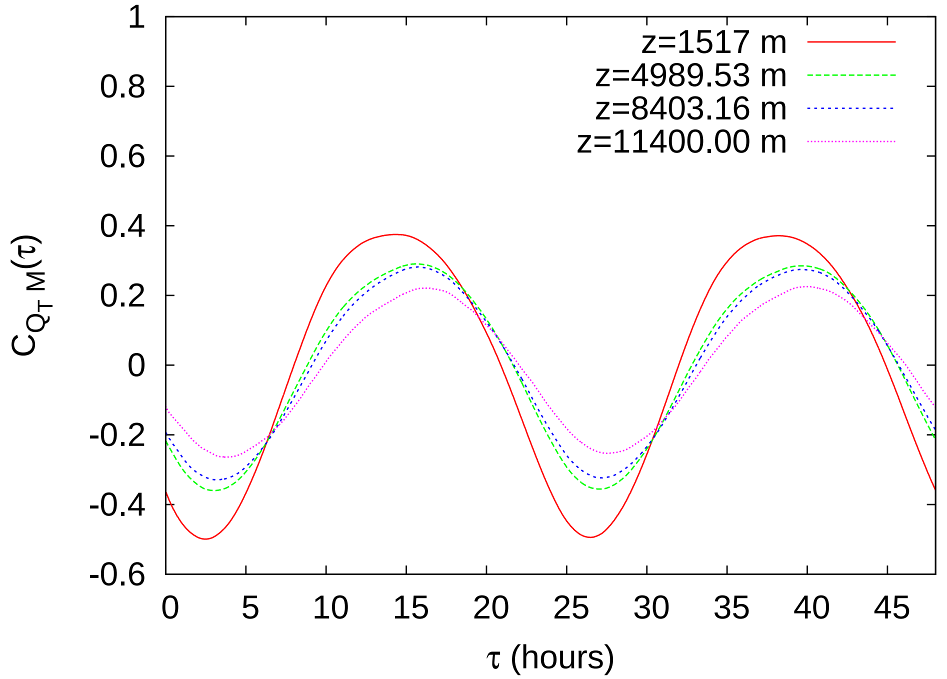
Short and long wave heating rates







Adjustment time

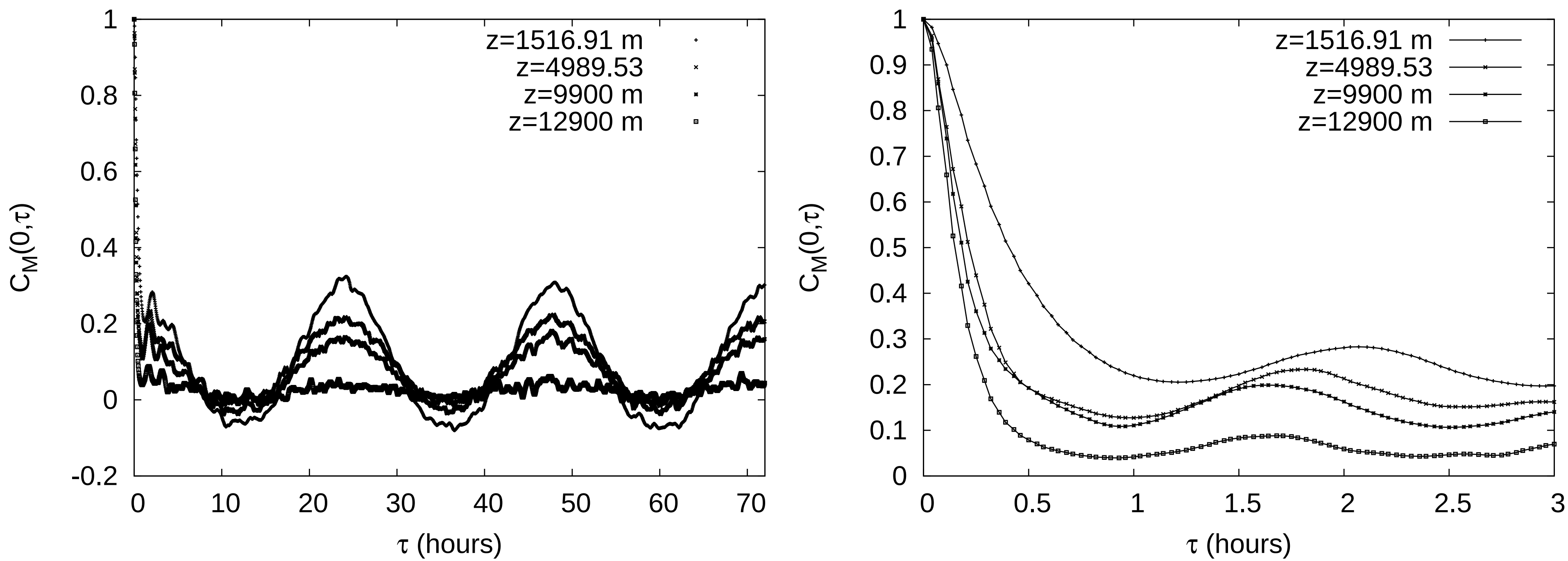


$$C_{Q_T M}(\tau) = \frac{\langle \tilde{Q}_T(t)\tilde{M}(t+\tau)\rangle}{\sigma_{Q_T}\sigma_M}$$

The adjustment time of the total heating rate $Q_T = Q_S + Q_L$ and the mass flux at various altitudes.

The τ_{adj} varies in the range of $\simeq 2 - 4$ hours .

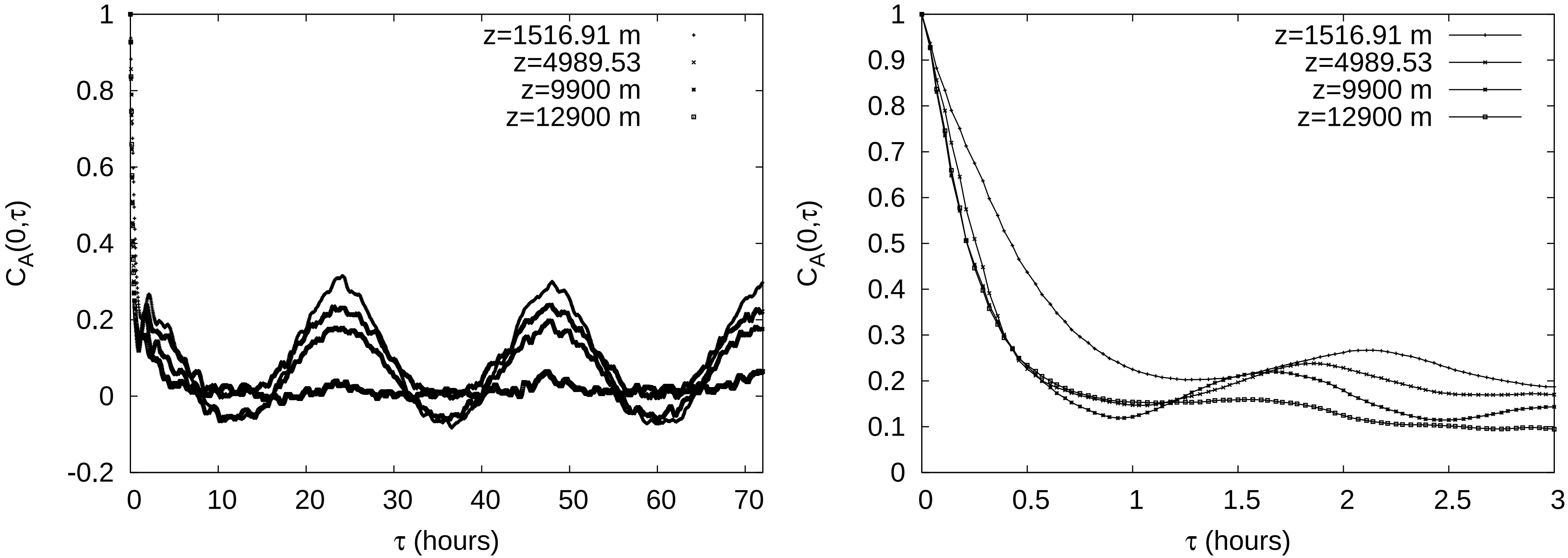
Auto-correlation of up-drafts M



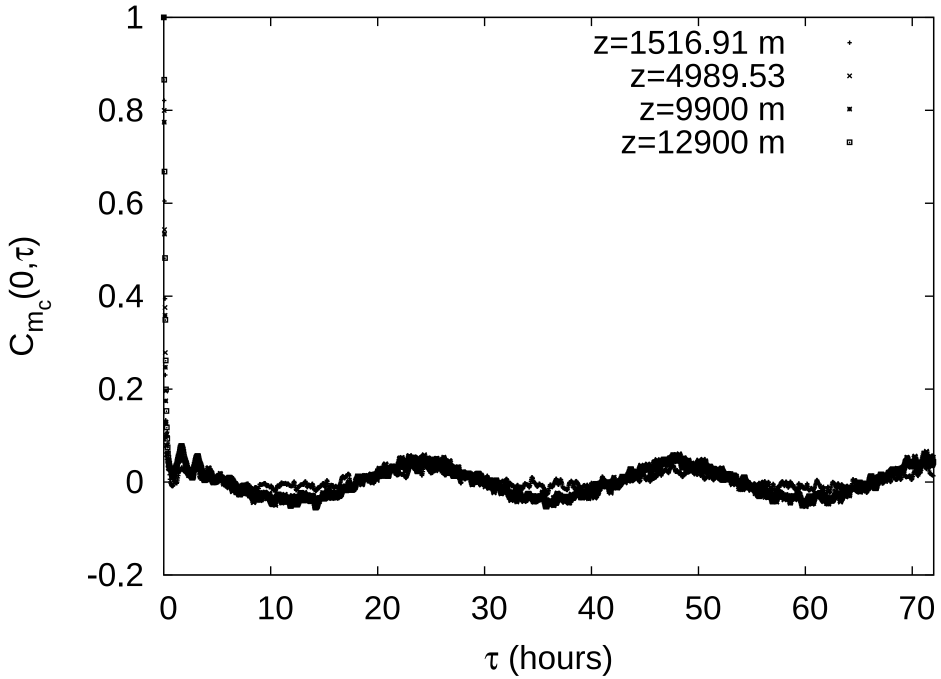
$$C_M(\delta, \tau) = \frac{\langle \tilde{M}(z,t)\tilde{M}(z+\delta, t+\tau)\rangle}{\sigma_M(z)\sigma_M(z+\delta)}$$

