Elements of a GraviGUT

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Outline

Wu-Yang Dictionary

	Physics	Mathematics
A, ∇	Yang-Mills field	connection in P
ψ	matter field in irrep V	section of $P \times_G V$
G	gauge group	structure group of P
$\mathcal{G}$	gauge group	vertical automorphisms of P
σ	Goldstone bosons	sections of $P \times_G G/H$

Wu-Yang Dictionary

An H structure is a principal H subbundle $Q \subset P$.
If P has an H structure, one can find a bundle atlas with transition functions in H.
P has an H structure iff there exists a global section $\sigma_0 \in C^\infty(P \times_G G/H)$.
Its local representative is $\sigma_0(x) = eH$ in all charts.

Remark: $\mathcal{G}$ acts transitively on $C^\infty(P \times_G G/H)$.
I will call *Goldstone boson* a field whose configuration space carries a transitive $\mathcal{G}$ action.

Higgs mechanism I

Certain gauge fields are experimentally seen to be massive.
How to reconcile a mass with gauge invariance?

- Higgs field $\phi = C^\infty(P \times_G V)$
- W a G-invariant potential with minimum in $G/H \subset V$
- $P \times_G G/H \subset P \times_G V$
- choose coordinates in V: $\phi = (\rho, \sigma)$
- dynamics gives $\langle\phi\rangle = (\rho_0, \sigma)$
- can choose "unitary" gauge $\phi = (\rho, \sigma_0)$

Higgs mechanism II

- $\mathcal{L}(G) = \mathcal{L}(H) \oplus \mathcal{P}$
- $A = A|_{\mathcal{L}(H)} + A|_{\mathcal{P}}$
- $D\phi = \partial\phi + A\phi = (\partial\rho, D\sigma)$
- with $D\sigma = \partial\sigma + A^i K_i(\sigma)$
- in particular $D\sigma_0 = A^i|_{\mathcal{P}} K_i(\sigma_0)$
- in unitary gauge $(D\phi)^2 \mapsto (\partial\rho)^2 + \rho_0^2 (A|_{\mathcal{P}})^2$
- Higgs field $\rho - \rho_0$ also has mass $\approx \rho_0$

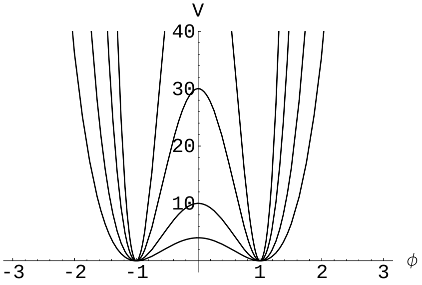
Higgsless Higgs mechanism

- only Goldstone bosons are necessary for Higgs mechanism
- Higgs particle only necessary for perturbative renormalizability

Strong Coupling limit

$W = \frac{\lambda}{4}(\phi^2 - \rho_0^2)^2$

$\lim_{\lambda \to \infty} W$ with ρ_0=const



Low Energy EFT

at momenta $p \ll \rho_0$:

- $\rho = \rho_0$
- $A|_{\mathcal{P}} = 0$ or $D\sigma = 0$

Let P have an H-structure Q and σ be the corresponding section of $P \times_G G/H$. A connection A in P is an H connection iff $D\sigma = 0$.

Grand Unification

use Higgs phenomenon with $G_1 \times G_2 \subset G$
to do list:

- identify GUT group G
- fit particles in irreps of G
- write $\mathcal{G}$-invariant action
- explain symmetry breaking (select order parameter, orbit, potential)
- check that new particles not seen at low energy have high mass

Grand unification: $SO(10)$

$$(e_L\ ,\nu_L\ ,e_R\ ,\nu_R\ ,u_L^{r,g,b}\ ,d_L^{r,g,b}\ ,u_R^{r,g,b}\ ,d_R^{r,g,b})$$

16 complex 2 component Weyl spinors of Lorentz
4 doublets and 8 singlets of $SU(2)_L$
Repeat three times. ($n_\nu = 2.984 \pm 0.008$ measured at LEP)

Fit exactly in the **16** of $SO(10)$!

