BIRS Workshop 09w5082
Frames from first principles:
Error correction, symmetry goals, and numerical efficiency
March 15-20, 2009

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00	Lectures: Kutyniok, Bownik
10:25–10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45	Lecture: Calderbank
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:10–14:45	Lecture: Cohn
14:45-15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15	Lectures: Weber, Lammers, Bodmann
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00	Lectures: Gunturk, Powell
10:25–10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45	Lecture: Benedetto
11:30–13:30	Lunch
13:30	Lectures: Pfander, Han
14:45–15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15	Lectures: Jain, Steidl, Li
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00	Lectures: Gröchenig, Christensen
10:25-10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45	Discussion
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00	Lectures: Tropp, Fickus
10:25-10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45	Lecture: Casazza
11:30–13:30	Lunch
13:30	Lectures: Rauhut, Boufounos
14:45-15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15	Lectures: Saab, Larson
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00	Informal Discussions
10:00-11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

BRIEF SCHEDULE

Time/Day	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-9:40	Kutyniok	Gunturk	Gröchenig	Tropp	Discussion
9:45-10:25	Bownik	Powell	Christensen	Fickus	Discussion
Coffee Break					
10:45-11:30	Calderbank	Benedetto		Casazza	
Lunch					
1:30-2:05	1pm: Tour+Pic	Pfander		Rauhut	
2:10-2:45	Cohn	Han		Boufounos	
Coffee Break					
3:15-3:50	Weber	Jain		Saab	
3:55-4:30	Lammers	Steidl		Larson	
4:35-5:10	Bodmann	Li			

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **John Benedetto** (University of Maryland)
Title: *Some non-linear frame theoretic problems*
Abstract: The theory of frames is a topic in linear algebra and linear functional analysis. However, there have been some useful applications to non-linear problems. We consider two applications. In the first, we give a precise quantitative relationship between Sigma-Delta quantization and Pulse Code Modulation. This answers a recent question of Matthew Fickus. In the second, we develop a theory of vector valued ambiguity functions in order to deal with MIMO and multi-sensor environments. The theory of frames arises necessarily in this theory. The two applications are related by problems related to the construction of low autocorrelation codes arising in quantization schemes.

Speaker: **Bernhard Bodmann** (University of Houston)
Title: *Correcting erasures of quantized frame coefficients*
Abstract: In this talk we investigate an algorithm for the suppression of errors caused by quantization of frame coefficients and by erasures in their subsequent transmission. The erasures are assumed to happen independently, modeled by a Bernoulli experiment. The algorithm for error correction in this study embeds check bits in the quantization of frame coefficients, causing a possible, but controlled quantizer overload. If a single-bit quantizer is used in conjunction with codes which satisfy the Gilbert Varshamov bound, then the contributions from averaging over erasures and from the quantization error are shown to have bounds with the same asymptotics in the limit of large numbers of frame vectors. Joint work with Pete Casazza, Gitta Kutyniok and Steven Senger.

Speaker: **Petros Boufounos** (Rice University)
Title: *Compressive-Domain Filtering*
Abstract: We consider the scenario where a compressive sensing system acquires a signal of interest corrupted by an interfering signal. Under mild sparsity and orthogonality conditions on the signal and interference, we demonstrate that it is possible to efficiently filter out the interference from the compressive measurements in a manner that preserves our ability to recover the signal of interest. We develop a filtering method that nulls out the interference while maintaining the restricted isometry property (RIP) on the set of potential signals of interest. We further demonstrate that filtering out the interference before reconstruction exhibits better performance compared to reconstruction of the whole signal followed by filtering of the interference. The filter operates directly in the compressive domain and has computational complexity that is polynomial in the number of measurements. Joint work with Mark A. Davenport and Richard G. Baraniuk.

Speaker: **Marcin Bownik** (University of Oregon)
Title: *Characterization of sequences of frame norms*
Abstract: In this talk we give a characterization of all sequences of norms of frames with optimal frame bounds A and B. This extends known results for finite tight frames by Casazza et al. and infinite Parseval frames by Kadison. We reformulate this problem in terms of diagonals of positive operators with prescribed spectrum. Then the case of finite frames can be reduced to the well-known Schur-Horn theorem.

The infinite case is much more interesting as it breaks naturally into two distinct parts. In the terminology of Kadison these are Pythagorean and Carpenter's Theorems. This talk is based on a joint work with John Jasper.

Speaker: **Robert Calderbank** (Princeton University) Title: *Fast Reconstruction Algorithms for Deterministic Sensing Matrices and Various Applications*

Speaker: **Pete Casazza** (University of Missouri)
Title: *Five Deep Problems in Frame Theory: A Progress Report.*
Abstract: In just the last four months, serious progress has been made on five of the deepest problems in frame theory. We will look at the problems and give a progress report.