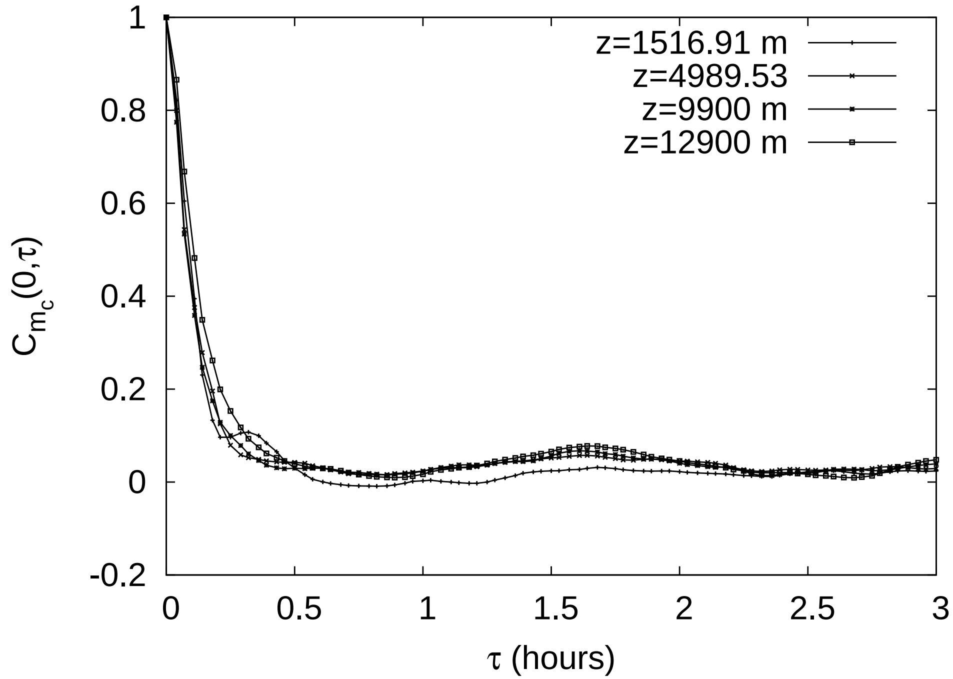
where $\tilde{M} \equiv M - \langle M \rangle$ and $\langle \cdots \rangle$ is a time average.

