Modeling annual maximum 24 hour precipitation in Iceland

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Introduction

A Bayesian hierarchical model is developed for the prediction of annual maximum 24 hour precipitation in Iceland. This model is applied to observed data from 86 observation sites in Iceland over the years 1961 to 2006 and it uses other meteorological explanatory variables.

Model assumptions

The data are modeled with a hierarchical linear model assuming a log-normal distribution for the observations.
Let y_{it} be the log-transformed annual maximum 24 hour precipitation at station i at year t. Let α_i be the site effect for each site i and β be a common trend parameter. Let γ_t be the overall time effect at time t. The following model is proposed

$$y_{it} = \alpha_i + \beta(t - t_0) + \gamma_t + \epsilon_{it} \qquad i = 1, \dots, J \quad t = 1, \dots, T$$

where J is the number of sites, T is the number of years, t_0 is the median of the time period. And the terms ϵ_{it} are independent deviation terms for station i at time t where

$$\epsilon_{it} \sim \mathcal{N}(0, \sigma_\varepsilon^2).$$

The distributional assumptions can be written as

$$y_{it} \sim \mathcal{N}\left(\alpha_i + \beta(t - t_0) + \gamma_t, \sigma_\varepsilon^2\right)$$

Bayesian inference

On the second level in the hierarchical model we assume the following prior distributions

$$\alpha \sim \mathcal{N}(X\eta, \Sigma_\alpha) \quad \text{where} \quad \Sigma_{\alpha,ij} = \sigma_\alpha^2 \exp\left(-\phi_\alpha^* d_{ij}\right)$$
$$\gamma \sim \mathcal{N}(0, \Sigma_\gamma) \quad \text{where} \quad \Sigma_{\gamma,kl} = \sigma_\gamma^2 \exp\left(-\phi_\gamma^* |t_k - t_l|\right)$$
$$\beta \sim \mathcal{N}\left(\mu_\beta, \tau_\beta^2\right).$$

where X is a matrix which contains the meteorological explanatory variables, η is a vector of parameters corresponding to these varibles and d_{ij} denotes the distance in kilometers between site i and j. The parameters $\sigma_\varepsilon^2, \sigma_\alpha^2, \sigma_\gamma^2, \eta, \phi_\alpha^*$ and ϕ_γ^* are unknown need to be given a prior distribution.
On the third level in the hierarchical model we assume the following prior distributions

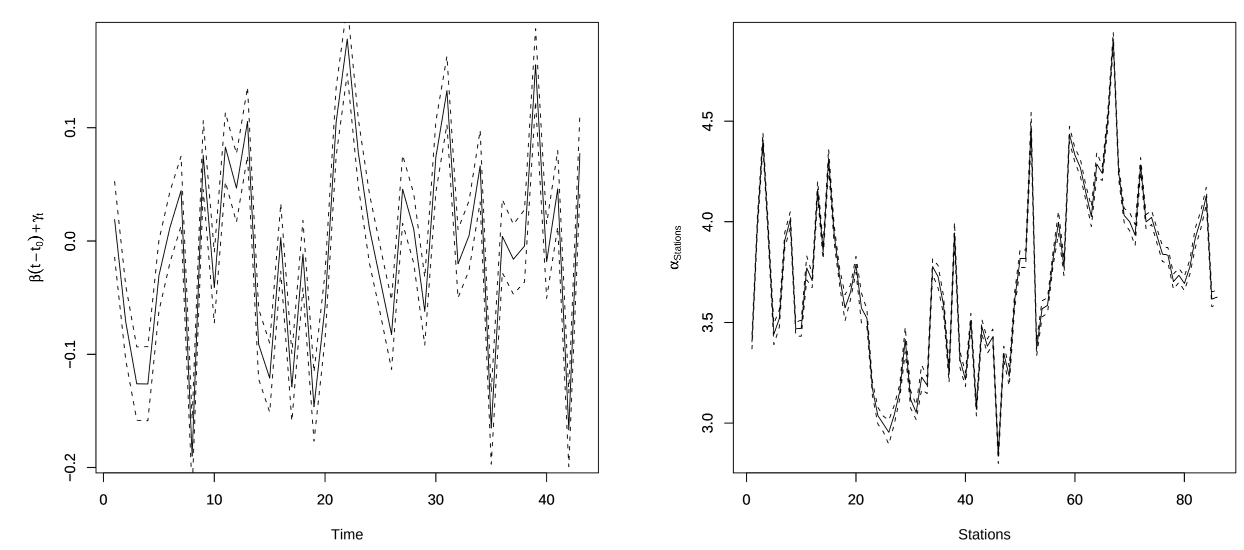
$$\sigma_\varepsilon^2 \sim \text{Inv}_{\chi^2}\left(V_\epsilon, S_\epsilon^2\right), \; \sigma_\alpha^2 \sim \text{Inv}_{\chi^2}\left(V_\alpha, S_\alpha^2\right),$$
$$\sigma_\gamma^2 \sim \text{Inv}_{\chi^2}\left(V_\gamma, S_\gamma^2\right), \eta \sim \mathcal{N}(\mu_\eta, diag(\tau_\eta^2))$$
$$\log(\phi_\alpha^*) \sim \mathcal{N}(\mu_{\phi\alpha}, \tau_{\phi\alpha}^2), \; \log(\phi_\gamma^*) \sim \mathcal{N}(\mu_{\phi\gamma}, \tau_{\phi\gamma}^2).$$