Speaker: **Ole Christensen** (Technical University of Denmark)
Title: *Pairs of dual Gabor frames*
Abstract: Consider a bounded function g supported on $[-1, 1]$ and a modulation parameter $b \in]1/2, 1[$ for which the Gabor system $\{E_{mb}T_n g\}_{m,n\in\mathbb{Z}}$ is a frame. We show that such a frame always has a compactly supported dual window. More precisely, we show that if $b < \frac{N}{N+1}$ for some $N \in \mathbb{N}$, it is possible to find a dual window supported on $[-N, N]$. Under the additional assumption that g is continuous and only has a finite number of zeros on $] - 1, 1[$, we characterize the frame property of $\{E_{mb}T_n g\}_{m,n\in\mathbb{Z}}$. As a consequence we obtain easily verifiable criteria for a function g to generate a Gabor frame with a dual window having compact support of prescribed size.

Speaker: **Matt Fickus** (Air Force Institute of Technology)
Title: *Classifying compact convex sets with frames*
Abstract: Classification is a fundamental image processing task. Recent empirical evidence suggests that classification algorithms which make use of redundant linear transforms will regularly outperform their nonredundant counterparts/ We provide a rigorous explanation of this phenomenon in the single class case. We begin by developing a measure-theoretic analysis of the set of points at which a given decision rule will have an intolerable chance of making a classification error. We then apply this general theory to the special case where the class is compact and convex, showing that such a class may be arbitrarily well-approximated by frame sets, namely, preimages of hyperrectangles under frame analysis operators. This leads to a frame-based classification scheme in which frame coefficients are regarded as features. We show that indeed the accuracy of such a classification scheme approaches the ideal as the redundancy of the frame grows large.

Speaker: **Karlheinz Groechenig** (University of Vienna)
Title: *New results on Gabor frames*
Abstract: While there are many structural results about Gabor frames, it remains difficult to determine when a given function and lattice generate a Gabor frame. We present two new types of results. On the one hand, we discuss a new criterion for Gabor frames that does not involve any inequality. In this criterion it is not necessary to establish the existence of a lower frame bound, but it suffices to verify that the coefficient operator is one-to-one on a space of distributions. On the other hand, we discuss the known results for which lattices the Hermite functions generate Gabor frames. This part is joint work with Yura Lyubarski.

Speaker: **Sinan Güntürk** (Courant Institute)
Title: *Recent advances in sigma-delta modulation*
Abstract: In this talk, we will describe some recent results on the best achievable accuracy for sigma-delta encoding of bandlimited functions. Joint work with Felix Krahmer and Percy Deift.

Speaker: **Bin Han** (University of Alberta)
Title: *Matrix Extension with Symmetry and Symmetric Orthonormal Complex M- wavelets*

Abstract: Let F denote a subfield of the complex field C such that F is closed under the operations of complex conjugate of F and square roots of positive numbers in F. For example, F = R or C. The matrix extension problem with symmetry is to find a unitary square matrix P of trigonometric polynomials with symmetry and with coecients in F such that the first row of P is a given row vector p of trigonometric polynomials with symmetry and coeficients in F satisfying $\|p\| = 1$. Matrix extension plays a fundamental role in many areas. For example, the matrix extension with symmetry plays a central role in the construction of symmetric tight M-wavelet frames from symmetric M-refinable functions. In this talk, we shall solve the matrix extension problem with symmetry by developing a step-by-step simple algorithm to derive a desired square matrix P from a given row vector p of trigonometric polynomials with complex coecients and symmetry. As an application of our algorithm for the matrix extension with symmetry, for any dilation factor M, we shall present two families of compactly supported symmetric orthonormal complex M-wavelets with arbitrarily high vanishing moments. Wavelets in the first family have the shortest possible supports with respect to their orders of vanishing moments; their existence relies on the establishment of nonnegativity on the real line of certain associated polynomials. Wavelets in the second family have increasing orders of linear-phase moments and vanishing moments. Symmetric complex orthonormal M-wavelets also have a natural connection to symmetric tight M-wavelet frames.