Auto-correlation of up-drafts A

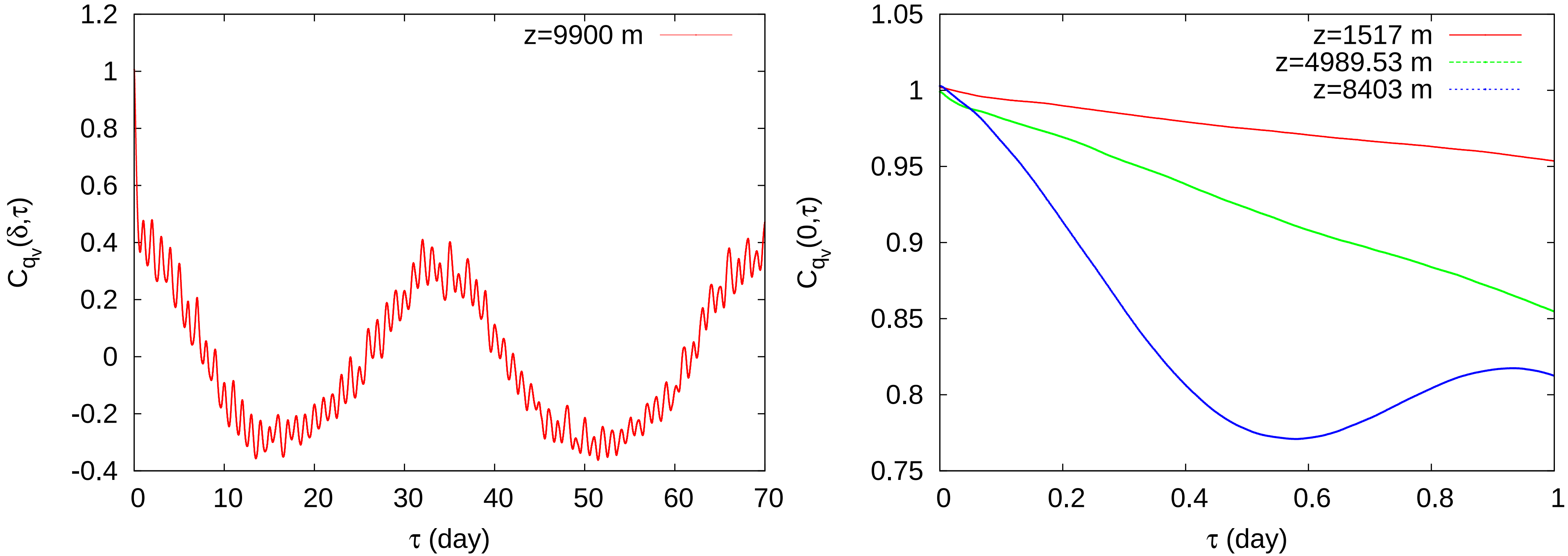


Auto-correlation of up-drafts m_c

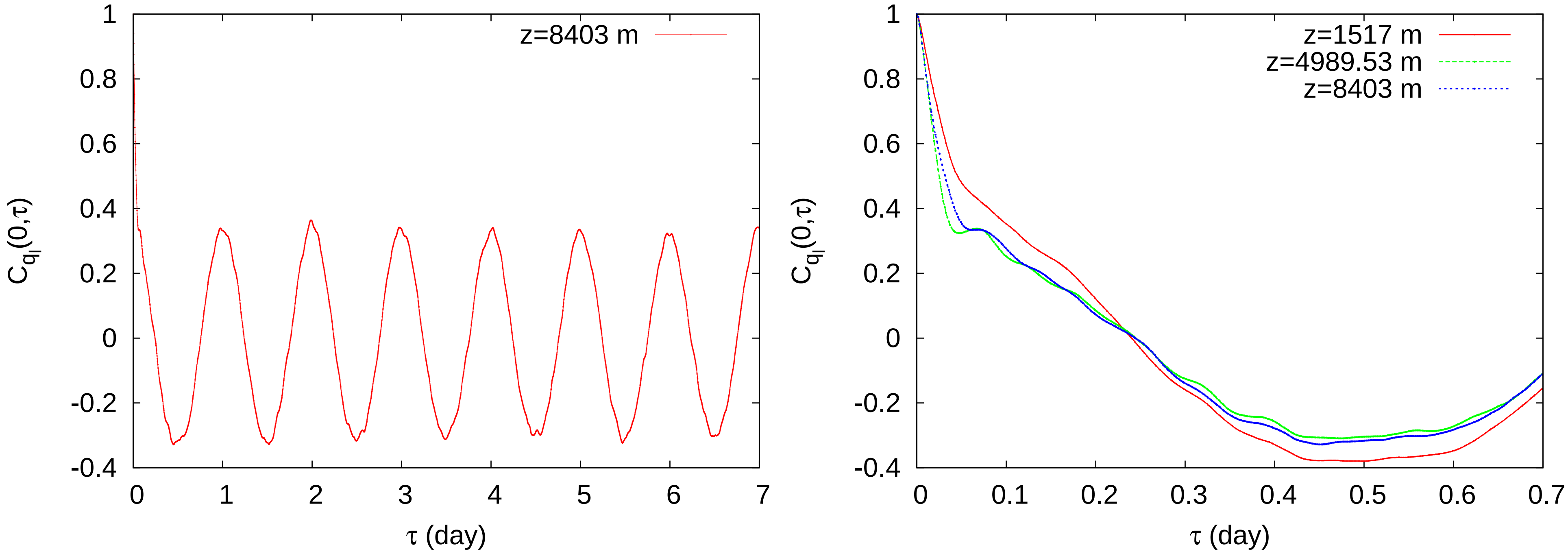




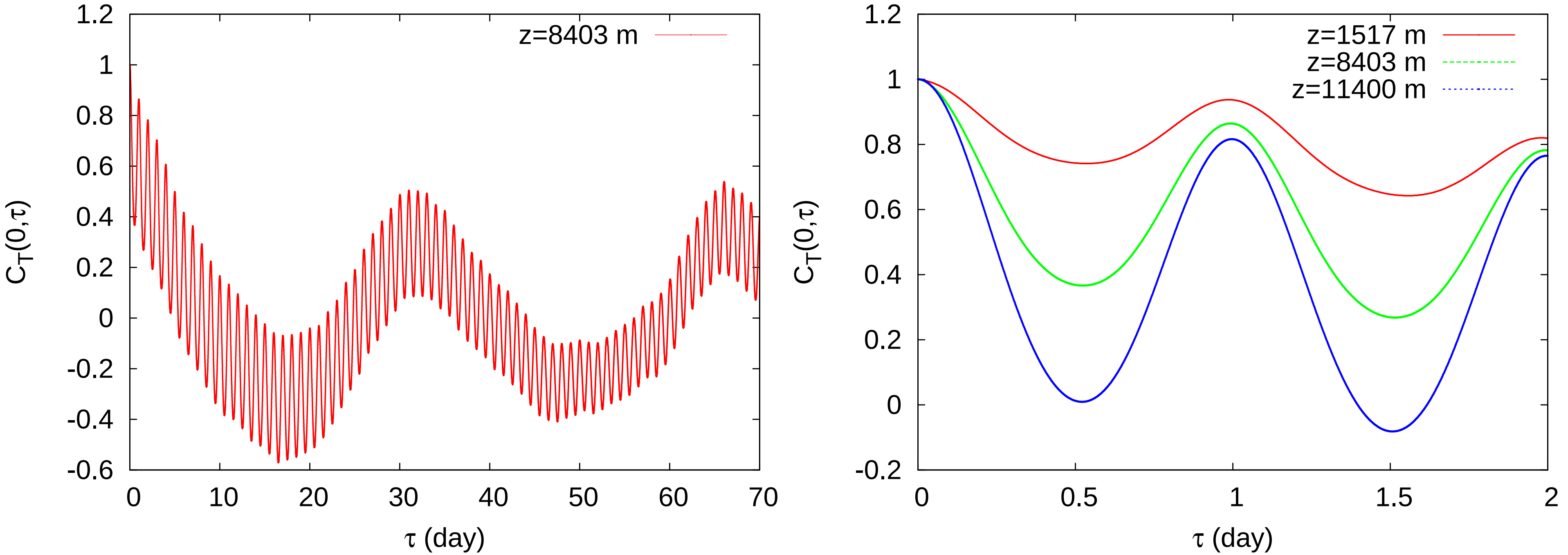
Auto-correlation of q_v



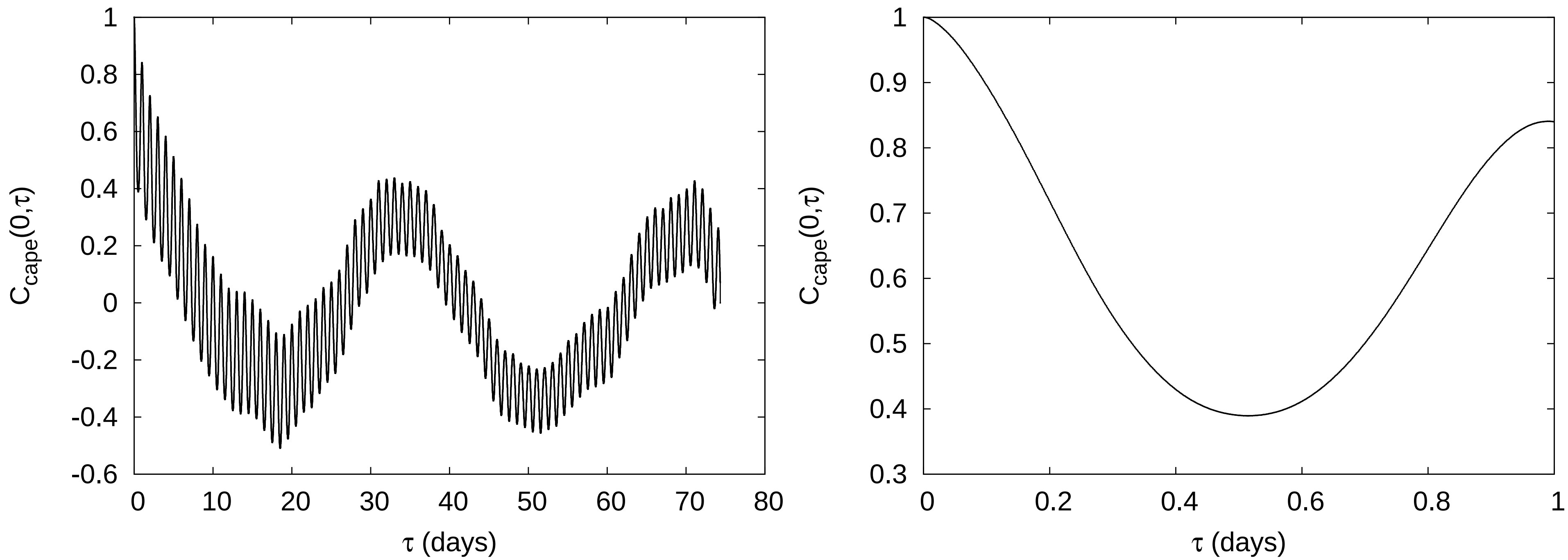
Auto-correlation of q_l



Auto-correlation of T



CAPE auto-correlation

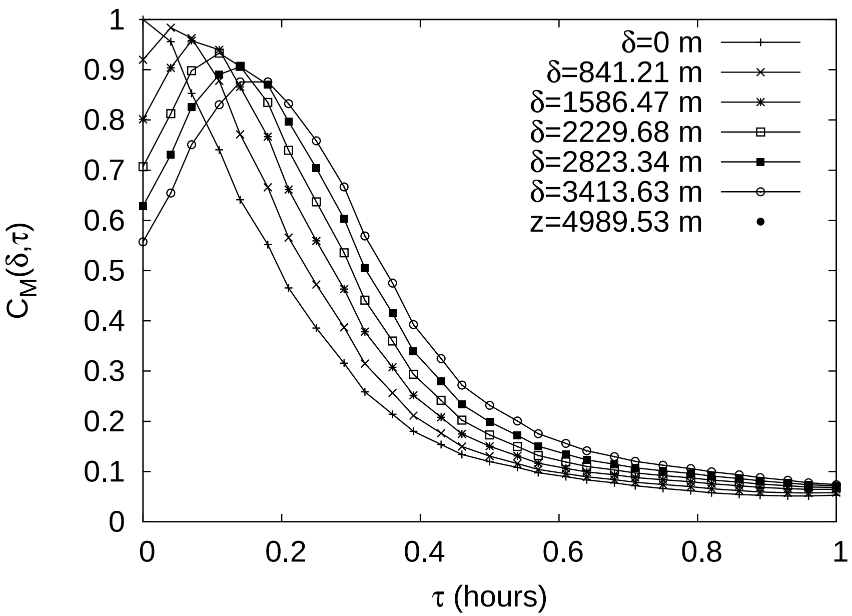


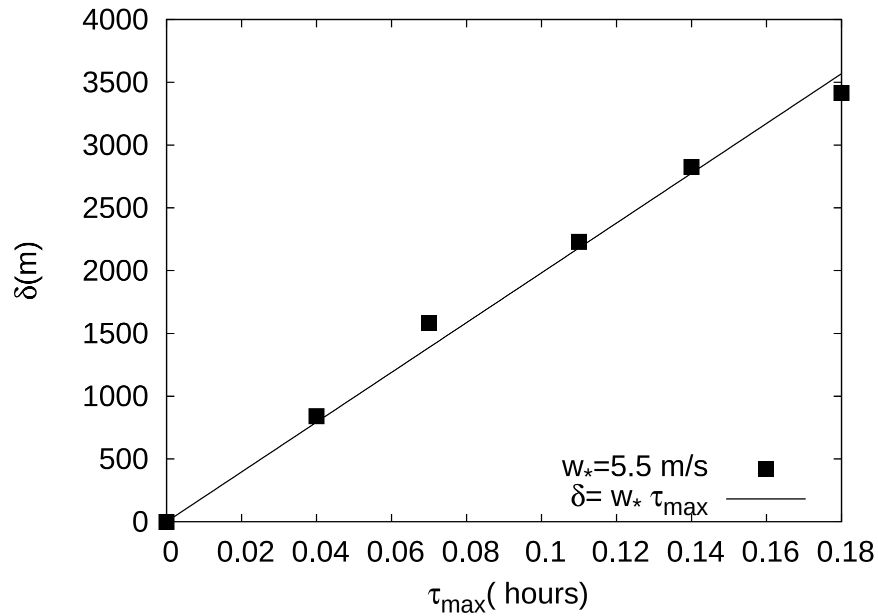
Speculation:

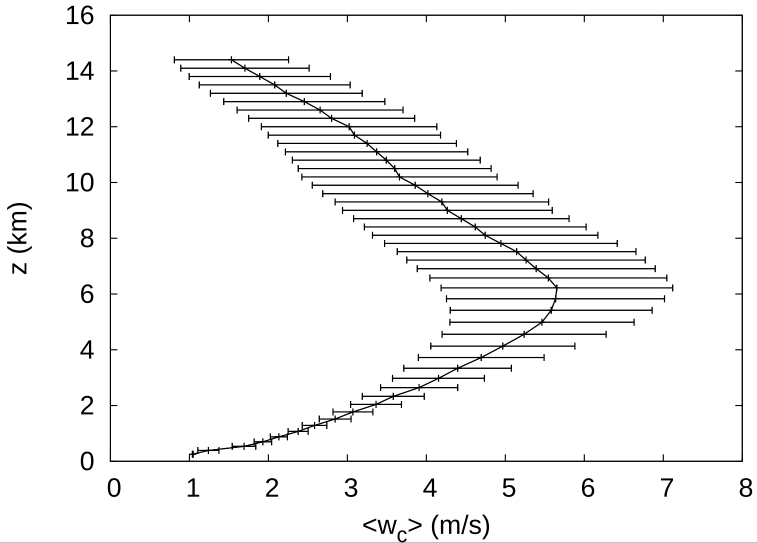
$$\tau_{rad} = \frac{H^2 \rho c_p (d\theta/dz)}{\langle Q_{rad} \rangle}$$

Assuming $\langle Q_{rad} \rangle = 125 W m^{-2}$, $H = 15 Km$, $\rho = 0.6 kg m^{-3}$, $d\theta/dz = 3 K km^{-1}$ gives $w_s \simeq 0.005 cm s^{-1}$ and $\tau_{rad} \simeq 30$ days.

