

Derived Category Methods in Commutative Algebra II

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This workshop was the organizers’ second meeting at BIRS to work on their book *Derived Category Methods in Commutative Algebra* which will be published by Springer-Verlag. Unfortunately, illness hindered Foxby’s participation in the workshop.

1 Background

In our report on the first “Derived Category Methods in Commutative Algebra” workshop (2008) we wrote:

“Derived category methods have proved to be very successful in ring theory, in particular in commutative algebra. Evidence is provided by [1, 8, 4, 5, 6, 7, 11, 12, 15, 16, 19, 20, 23, 24, 27], to list some work of considerable importance.

Surprisingly, there is no accessible introduction or reference to the applications of derived category methods in commutative algebra, or in general ring theory for that matter. To be an effective practitioner of these methods, one must be well-versed in a series of research articles and lecture notes, including unpublished ones: [10, 14, 17, 16, 22, 25, 28, 13, 3, 2, 9, 18, 29]. To get an overview of their applications in commutative algebra, the list grows further. The purpose of the BIRS workshop was to make progress on a book manuscript—authored by L.W. Christensen, H.-B. Foxby, and H. Holm—that will remedy this deficiency.

As implied in the discussion above, the book has no direct competition. Many books cover applications of classical homological algebra in (commutative) ring theory, but only a few books address derived category methods and their applications in this field: In *Homological Algebra* [9] by Cartan and Eilenberg, resolutions of complexes and derived functors are briefly discussed in the final chapter; no applications are given. In Weibel’s *An introduction to homological algebra* [30], derived categories are introduced in the final chapter; a few applications to ring theory are included as exercises. Derived categories are also covered in *Methods of Homological Algebra* [21] by Gelfand and Manin, but applications to ring theory are not. A very thorough construction of derived categories is given in *Categories and Sheaves* [26] by Kashiwara and Schapira. However, the aim of [26] is sheaf theory, so beyond the construction of derived categories, there is barely any overlap with this book. Finally, Christensen’s *Gorenstein Dimensions* [10] has an appendix on derived category methods. It provides a rudimentary and incomplete survey of technical results without proofs. The fact that it has, nevertheless, become a frequently cited reference betrays a significant gap in the existing literature.”

Goals and results

The purpose of the workshop was for the authors to finalize the central chapters of their book “Derived Category Methods in Commutative Algebra”. As the authors live on different continents, and in different

time zones, the extended face-to-face interaction afforded by the workshop was extremely important for resolving scientific as well as editorial questions. It is a great pleasure to thank BIRS for providing this opportunity.

The workshop served three main purposes:

- (A) To coalesce the material contributed by each author since their previous meeting
- (B) To finalize the construction of index, glossary and other structural elements of the book
- (C) To distribute exercises between the sections in the book

Ad (A). Since the authors last met in May 2010, they had rewritten the central chapters on “Modules and Homomorphisms”, “Complexes and Morphisms”, “Derived Functors”, and “A Brief for Commutative Ring Theorists.” In the course of the workshop, the authors discussed this material, and decided on the final contents.

Ad (B). Principles for indexing were decided on and implemented and guidelines for the List of Symbols and the Glossary were “tested”.

Ad (C). As the manuscript is shaping up to be a genuine graduate textbook, it was decided to add a significant number of exercises. The process of choosing exercises was started this summer and continued at BIRS.

By the end of the workshop, the authors distributed new tasks and revised the time table for completion of the book. The book will be completed within the next year, and it will be published by Springer-Verlag. Finally, the latest version of the manuscript and a revised schedule were sent to Springer-Verlag.

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