

***t*-motives: Hodge structures, transcendence and other motivic aspects  
September 27 – October 2, 2009**

**LECTURE HALL and MEETING ROOMS**

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. The meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159.

**SCHEDULE**

**Sunday**

- 16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)  
**17:30–19:30** Buffet Dinner, Sally Borden Building  
**20:00** Informal gathering in 2nd floor lounge, Corbett Hall  
Beverages and small assortment of snacks available on a cash honour-system.

**Monday**

- 7:00–8:45** Breakfast  
**8:45–9:00** Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159  
**9:00–9:50** Dale Brownawell (Penn State University), *Introduction to Drinfeld modules and *t*-motives*  
**9:50–10:20** Coffee Break, 2nd floor lounge, Corbett Hall  
**10:20–11:10** Jing Yu (National Taiwan University), Transcendence Lecture #1 *On transcendence theory of Drinfeld modules*  
**11:20–12:10** Richard Pink (ETH Zürich), Hodge Structures Lecture #1 *TBA*  
**12:10–13:00** Lunch  
**13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall  
**14:00** Group Photo; meet on the front steps of Corbett Hall  
**14:20–15:00** Seidai Yasuda (Kyoto University), *Regulators, periods, and special values of automorphic *L*-functions over function fields*  
**15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall  
**15:30–16:20** Lenny Taelman (Mathematisch Instituut Leiden), *The Mordell-Weil and Tate-Shafarevich modules of a Drinfeld module*  
**17:30–19:30** Dinner

**Tuesday**

- 7:00–9:00** Breakfast  
**9:00–9:50** Dinesh Thakur (University of Arizona), Transcendence Lecture #2 *Automata methods in transcendence*  
**9:50–10:30** Coffee Break, 2nd floor lounge, Corbett Hall  
**10:30–11:20** Richard Pink (ETH Zürich), Hodge Structures Lecture #2 *TBA*  
**11:30–12:10** Mihran Papikian (Penn State University), *On the arithmetic of modular curves of  $\mathcal{D}$ -elliptic sheaves*  
**12:10–13:30** Lunch  
**14:20–15:00** Ambrus Pal (Imperial College London), *Constructing rational points on genus one curves*

15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall  
15:30–16:20 Dinesh Thakur (University of Arizona), *TBA*  
17:30–19:30 Dinner

### Wednesday

7:00–9:00 Breakfast  
9:00–10:00 Urs Hartl (Universität Münster), Hodge Structures Lecture #3 *Divisible local t-motives, local shtukas and Hodge structures*  
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall  
10:30–11:20 Federico Pellarin (Université Jean Monnet, St. Etienne), Transcendence Lecture #3 *On Mahler's method*  
11:30–12:20 Florian Breuer (University of Stellenbosch), *The Andre-Oort Conjecture for Drinfeld modular varieties*  
12:20–13:30 Lunch  
Free Afternoon  
17:30–19:30 Dinner

### Thursday

7:00–9:00 Breakfast  
9:00–10:00 Wansu Kim (Imperial College, London), Hodge Structures Lecture #4 *Weak admissibility, Galois Representations, and Deformations*  
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall  
10:30–11:20 Chieh-Yu Chang (NCTS, Taiwan), Transcendence Lecture #4 *Frobenius difference equations and difference Galois groups*  
11:30–12:10 Ignazio Longhi (National Taiwan University), *On Iwasawa theory over function fields*  
12:10–13:30 Lunch  
14:20–15:00 Toshiro Hiranouchi (RIMS, Kyoto University), *Smallness of fundamental groups for varieties over finite fields*  
15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall  
15:30–16:20 Vincent Bosser (Université de Caen), *On Drinfeld quasi-modular forms*  
17:30–19:30 Dinner

### Friday

7:00–9:00 Breakfast  
9:00–9:40 Chieh-Yu Chang (NCTS, Taiwan), *Algebraic relations among periods and logarithms for Drinfeld modules*  
9:50–10:30 Imin Chen (Simon Fraser University), *Newton polygons of exponential functions attached to Drinfeld modules of rank 2*  
10:30–10:50 Coffee Break, 2nd floor lounge, Corbett Hall  
10:50–11:30 Ernst-Ulrich Gekeler (Universität des Saarlandes), *Frobenius actions on the cohomology of Drinfeld modules*  
11:30–13:30 Lunch

**Checkout by 12 noon.**

\*\* 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

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**September 27 – October 2, 2009**

**ABSTRACTS**

Speaker: **Vincent Bosser** (Université de Caen)

Title: *On Drinfeld quasi-modular forms*

Abstract: In this talk, I will discuss some recent results involving Drinfeld quasi-modular forms that were obtained in a joint work with F. Pellarin. In particular, I will describe new advances in the problem of estimating the order of vanishing at infinity of such forms.