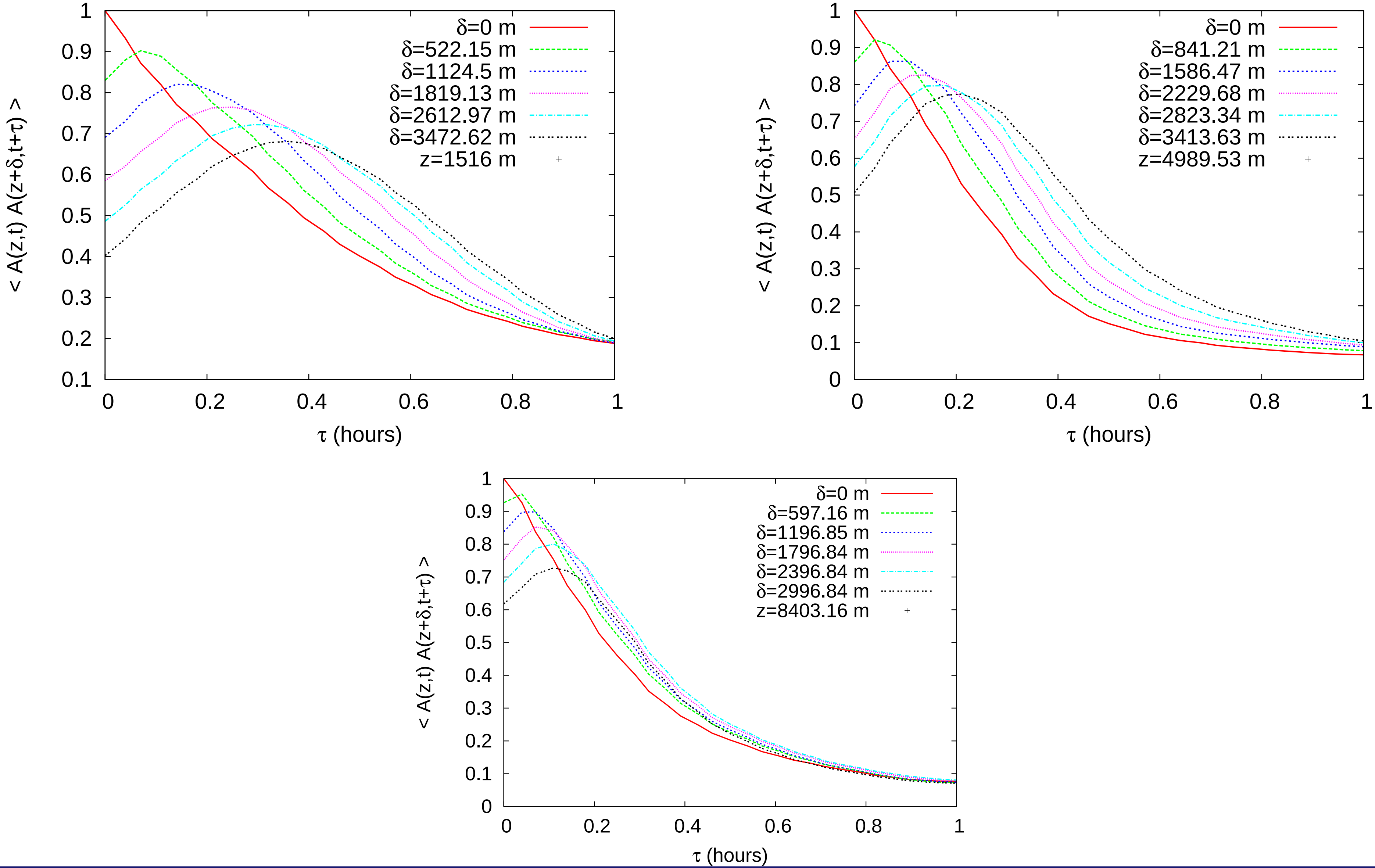
Spatio-temporal delayed correlation function