A symmetry breaking chain

$$
\begin{array}{ccc}
SO(10) & & \\
\downarrow & & \\
SO(4) \times SO(6) & \approx & SU(2)_R \times SU(2)_L \times SU(4) \\
& & \downarrow \\
& & SU(2)_R \times SU(2)_L \times SU(3)_C \times U(1)_{B-L} \\
& & \downarrow \\
& & U(1)_{EM} \times SU(2)_L \times SU(3)_C \\
& & \downarrow \\
& & U(1)_{EM} \times SU(3)_C
\end{array}
$$

requires *at least* **45**, **16** and **10**

Gravity with more Variables

Spacetime manifold M, dim$M = 4$
E real vectorbundle with fiber dimension 4
local bases $\{\partial_\mu\}$ in TM and $\{e_a\}$ in E

- pseudo-fiber metric in E, γ_{ab} signature $+,+,+,-$
- linear connection in E, $A_\mu{}^a{}_b$
- soldering form $\theta^a{}_\mu$, det$\theta \neq 0$

Induced structures in *TM*

- $g_{\mu\nu} = \theta^{a}{}_{\mu}\,\theta^{b}{}_{\nu}\,\gamma_{ab}$
- $\Gamma_{\lambda}{}^{\mu}{}_{\nu} = \theta^{-1}{}_{a}{}^{\mu} A_{\lambda}{}^{a}{}_{b} \theta^{b}{}_{\nu} + \theta_{a}^{-1\,\mu} \partial_{\lambda} \theta^{a}{}_{\nu}$

Torsion and Nonmetricity

- $\Theta_\mu{}^a{}_\nu = \partial_\mu \theta^a{}_\nu - \partial_\nu \theta^a{}_\mu + A_\mu{}^a{}_b\, \theta^b{}_\nu - A_\nu{}^a{}_b\, \theta^b{}_\mu$
- $\Delta_{\lambda ab} = -\partial_\lambda \gamma_{ab} + A_\lambda{}^c{}_a\, \gamma_{cb} + A_\lambda{}^c{}_b\, \gamma_{ac}$

Gauge invariance

$$\mathcal{G} = \mathcal{A}ut^{GL(4)}E$$

$$\begin{aligned}
\theta^a{}_\mu(x) &\mapsto \theta'^a{}_\mu(x') = \Lambda^{-1a}{}_b(x)\,\theta^b{}_\nu(x)\frac{\partial x^\nu}{\partial x'^\mu} \\
\gamma_{ab}(x) &\mapsto \gamma'_{ab}(x') = \Lambda^c{}_a(x)\,\Lambda^d{}_b(x)\,\gamma_{cd}(x) \\
A_\mu{}^a{}_b(x) &\mapsto A'_\mu{}^a{}_b(x') = \frac{\partial x^\nu}{\partial x'^\mu}\big(\Lambda^{-1a}{}_c(x)A_\nu{}^c{}_d(x)\Lambda^d{}_b(x) \\
&\qquad +\Lambda^{-1a}{}_c(x)\partial_\nu\Lambda^c{}_b(x)\big)
\end{aligned}$$

$$0 \to \mathcal{A}ut^{GL(4)}_M E \to \mathcal{A}ut^{GL(4)}E \to \mathcal{D}iff M \to 0$$

is split: $\theta_* : \mathcal{D}iff M \to \mathcal{A}ut^{GL(4)}E$
$\theta_*(f) = \theta \circ Tf \circ \theta^{-1}$

Goldstone Bosons

$\mathcal{A}ut^{GL(4)}E$ acts transitively on metric and soldering

$\gamma(x) \in GL(4)/SO(3,1)$

$\gamma \in \{\text{fibermetrics}\} \approx \mathcal{A}ut^{GL(4)}E/\mathcal{A}ut^{SO(3,1)}E$

$\theta \in \{\text{isomorphisms } TM \to E\} \approx \mathcal{A}ut^{GL(4)}E/\mathcal{D}iffM$

Metric gauge

$$\theta^a{}_\mu = \delta^a_\mu$$

unbroken group $\mathcal{D}iffM$

$g_{\mu\nu} = \gamma_{\mu\nu}$, $\Gamma_\lambda{}^\mu{}_\nu = A_\lambda{}^\mu{}_\nu$