Speaker: **Florian Breuer** (University of Stellenbosch)

Title: *The Andre-Oort Conjecture for Drinfeld modular varieties.*

Abstract: We explore an analogue of the André-Oort conjecture for subvarieties of Drinfeld modular varieties. The conjecture states that a subvariety  $X$  of a Drinfeld modular variety contains a Zariski-dense set of complex multiplication (CM) points if and only if  $X$  is a “special” subvariety (i.e.  $X$  is defined by requiring additional endomorphisms). We prove this conjecture in two cases. Firstly when  $X$  contains a Zariski-dense set of CM points with a certain behaviour above a fixed prime (which is the case if these CM points lie in one Hecke orbit), and secondly when  $X$  is a curve containing infinitely many CM points without any additional assumptions.

Speaker: **Dale Brownawell** (Penn State University)

Title: *Introduction to Drinfeld modules and *t*-motives*

Speaker: **Chieh-Yu Chang** (NCTS, Taiwan)

Title: *Frobenius difference equations and difference Galois groups*

Speaker: **Chieh-Yu Chang** (NCTS, Taiwan)

Title: *Algebraic relations among periods and logarithms for Drinfeld modules*

Abstract: Given a Drinfeld  $\mathbb{F}_q[t]$ -module  $\rho$  defined over an algebraic function field, we consider its period matrix  $P$ . We prove that all the algebraic relations among the entries of  $P$  are those linear relations induced from the endomorphisms of  $\rho$ . We further prove the algebraic independence of  $\rho$ -logarithms of algebraic points that are linearly independent over the endomorphism ring of  $\rho$ . (This is a joint work with Matt Papanikolas). We will also present an application to the transcendence of certain special values of Drinfeld quasi-modular forms.

Speaker: **Imin Chen** (Simon Frazer University)

Title: *Newton polygons of exponential functions attached to Drinfeld modules of rank 2*

Abstract: Jointly with Yoonjin Lee we explicitly determine the Newton polygons of exponential functions attached to Drinfeld modules of rank 2 defined over  $\mathbb{F}_q(T)$ . The method is mostly elementary but nonetheless reveals some interesting closed form patterns.

Speaker: **Ernst-Ulrich Gekeler** (Universitat des Saarlandes)

Title: *Frobenius actions on the cohomology of Drinfeld modules*

Abstract: With (the motive of) a Drinfeld module one associates several modules or vector spaces that - in the familiar analogy of Drinfeld modules with abelian varieties - play the parts of singular, Betti, or deRham (co-)homology modules. We report on the relations between these, and study the various Frobenius actions in case the Drinfeld module is defined in finite characteristic.

Speaker: **Urs Hartl** (Universität Münster)

Title: *Divisible local  $t$ -motives, local shtukas and Hodge structures*

Speaker: **Toshiro Hiranouchi** (RIMS, Kyoto University)

Title: *Smallness of fundamental groups for varieties over finite fields*

Abstract: It is known that the Hermite-Minkowski type theorem for curves over finite fields. It asserts that on a function field of one variable over a finite field, there exist only finitely many separable extensions of bounded degree and discriminant. In this talk, we will deal with higher dimensional analogue of such theorem and its applications (joint work with S. Harada).

Speaker: **Wansu Kim** (Imperial College, London)

Title: *Weak admissibility, Galois Representations, and Deformations*

Abstract: The first half of this lecture will be a continuation of Professor Hartl's lecture, focusing on the definition and foundational results on weak admissibility for isocrystals with Hodge-Pink structures and its relation with Galois representations, due to Genestier-Lafforgue, Hartl, etc.

In the second half of this lecture, I will discuss the existence and the local structure of the equi-characteristic analogue of crystalline deformation rings, as discussed in my thesis. Mind that the existence of such equi-characteristic deformation rings may be surprising because a 'usual' deformation functor (with no deformation condition) has an infinite-dimensional tangent space in the equi-characteristic case, hence unlike the  $p$ -adic case we have no 'unrestricted' universal deformation ring to start with (among complete local noetherian rings). Thus, the key step in the proof of the existence is to show the finiteness of the tangent space of the deformation subfunctor defined a deformation condition analogous to torsion crystalline deformations with all weights within a fixed bounded interval.