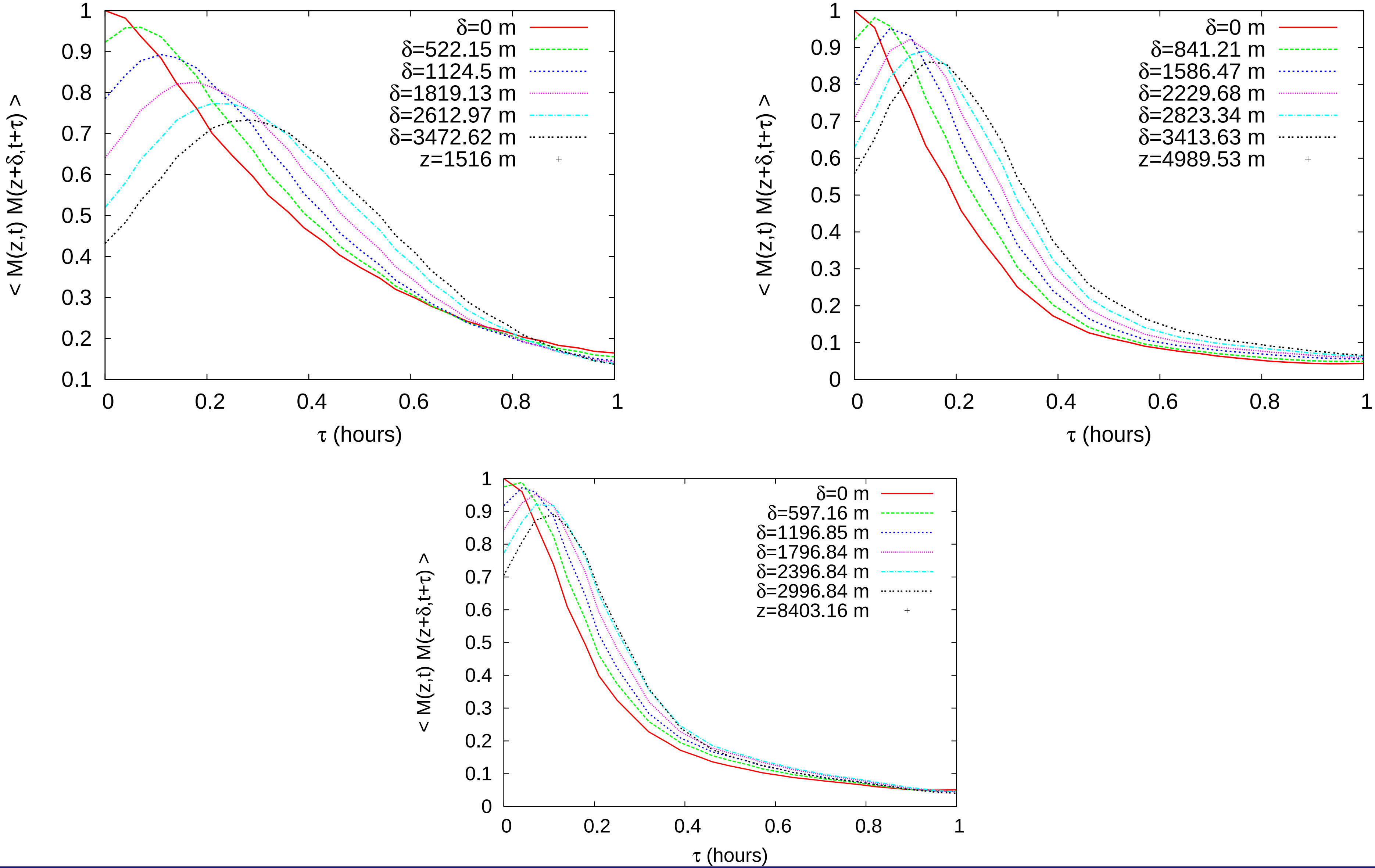




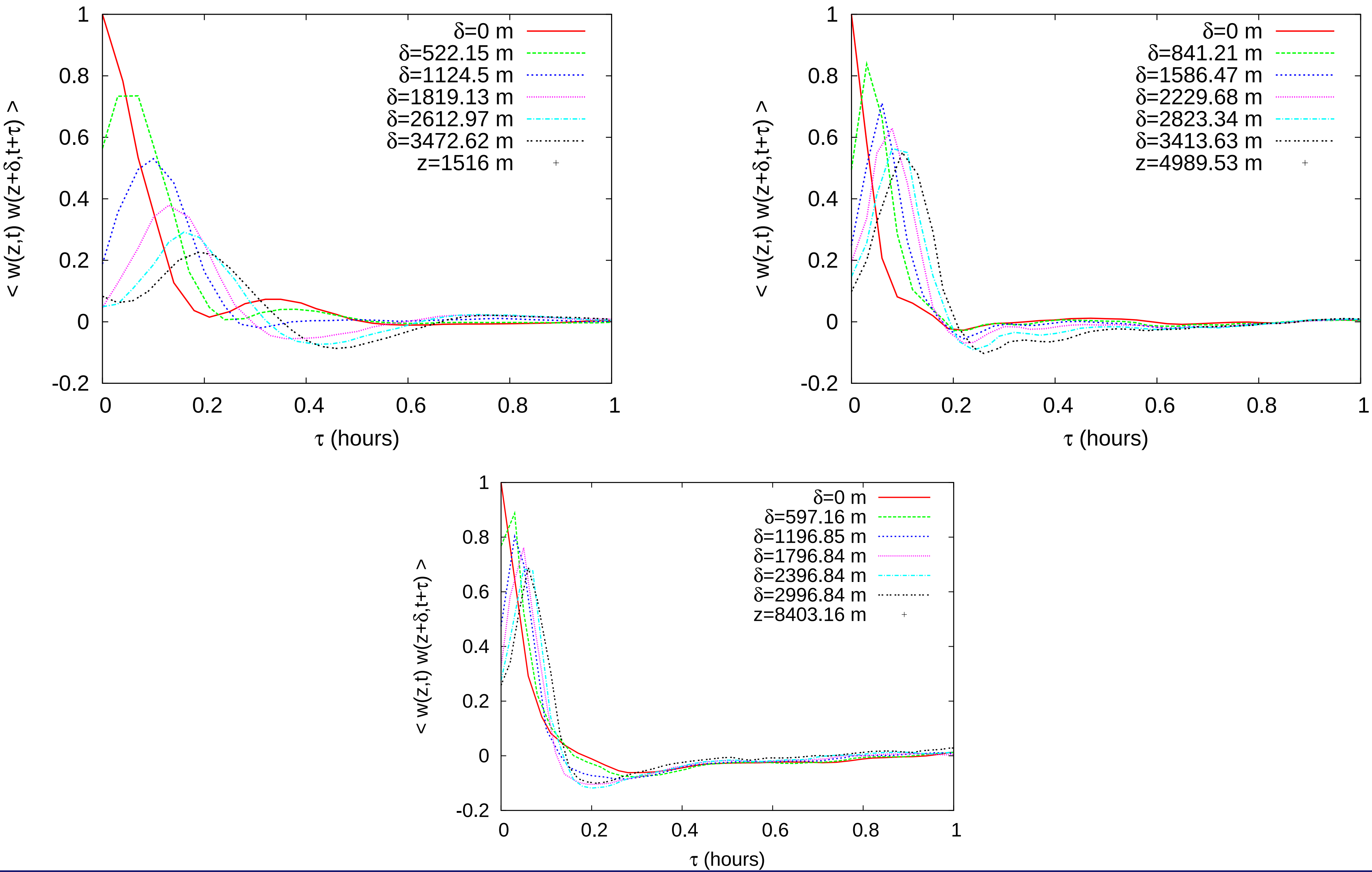
Information transport in A_u



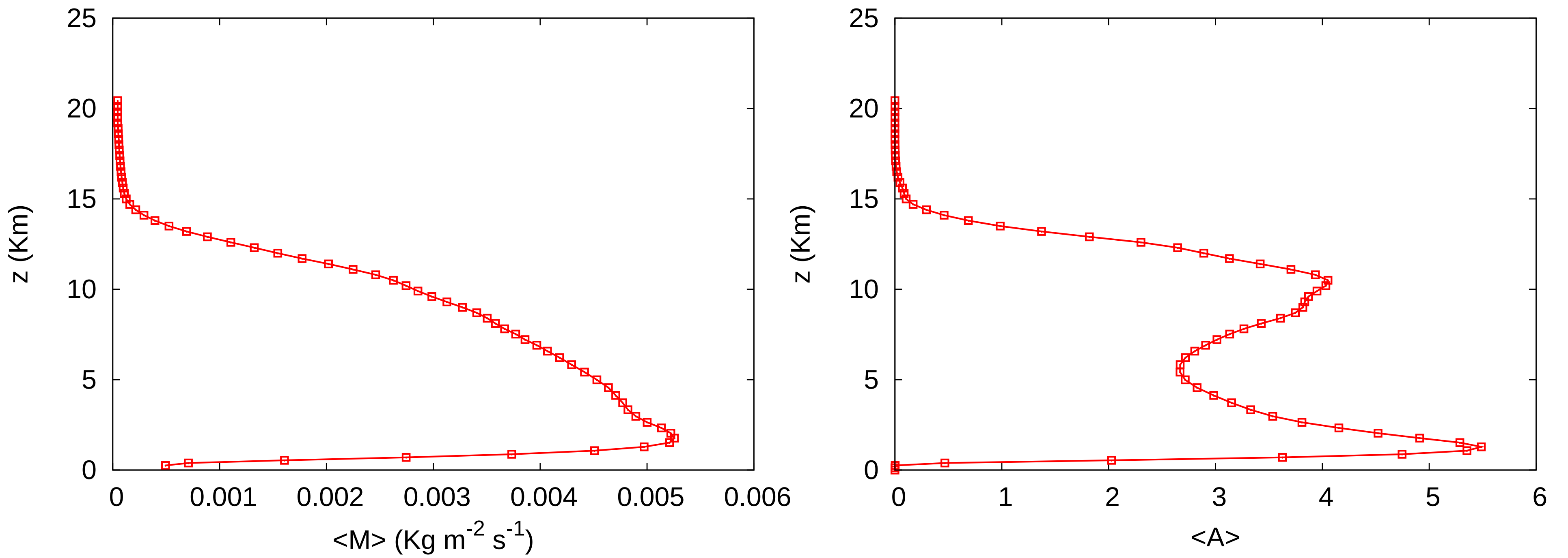
Information transport in M_u



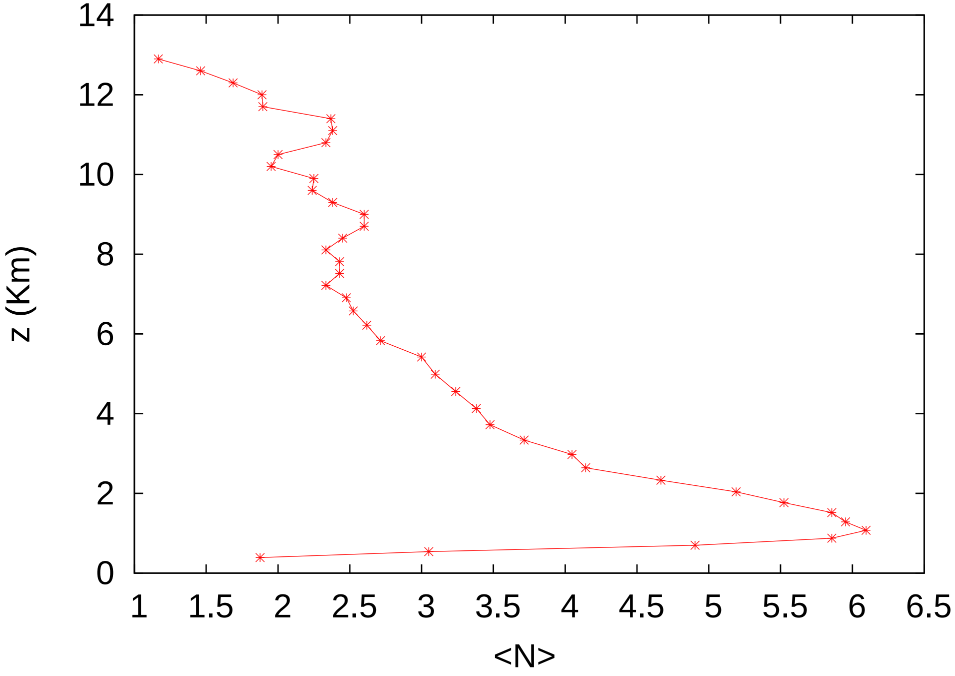
Information transport in W_u

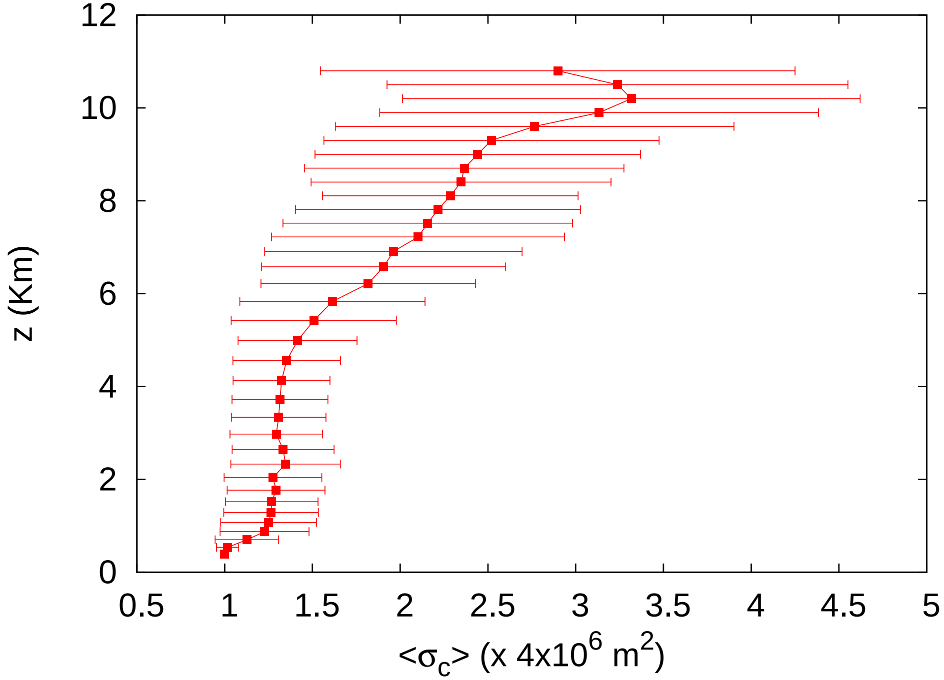


Mean characteristics