Speaker: **Saurabh Jain** (University of Houston)
Title: *Isotropic Multiresolution Analysis: Theory and Applications*
Abstract: Directionally unbiased processing of 2D and 3D data is highly desirable in biomedical and seismic imaging. This consideration led to the development of Isotropic Multiresolution Analysis (IMRA) and the associated isotropic wavelet transforms. I begin with a brief overview of the applications that motivated Papadakis and Bodmann to introduce isotropic filter banks. A quest for a multiresolution theory behind these filter banks led us to give a formal definition of IMRA. We incorporate rotational invariance of each resolution space in the definition of IMRA. This definition has interesting consequences on the structure of the resolution spaces and refinable functions associated with IMRA. One of the consequences is that the IMRAs can have a frame of translates of a single refinable function only if this function is discontinuous which is not useful in practice. In order to use smooth filters, we derive a version of the Unitary Extension Principle that also takes into account the low pass component i.e. the translates of the scaling function. In this version, we characterize the families consisting of translates of the scaling function and only the positive dilates of the wavelets that form a frame for $L^2(R^d)$ in terms of conditions on the associated filters. This is useful in applications because often we do not want to throw away the low pass component after a few levels of decomposition of the signal.

Speaker: **Mark Lammers** (University of North Carolina, Wilmington)
Title: *Uncertainty in Finite Frames with application to Quantization*
Abstract: We begin by motivating a characterization of localization/uncertainty in the finite setting using some recent work in Sigma Delta quantization. This work exploits an alternate dual, ie. a non-canonical dual, to reduce the error of the quantization process. This alternate dual is found by minimizing the ℓ^2 norms of the dual frame vectors after one applies a finite difference matrix. Using a finite difference Matrix (D) and the Discrete Fourier Transform (F) leads to a natural representation of the Heisenberg product , $\|Dv\|\|DFv\|$, in the finite setting. A number of authors have used these matrices to develop finite versions of the Gauss and Hermite functions as eigenvectors of the Discrete Fourier Transform, as well as some finite versions of the classical uncertainty principle. Inspired by the Balian-Low theorem, we present some initial findings for both general finite frames and finite Gabor systems. (Joint work with Blum/Powell/Yilmaz and Fickus/Powell)

Speaker: **David Larson** (Texas A&M University)
Title: *Perturbations and surgery on frames*
Abstract: We discuss some results concerning perturbing and otherwise modifying existing frames to obtain

new frames which may have better analytic and geometric properties. This represents work that has been done over the past few years by the speaker with several students and colleagues.

Speaker: **Shidong Li** (San Francisco State University)
Title: *Frame fundamental image fusion, one-side frame perturbation and its stability and a dimension invariance principle*
Abstract: Three connected topics will be reported, all resulting from frame fundamental image fusion applications. From the modeling of imaging devices, a one-side frame perturbation (OSFP) naturally arises. Stability of OSFP in terms of a reconstruction error is established; While image fusion through sensory frame formulation brings in more realistic physical factors into the observation model, the fusion reconstruction becomes more involved as no (bi)orthogonal structures exist. An iterative reconstruction procedure is proposed whose convergence is also easily established. Whats more flexible is that a variety of regularization operators can be applied in this (believed) new iterative procedure, which is demonstrated as substantially beneficial; Finally, the dimension invariance principle maybe indispensable in the inversion of a more general class of circulant matrices, certainly also arises from sensory frame modeling of sensors. We show naturally that the iterative procedure we proposed results in a generalized Tikhonov regularization mechanism. This in turn amounts to solving a system of equations. Though FFT based diagonalization is possible, when image fusion application is the subject, matrices easily goes to a size of the order $10^18 \times 10^18$ for fusing an image of (merely) the size of 256×256. Thats when dimension invariance principle comes in handy. It states that if the sensory frame and its dual are compactly supported (which they are essentially so in all practical applications), compact duals can be evaluated from a subspace X with a fraction of the actual dimension (of the image space H), and stay valid while naturally embedded to H. This dimension invariance principle coupled with FFT based method make the implementation a lot more feasible. Error bounds are also established when approximate duals are utilized. Fusion examples from simulated images and actual inhomogeneous images (taken by different cameras and/or at different times) will be demonstrated.

Speaker: **Götz Pfander** (Jacobs University Bremen)
Title: *Gabor frames for $\mathbb{C}^d$ and some applications*
Abstract: This talk reviews recent results on Gabor systems in finite dimensions. We will focus on the linear independence of subsets of Gabor systems but also touch upon the the coherence of Gabor systems and the condition number of matrices formed by a small number of vectors from a Gabor system. The results discussed have been obtained in collaborations with Jim Lawrence, Felix Krahmer, Peter Rashkov, Jared Tanner, Holger Rauhut, and David Walnut.