The local structure of such equi-characteristic deformation rings is very similar to that of crystalline deformation rings studied by Kisin, and the proofs are also similar (except that in the equi-characteristic case we should work with weakly admissible isocrystals with Hodge-Pink structures instead of weakly admissible filtered isocrystals).

If time allows, I would like to discuss some interesting open questions.

Speaker: **Ignazio Longhi** (National Taiwan University)

Title: *On Iwasawa theory over function fields*

Abstract: The goal is to present a short account of some recent developments in (abelian) Iwasawa theory for function fields in positive characteristic. In this setting, the role cyclotomic towers play over number fields is taken by the extensions arising from the torsion of Drinfeld-Hayes modules. The focus shall be on Iwasawa theoretic interpretations of  $p$ -adic  $L$ -functions for automorphic abelian varieties (with most emphasis on the elliptic curves case) and/or for Drinfeld-Hayes modules.

Speaker: **Ambrus Pal** (Imperial College London)

Title: *Constructing rational points on genus one curves*

Speaker: **Mihran Papikian** (Penn State University)

Title: *On the arithmetic of modular curves of  $\mathcal{D}$ -elliptic sheaves*

Abstract: The notion of  $\mathcal{D}$ -elliptic sheaf is a generalization of the notion of Drinfeld module.  $\mathcal{D}$ -elliptic sheaves and their moduli schemes were introduced by Laumon, Rapoport and Stuhler in their proof of certain cases of Langlands conjecture over function fields.

We discuss basic arithmetic properties of modular curves of  $\mathcal{D}$ -elliptic sheaves and draw parallels with the theory of Shimura curves. In particular, we produce a genus formula for modular curves of  $\mathcal{D}$ -elliptic sheaves, examine the existence of rational points on these curve, compute their fundamental domains in Bruhat-Tits trees, and determine the cases when these curves are hyperelliptic. As applications of previous results, we construct new asymptotically optimal sequences of curves over finite fields (such sequences are

important in coding theory), and find presentations for certain arithmetic groups arising from quaternion algebras over function fields.

Speaker: **Federico Pellarin** (Universite Jean Monnet, St. Etienne)

Title: *On Mahler's method*

Abstract: *Mahler's method* is a technique to show the transcendency of values of certain transcendental functions at algebraic complex numbers that was introduced by Mahler in 1929. These functions must satisfy functional equations in a certain class. Although transcendency of values of these functions can be also reached by other techniques (eg. Schmidt's subspace theorem), a remarkable feature of Mahler's method is that it also delivers algebraic independence in quite a precise way.

Moreover, the method extends to values of certain functions at points which are algebraic over the field  $\mathbf{F}_q(\theta)$ . Surprisingly, it turns out in this context that certain values of these functions are also "periods" of Anderson's  $t$ -modules. This was first remarked by Denis, and used to show algebraic independence of Carlitz logarithms of elements of  $\mathbf{F}_q(\theta)$ .

The aim of this lecture is to give a survey of this method, its interesting features, and its limits.

Speaker: **Richard Pink** (ETH Zürich)

Title: *TBA*

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Title: *TBA*

Speaker: **Lenny Taelman** (Mathematisch Instituut Leiden)

Title: *The Mordell-Weil and Tate-Shafarevich modules of a Drinfeld module*

Abstract: I will propose a definition of the Mordell-Weil and Tate-Shafarevich modules of a Drinfeld module with everywhere good reduction. These will be finitely generated, and finite, respectively. Moreover, I will explain how these can be used to give a class-number style formula for a certain special value of the Goss  $L$ -function of the Drinfeld module.

Speaker: **Dinesh Thakur** (University of Arizona)

Title: *Automata methods in transcendence*

Speaker: **Dinesh Thakur** (University of Arizona)

Title: *TBA*

Speaker: **Seidai Yasuda** (Kyoto University)

Title: *Regulators, periods, and special values of automorphic  $L$  functions over function fields.*

Abstract: In this talk, we give two formulas on special values on automorphic  $L$  functions over  $GL_n$  over function fields. The first formula, obtained in a joint work with S. Kondo, involves a regulator map on a Drinfeld modular variety which establishes a function-field analogue of Beilinson's conjecture. The second one involves an integral over the diagonal torus of a cusp form on  $GL_n$  over the rational function field.

Speaker: **Jing Yu** (National Taiwan University)

Title: *On transcendence theory of Drinfeld modules*