$\Theta_\mu{}^a{}_\nu = \Gamma_\mu{}^a{}_\nu - \Gamma_\nu{}^a{}_\mu$

Vierbein gauge

$$\gamma_{ab} = \eta_{ab}$$

unbroken group $\mathcal{A}ut^{SO(3,1)}M$

$g_{\mu\nu} = \theta^a{}_\mu \, \theta^b{}_\nu \, \eta_{ab}$

$\Delta_{\lambda ab} = A_{\lambda ab} + A_{\lambda ba}$

not enough freedom to fix both simultaneously

First Hint of a Higgs Mechanism

flat background: $A = 0$, $\theta = \mathbf{1}$, $\gamma = \eta$

Palatini action

$$S_P(A, \gamma, \theta) = \frac{1}{16\pi G} \int d^4x \, \sqrt{|\det g|} \, {\theta^{-1}}_a{}^\mu \theta^b{}_\rho \, g^{\rho\nu} F_{\mu\nu}{}^a{}_b$$

contains

$$\frac{1}{16\pi G} \int d^4x \, \delta_a{}^\mu \delta^b{}_\rho \, \delta^{\rho\nu} (A_\mu{}^a{}_c A_\nu{}^c{}_b - A_\nu{}^a{}_c A_\mu{}^c{}_b)$$

Levi–Civita Connection

given θ, γ, there is a unique $\bar{A}$ s.t. $\bar{\Theta} = 0$, $\bar{\Delta} = 0$

$$\bar{A} = \frac{1}{2}\left(\theta^{-1}{}_c{}^\lambda\, \partial_\lambda \kappa_{ab} + \theta^{-1}{}_a{}^\lambda\, \partial_\lambda \kappa_{bc} - \theta^{-1}{}_b{}^\lambda\, \partial_\lambda \kappa_{ac}\right) + \frac{1}{2}\left(C_{abc} + C_{bac} - C_{cab}\right)$$

where $C_{abc} = \gamma_{ad}\, \theta^d{}_\lambda \left(\theta^{-1}{}_b{}^\mu\, \partial_\mu \theta^{-1}{}_c{}^\lambda - \theta^{-1}{}_c{}^\mu\, \partial_\mu \theta^{-1}{}_b{}^\lambda\right)$

Any connection A can be split uniquely in $A = \bar{A} + \Phi$

then $S(A, \gamma, \theta) = S(\bar{A}(\theta, \gamma) + \Phi, \theta, \gamma) = S'(\Phi, \theta, \gamma)$

Example: Einstein Theory

$$S(A,\gamma,\theta) = S_P(A,\gamma,\theta) + S_{\mathrm{m}}(A,\gamma,\theta)$$

where

$$\begin{aligned} S_{\mathrm{m}} &= \int d^4x \sqrt{|\det g|}\big[A^{\mu}{}_{a}{}^{\nu\rho}{}_{b}{}^{\sigma}\Theta_{\mu}{}^{a}{}_{\nu}\Theta_{\rho}{}^{b}{}_{\sigma} \\ &\quad + B^{\mu ab\nu cd}\Delta_{\mu ab}\Delta_{\nu cd} + C^{\mu}{}_{a}{}^{\nu\rho cd}\Theta_{\mu}{}^{a}{}_{\nu}\Delta_{\rho cd}\big] \end{aligned}$$

Rewrite

$$\begin{aligned}
\Theta_{\mu}{}^{a}{}_{\nu} &= \Phi_{\mu}{}^{a}{}_{\nu} - \Phi_{\nu}{}^{a}{}_{\mu} \\
\Delta_{\mu ab} &= \Phi_{\mu ab} + \Phi_{\mu ba} \\
F_{\mu\nu}{}^{a}{}_{b} &= \bar{F}_{\mu\nu}{}^{a}{}_{b} + \bar{\nabla}_{\mu}\phi_{\nu}{}^{a}{}_{b} - \bar{\nabla}_{\nu}\phi_{\mu}{}^{a}{}_{b} + \phi_{\mu}{}^{a}{}_{c}\,\phi_{\nu}{}^{c}{}_{b} - \phi_{\nu}{}^{a}{}_{c}\,\phi_{\mu}{}^{c}{}_{b}
\end{aligned}$$

therefore

$$S(A,\gamma,\theta) = S(\bar{A}+\Phi,\gamma,\theta) = S_H(\gamma,\theta) + S_Q(\Phi,\gamma,\theta)$$

where

$$S_Q(\phi,\gamma,\theta) = \frac{1}{2}\int d^4x\ \sqrt{|\det g|}\ Q^{\mu}{}_{a}{}^{b\nu}{}_{c}{}^{d}\,\Phi_{\mu}{}^{a}{}_{b}\,\Phi_{\nu}{}^{c}{}_{d}$$