Speaker: **Alex Powell** (Vanderbilt)
Title: *Error bounds for consistent reconstruction by soft thresholding*
Abstract: We consider the problem of signal reconstruction from quantized frame coefficients under Bennett's white noise model. The Rangan-Goyal (RG) algorithm addresses this problem with a recursive soft thresholding procedure based on consistent reconstruction; the RG algorithm may be viewed as a generalization of the Kaczmarz algorithm that is specifically adapted to bounded noise. We shall derive refined mean squared error bounds for the Rangan-Goyal algorithm in the settings of random and deterministic frame measurements. In particular, it is shown that the RG algorithm achieves MSE of order $1/N^2$, where N is the number of measurements. Frame ordering issues play an important role in the analysis.

Speaker: **Holger Rauhut** (University of Bonn)
Title: *Circulant and Toeplitz matrices in Compressed Sensing*
Abstract: Compressed sensing seeks to recover a sparse vector from a small number of linear and non-adaptive measurements. While most work so far focuses on Gaussian or Bernoulli random measurements we investigate the use of partial random circulant and Toeplitz matrices in connection with recovery by ℓ_1-minization. In contrast to recent work in this direction we allow the use of an arbitrary subset of rows of

a circulant and Toeplitz matrix. Our recovery result predicts that the necessary number of measurements to ensure sparse reconstruction by ℓ_1-minimization with random partial circulant or Toeplitz matrices scales linearly in the sparsity up to a log-factor in the ambient dimension. This represents a significant improvement over previous recovery results for such matrices. As a main tool for the proofs we use a new version of the non-commutative Khintchine inequality.

Speaker: **Rayan Saab** (University of British Columbia)
Title: *Sparse recovery via non-convex optimization: instance optimality*
Abstract: It has been recently shown that one can recover/decode estimates of sparse and compressible signals from an "incomplete" set of noisy measurements via l1-norm minimization methods under certain conditions on the "measurement matrix". For example, these conditions are satisfied when the matrix is a random matrix whose entries are drawn i.i.d. from a Gaussian distribution.

In this talk, we present the theoretical recovery guarantees obtained when decoding by p-quasinorm minimization with $0 < p < 1$ in the setting described above, and we prove that the corresponding guarantees can be better than those one can obtain in the case of one-norm minimization. In particular, we show that decoders based on p-quasinorm minimization are (l2,l^p) instance optimal. Moreover, these decoders are (l2,l2) instance optimal in probability (this latter relates to a result on distances of p-convex bodies to their convex hulls). Finally, we comment on algorithmic issues.

Speaker: **Gabriele Steidl** (University of Mannheim)
Title: *The Continuous Shearlet Transform in Arbitrary Space Dimensions*
Abstract: We are concerned with the generalization of the continuous shearlet transform to higher dimensions. Similar to the two-dimensional case, our approach is based on translations, anisotropic dilations and specific shear matrices. We show that the associated integral transform again origi- nates from a square-integrable representation of a specific group, the full n-variate shearlet group. Moreover, we verify that by applying the coorbit theory, canonical scales of smoothness spaces and associated Banach frames can be derived. We also indicate how our transform can be used to characterize singularities in signals. This is joint work with G. Teschke (University Neubrandenburg, Germany) and S. Dahlke (Univer- sity Marburg, Germany).

Speaker: **Joel Tropp** (California Institute of Technology)
Title: *Column subset selection, matrix factorization, and eigenvalue optimization*
Abstract: Given a fixed matrix, the problem of column subset selection requests a column submatrix that has favorable spectral properties. Most research from the algorithms and numerical linear algebra communities focuses on a variant called rank-revealing QR, which seeks a well-conditioned collection of columns that spans the (numerical) range of the matrix. The functional analysis literature contains another strand of work on column selection whose algorithmic implications have not been explored. In particular, a celebrated result of Bourgain and Tzafriri demonstrates that each matrix with normalized columns contains a large column submatrix that is exceptionally well conditioned. Unfortunately, standard proofs of this result cannot be regarded as algorithmic.

Speaker: **Eric Weber** (Iowa State University)
Title: *Algebraic Aspects of the Paving and Feichtinger Conjectures*
Abstract: The Paving Conjecture in operator theory and the Feichtinger Conjecture in frame theory are both problems that are equivalent to the Kadison-Singer problem concerning extensions of pure states. In all three problems, one of the difficulties is that the natural multiplicative structure appears to be incompatible–the unique extension problem of Kadison-Singer is compatible with a linear subspace, but not a subalgebra; likewise, the pavable operators is known to be a linear subspace but not a subalgebra; the Feichtinger Conjecture does not even have a linear structure. The Paving Conjecture and the Feichtinger Conjecture both have special cases in terms of exponentials in $L^2[0, 1]$. We introduce convolution as a multiplication to demonstrate a possible attack for these special cases.