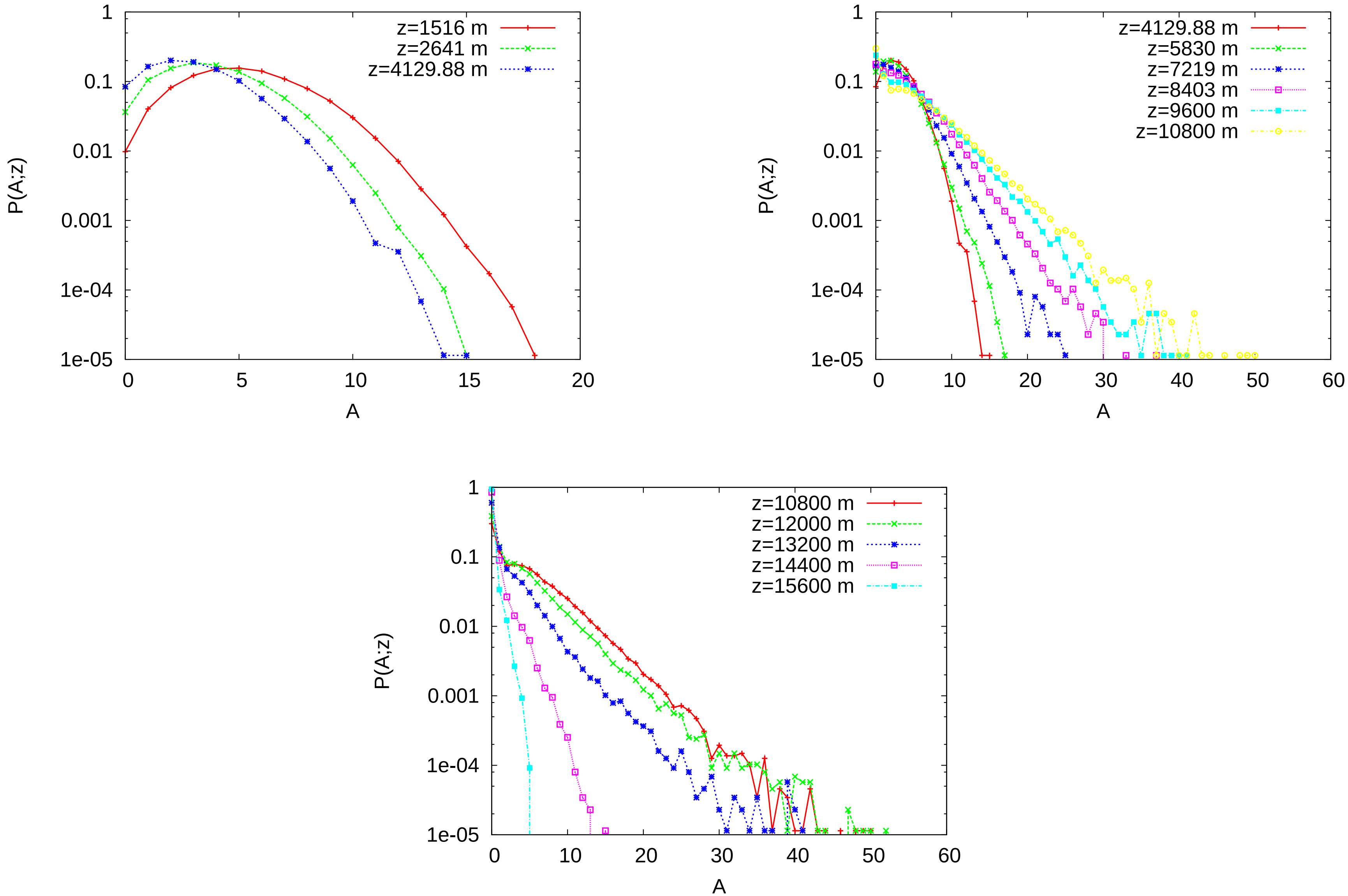


$\langle N \rangle$ and $\langle \sigma_c \rangle$

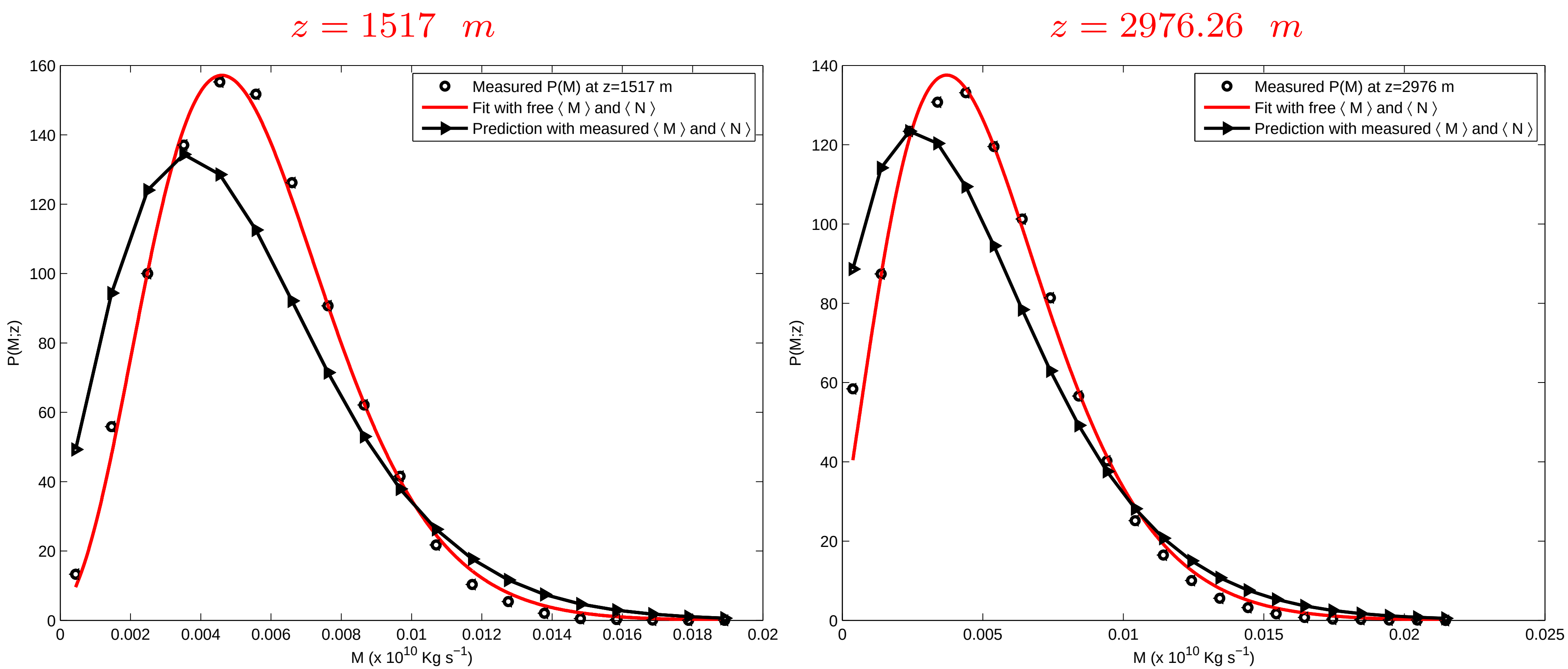




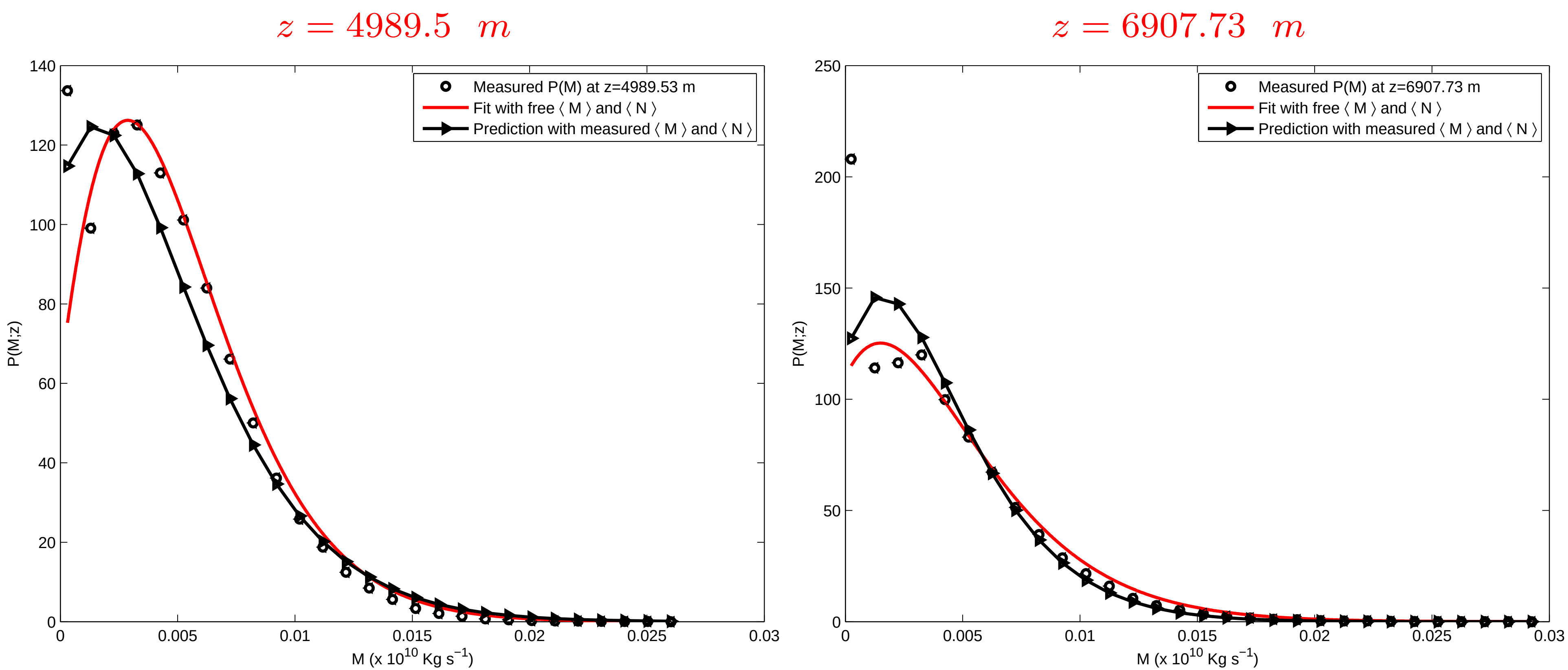
Fat tails of marginal PDF of up-draft area coverage



Altitude variability of total Mass flux PDF

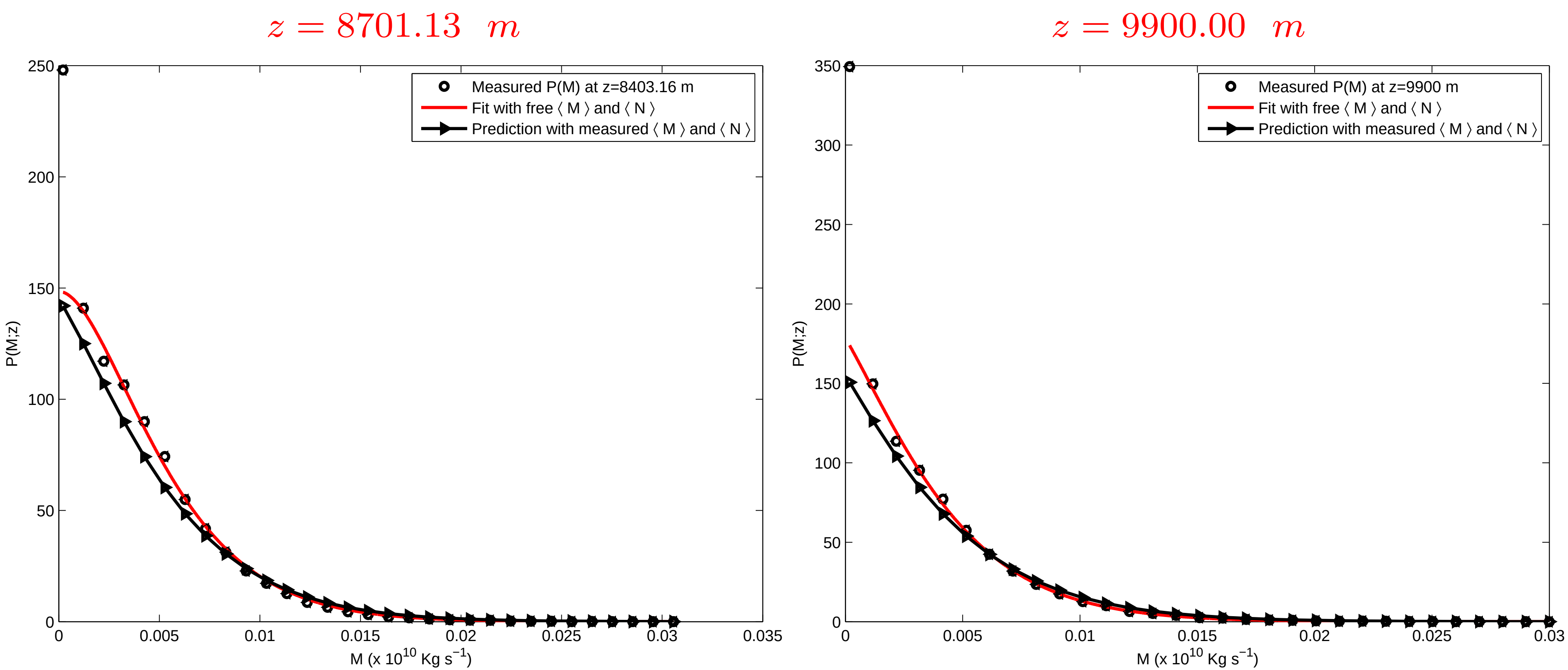


P(M,z)