Gravitational Higgs Phenomenon

- gravity is a gauge theory of $GL(4)$ with two Goldstone bosons
- there are two unitary gauges
- Higgs phenomenon occurs at Planck scale, giving mass to Φ
- at low energy $A = \bar{A}(\theta, \gamma)$
- $\Theta = 0$ and $\Delta = 0$ are like condition $D\sigma = 0$
- in a unitary gauge A is a function of the surviving Goldstone boson (e.g. in metric gauge $A_\mu{}^\lambda{}_\nu = \{_\mu{}^\lambda{}_\nu\}$) (similar to CP^n etc)
- if we add F^2 more complicated but still at low energy torsion and nonmetricity can be neglected

GraviGUT

use gravitational Higgs phenomenon to construct unified theory of gravity and all other interactions.
to do list:

- identify GraviGUT group G
- fit particles in irreps of G
- write $\mathcal{G}$-invariant action
- explain symmetry breaking (select order parameter, orbit, potential)
- check that new particles not seen at low energy have high mass

General Framework for Unification

keep dim$M$=4, enlarge fibers of E to have dimension $N > 4$

gauge theory of $GL(N)$

$$\theta = \begin{bmatrix} \mathbf{1}_4 \\ 0 \end{bmatrix} \quad , \quad \gamma = \begin{bmatrix} g & 0 \\ 0 & \mathbf{1}_{N-4} \end{bmatrix}$$

$$A_\lambda = \begin{bmatrix} A_\lambda^{(4)} & H_\lambda \\ K_\lambda & A_\lambda^{(N-4)} \end{bmatrix}$$

give mass to $A^{(4)}$, H, K, symmetric part of $A^{(N-4)}$

unbroken $O(N-4)$

Fermions I

F. Nesti, R.P., Phys. Rev. D 81, 025010 (2010) arXiv:0909.4537 [hep-th]

Assume $\gamma_{ab} = \eta_{ab}$, $G = SO(3, 11)$

In $SO(10)$ GUT one family is
$\eta \in \mathbf{2_C} \times \mathbf{16_C}$ of $SO(3, 1) \times SO(10)$

Fermions II

Let $B\Sigma_{ij}^* = \Sigma_{ij} B$ and $\psi^c = B\psi^*$. Define $\psi_\pm$ by $(\psi_\pm)^c = \pm\psi_\pm$ (Majorana spinors)
Define $\psi_{L/R}$ by $\hat{\gamma}\psi_{L/R} = \mp\psi_{L/R}$ (Weyl spinors).
In signature $(3, 11)$ $[\hat{\gamma}, B] = 0$ so we can define Majorana-Weyl spinors $\psi_{L/R\pm}$. These have 64 real components.
Decomposing ψ_{L+} under $SO(3, 1) \times SO(10) \subset SO(3, 11)$ we find it is equivalent to η.

Remark: $SO(1, 13)$ has Weyl $\mathbf{64_C}$
decomposing as $\mathbf{64_C} = \mathbf{2_C} \times \mathbf{16_C} + \overline{\mathbf{2}}_\mathbf{C} \times \overline{\mathbf{16}}_\mathbf{C}$

Fermions III

$$D_\mu \psi_{L+} = \left(\partial_\mu + \frac{1}{2} A_\mu^{ij} \Sigma_{L\, ij}^{(3,11)} \right) \psi_{L+}$$

let $\Sigma_{ij}^\dagger A = -A\, \Sigma_{ij}$

then $\psi_{L+}^\dagger (A\gamma^i)_L D\psi_{L+}$ is one-form in **14** of $SO(3,11)$

$$\mathcal{S} = \int \psi_{L+}^\dagger (A\gamma^i)_L D\psi_{L+} \wedge \theta^j \wedge \theta^k \wedge \theta^\ell\, \phi_{ijk\ell}\,.$$

Fermions IV

Assuming the following VEVs:

$$\begin{cases} \phi_{mnrs} = \epsilon_{mnrs} \\ \phi_{ijk\ell} = 0 \quad \text{otherwise} \end{cases} \qquad \begin{cases} \theta_\mu^m = M e^m{}_\mu \\ \theta_\mu^a = 0 \quad \text{otherwise} \end{cases}$$

we find after some work

$$S = \int d^4x\, \eta^\dagger \sigma^\mu \nabla_\mu \eta\,,$$

where now $\nabla_\mu = D_\mu^{(10)} = \partial_\mu + \frac{1}{2} A^{ab}_{\mu\,(10)} \Sigma^{(10)}_{ab}$

Where do we stand?

- kinematics well understood
- bosonic action for broken phase can be written
- hard to write action that works in both phases
- fermionic content and dynamics ok, with caveat
- further breaking of $SO(10)$ unaccounted for
- all this is still classical....