The remaining parameters in the distribution are constants which need to be specified.

Parameter estimation

The Gibbs sampler is used to estimate model parameters. The following posterior estimates are obtained for $\beta(t - t_0) + \gamma_t$ and α_i respectively.

$\beta(t - t_0) + \gamma_t$ $\qquad$ α_i



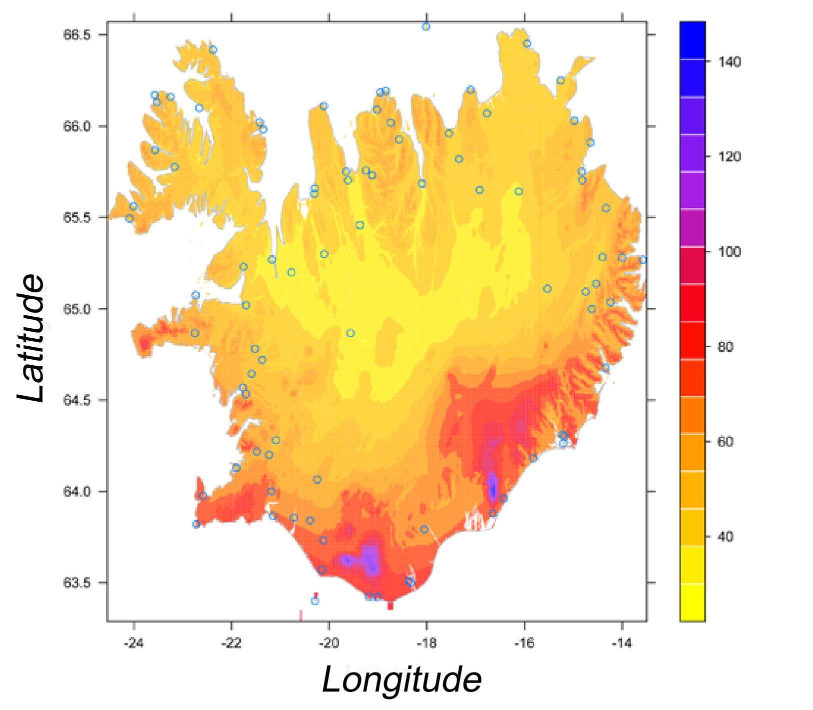
Posterior **2.5%** quantiles, means and **97.5%** quantiles are shown

parameters	**2.5%** quantile	mean	**97.5%** quantile
β	-0.00069	0.00021	0.00113
σ_ε^2	0.09268	0.09441	0.09613
σ_α^2	0.27255	0.37965	0.45551
σ_γ^2	0.05219	0.15821	0.20293
η_1 (intercept)	16.32845	17.35563	18.36950
η_3 (latitude)	-0.22144	-0.20571	-0.18996
η_4 (altitude)	-0.00020	0.00039	0.00101
η_5 (dst. sea)	-0.00928	-0.00700	-0.00477
$\log(\phi_\alpha^*)$	-0.67209	-0.04521	0.51864
$\log(\phi_\gamma^*)$	0.57592	0.92829	1.20800

Posterior estimates for other parameters in the model

Prediction

A posterior prediction for the mean of annual maximum 24 hour precipitation can now be obtained by using the posterior estimates of the parameters and known facts[1].



Predictions for the mean of annual maximum 24 hour precipitation for the year 2010 in Iceland

Future research

The proposed model works well when dealing with means, but when dealing with large extremes the model values deviate significantly from observed values, so improvements need to be made. In order to improve the model the following is proposed.

- Assume the general extreme value distribution for the observations, which is more suitable for dealing with extremes.
- Incorporate outputs from meteorological models.

References

1. Banerjee, Carlin, Gelfand. (2004). *Hierarchical Modeling and Analysis for Spatial Data*. Chapman & Hall/CRC.