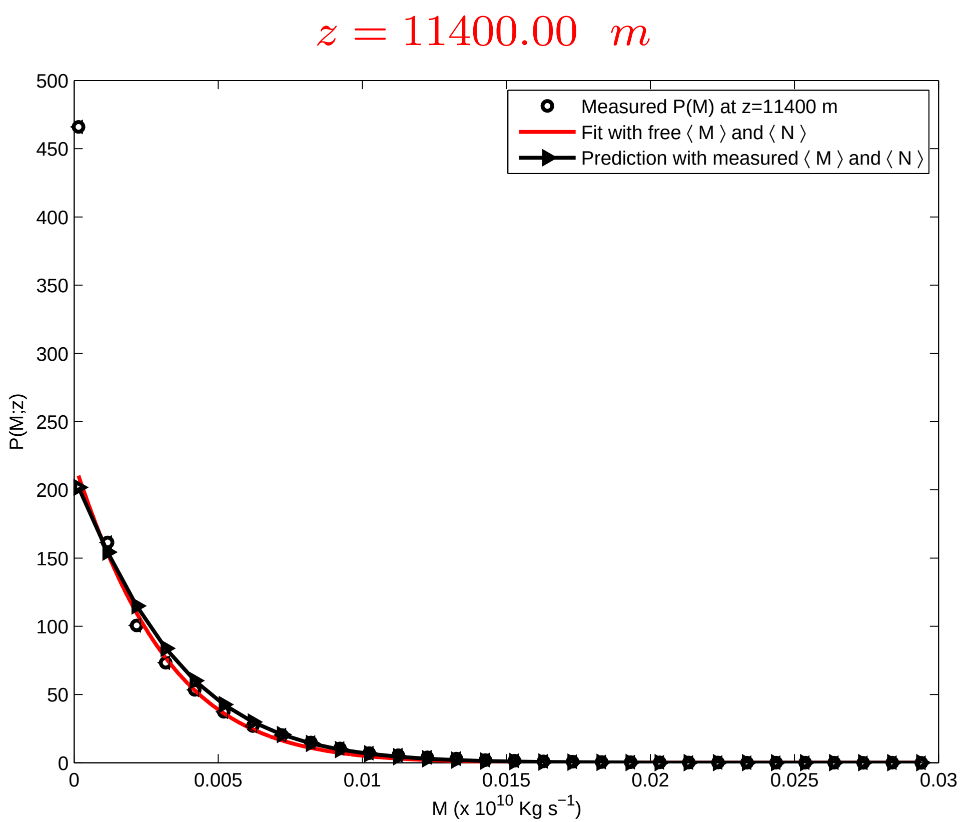
$z = 4989.5 \quad m$
Measured P(M) at z=4989.53 m
Fit with free ⟨ M ⟩ and ⟨ N ⟩
Prediction with measured ⟨ M ⟩ and ⟨ N ⟩
P(M;z)
M (x 10^{10} Kg s^{-1})
$z = 6907.73 \quad m$
Measured P(M) at z=6907.73 m
Fit with free ⟨ M ⟩ and ⟨ N ⟩
Prediction with measured ⟨ M ⟩ and ⟨ N ⟩
P(M;z)
M (x 10^{10} Kg s^{-1})

P(M,z)

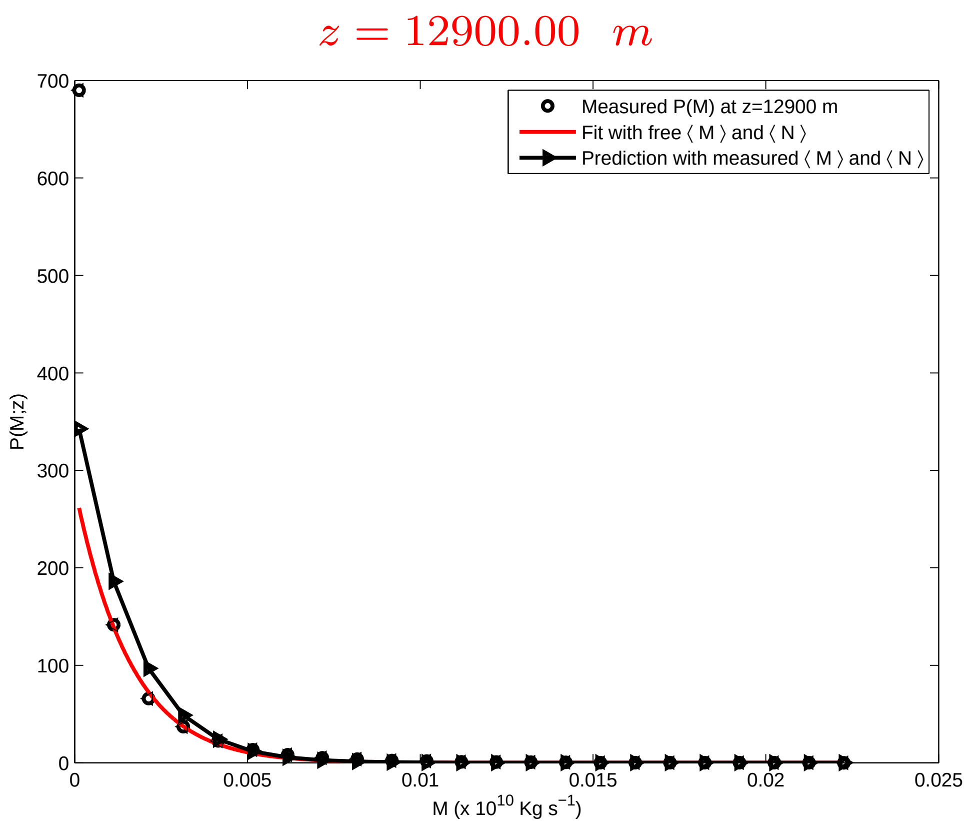


$z = 8701.13 \quad m$
Measured P(M) at z=8403.16 m
Fit with free ⟨ M ⟩ and ⟨ N ⟩
Prediction with measured ⟨ M ⟩ and ⟨ N ⟩
P(M;z)
M (x 10^{10} Kg s^{-1})
0
50
100
150
200
250
0.005
0.01
0.015
0.02
0.025
0.03
0.035
$z = 9900.00 \quad m$
Measured P(M) at z=9900 m
Fit with free ⟨ M ⟩ and ⟨ N ⟩
Prediction with measured ⟨ M ⟩ and ⟨ N ⟩
P(M;z)
M (x 10^{10} Kg s^{-1})
0
50
100
150
200
250
300
350
0.005
0.01
0.015
0.02
0.025
0.03

P(M,z)

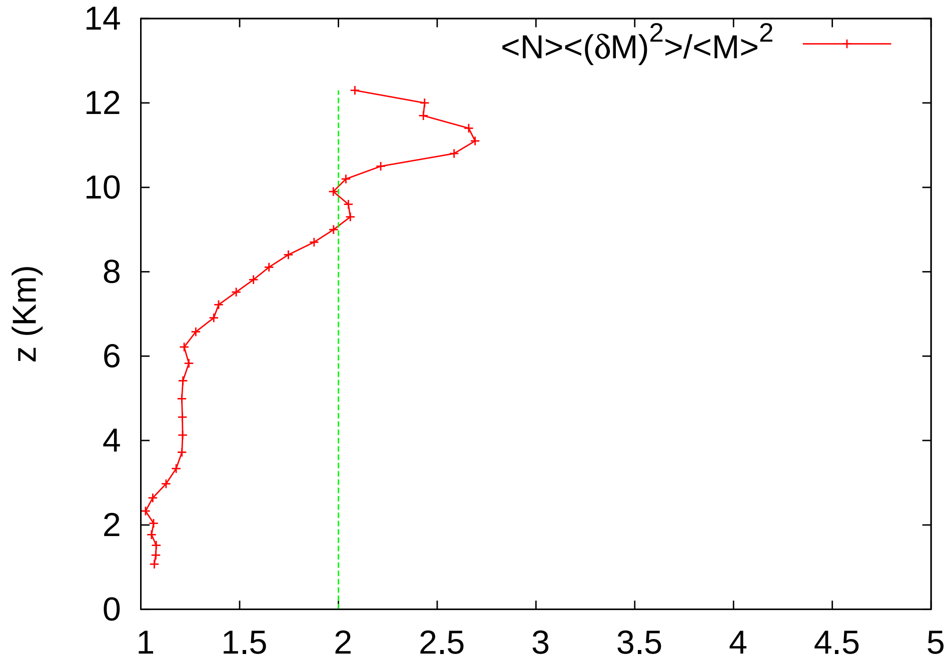


$z = 11400.00 \quad m$
Measured P(M) at z=11400 m
Fit with free ⟨ M ⟩ and ⟨ N ⟩
Prediction with measured ⟨ M ⟩ and ⟨ N ⟩
P(M;z)
M (x 10^10 Kg s^-1)



$z = 12900.00 \quad m$
Measured P(M) at z=12900 m
Fit with free ⟨ M ⟩ and ⟨ N ⟩
Prediction with measured ⟨ M ⟩ and ⟨ N ⟩
P(M;z)
M (x 10^10 Kg s^-1)

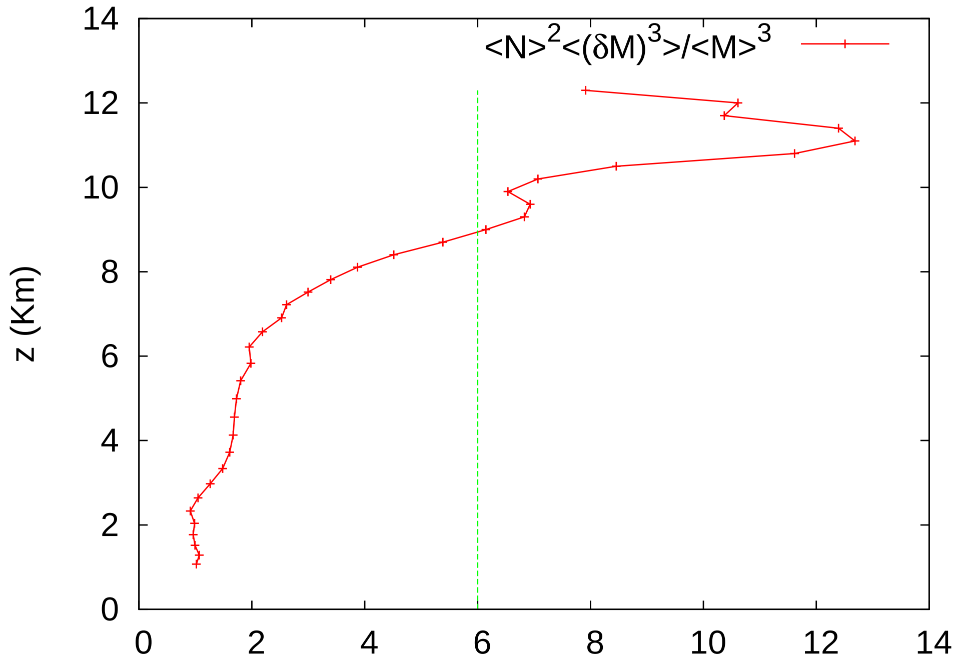
Mass flux variance



The scaling of the variance of total mass flux and number of active grids in different heights with the Craig and Cohen prediction:

$$\frac{\langle (\delta M)^2 \rangle}{\langle M \rangle^2} = \frac{2}{\langle N \rangle}$$

Mass flux Skewness



The prediction:

$$\frac{\langle(\delta M)^3\rangle}{\langle M\rangle^3} = \frac{6}{\langle N\rangle^2}$$

Clustering degree

If $N(A)$ is the number of clouds in any sub-area $A \subset S$ then $P_p[N(A) = k]$ is defined by

$$P_p[N(A) = k] = \frac{(\gamma|A|)^k e^{-\gamma|A|}}{k!} \qquad for \qquad k = 0, 1, \cdots ,$$

where $\gamma|A|$ is the average number of clouds in the sub-area with size $|A|$.

Centered on any arbitrary cloud we define the probability of finding the farthest neighboring cloud with a given Euclidean distance less than r, i.e. $\Pi_p^<(r)$.

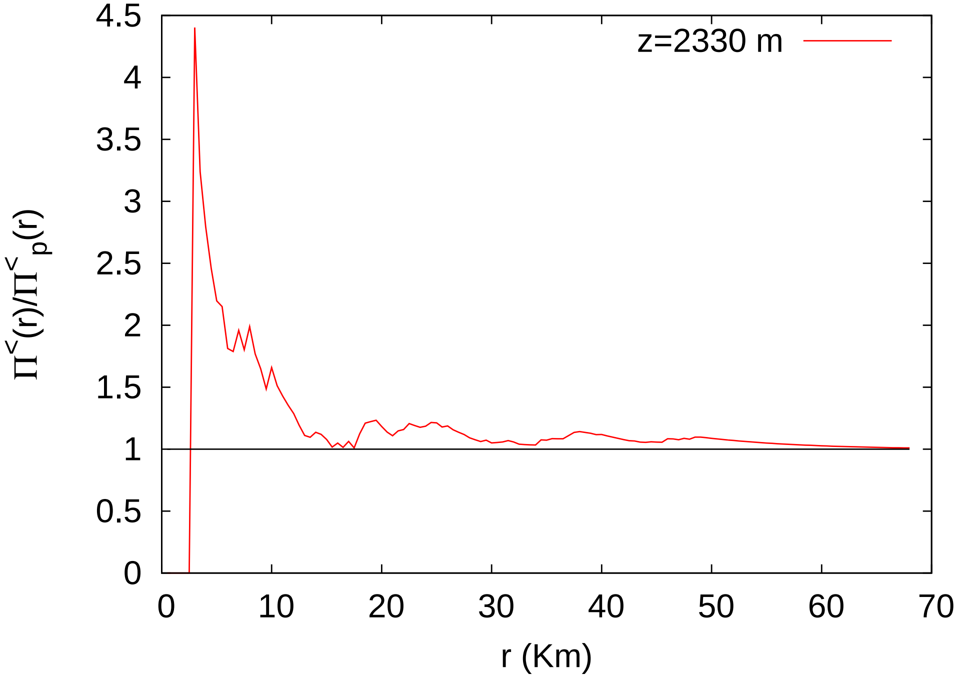
$$\begin{aligned} \Pi_p^<(r) &= 1 - \Pi_p^>(r) = 1 - P_p(N(A) = 0) \\ &= 1 - e^{-\gamma\pi r^2}, \end{aligned}$$

where $|A| = \pi r^2$ is used.

For obtaining the clustering degree one measures directly the cumulative probability of having k clouds inside a ball of radius r centered around any existing cloud in each altitude.
Then

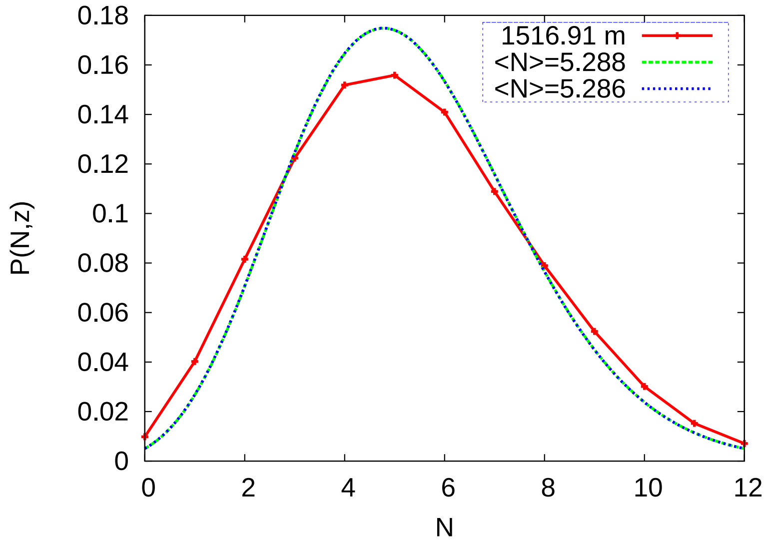
$$\chi(r) = \frac{\Pi^<(r)}{\Pi_p^<(r)}.$$

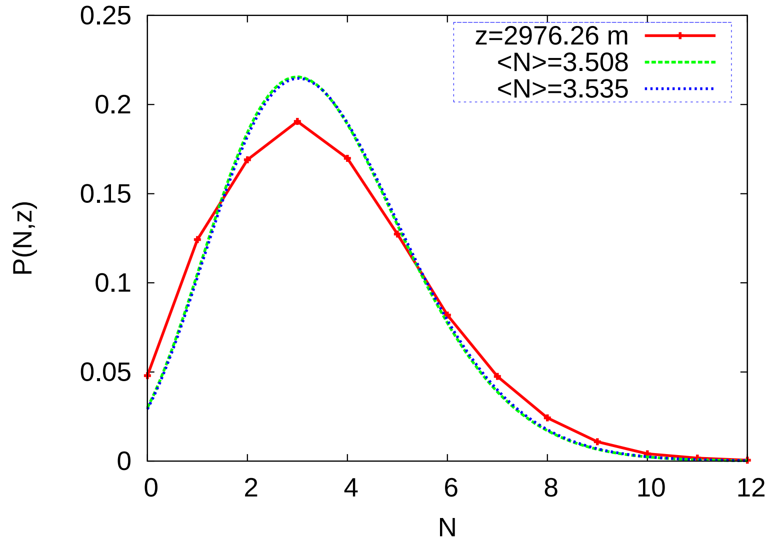
Clustering I

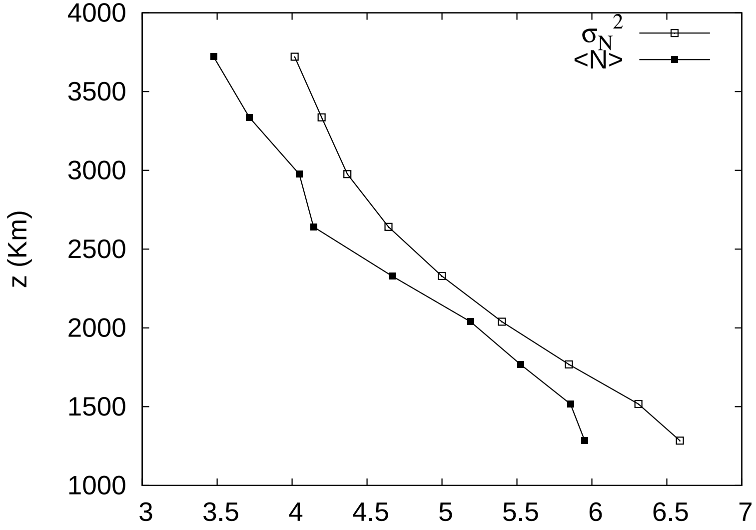


In a radius of around 10 km any cloud is surrounded with more neighboring clouds than a Poisson distribution predicts.

Clustering II







Summary

- Analysis of the time scales shows that a state of quasi-equilibrium establishes in our CRM simulation with diurnal forcing.
- The response of the total up-draft mass flux to the total heating rate at all heights indicates to a range of $2-4$ hours adjusting time.
- The statistics of the total up-draft mass flux is qualitatively consistent with the predictions of the Cohen and Craig (2006).
- The Gibbs theory under-estimates the variance and skewness of the total mass flux.
- Analysis of our CRM simulation shows that the non-interacting assumption employed in the Craig and Cohen theory does not hold as we demonstrate the clouds preferentially cluster .