Quantum Cohomology via the Linear Sigma Model

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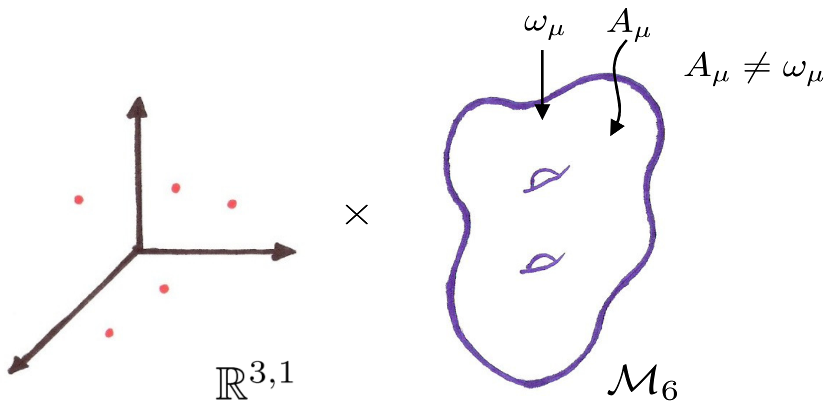
0712.3272, 0810.0012

Motivation

- String compactifications are an easy route to embedding four-dimensional physics in a ten-dimensional string theory.
- Cartoon of heterotic compactification:



- At large volume, physics amounts to choice of geometry and vector bundle $\mathcal{E}$. Supergravity description well-studied. What about the worldsheet?

Motivation

- Standard embedding ($A_\mu = \omega_\mu$) => we are in good shape:
  - Spacetime low energy effective field theory: unbroken $E_6 \times E_8$ gauge group, 27 and $\overline{27}$ matter multiplets, moduli
  - Worldsheet is (2,2) SCFT e.g. can compute $27^3, \overline{27}^3$ Yukawa couplings; special geometry; and mirror symmetry
- Quantum corrections are important in (2,2) models:
  - Lead to interesting physics. To name a few: topology change, resolution of singularities, modification of Yukawa couplings etc.
  - Cohomology rings are modified -> quantum cohomology rings. Interesting mathematics & important for considerations of mirror symmetry.
- What about quantum corrections in (0,2) models?

Motivation: (0,2) Compactifications

- Not known how to compute quantum corrections in many (0,2) theories

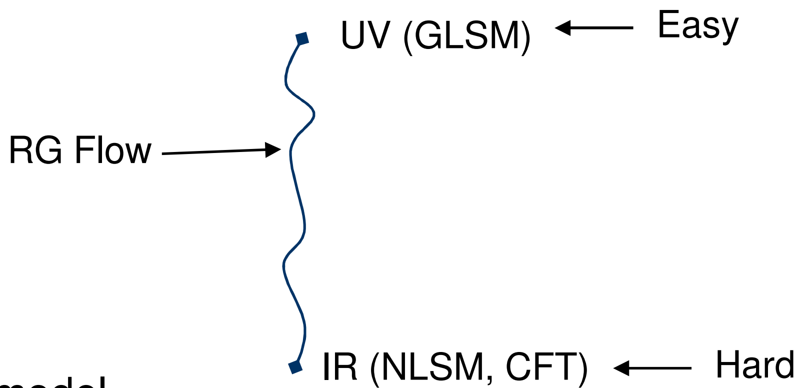
  Lots of open questions:
  - What is the moduli space of (0,2) SCFTs? Where are they singular?
  - What are the Yukawa couplings? What are the quantum cohomology rings?
  - Is there a notion of heterotic mirror symmetry? Is there special geometry?
- We analyze these questions for (0,2) models where bundle $\mathcal{E}$ is a small deformation from TV
- A priori expectations:
  - Worldsheet: Break (2,2) SUSY to (0,2) SUSY. How much control over dynamics do we retain?
  - Spacetime: a benign deformation, wiggling the bundle. Many results (e.g. Yukawa couplings) vary smoothly with moduli
- What works for (2,2) works for (0,2)?
  - Results indicate this is the case. Even though method of proof different
  - Deformations are finite, but still small. Picture is “local”

Outline

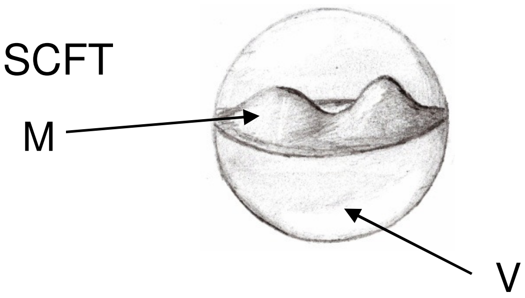
☑ 1. Motivation: How much do we know about the Heterotic String?
2. **(0,2) GLSMs**
3. A/2-Twist V-Model (toric varieties – a good warm-up)
4. A/2-Twist M-Model (Calabi-Yau's – Yukawa couplings)
5. B/2-Twist M-Model (LG theories)
6. Summary & Conclusion

Our Playground: Gauged Linear Sigma Model (GLSM)

- 2D abelian gauge theory
- Why is the GLSM useful?
  - GLSM quick route to generating and computing in CFTs and NLSMs
- Can do half-twist on the GLSM
  - (0,2) analogues of the A-model and B-model
  - Compute RG invariant properties of physical theories *exactly*



- We will consider two classes of models:
  - *V-Model:* Toric Variety V (e.g. $\mathbb{P}^4$) -> NLSM
  - *M-Model:* *CY* Hypersurfaces in V (e.g. quintic in $\mathbb{P}^4$) -> SCFT
- (0,2) Deformations come in two varieties:
  - E-deformations (deforming TV of toric variety V)
  - J-deformations (deformations not descending from TV)
- We'll compute the dependence of E and J in correlators, singularities

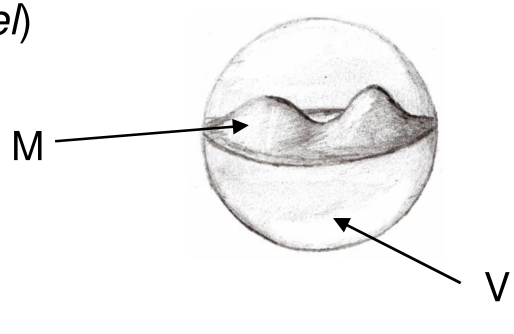


Recall the (2,2)-GLSM

- The (2,2) GLSM has an action $S = S_{\rm kin} + S_{\rm F-I} + S_{\rm W}$

$$
\begin{aligned}
S_{\rm kin} &= \int d^2y d^4\theta \sum_i \overline{\Phi}_i e^{2\sum_a Q_{i,a} V_a} \Phi^i - \sum_{a=1}^{r} \frac{1}{4} \int d^2 y d^4\theta \overline{\Sigma}_a \Sigma_a, \\
S_{\rm F-I} &= \tfrac{1}{4\pi} \int d^2 y d\theta^+ d\bar{\theta}^- \log(q_a) \Sigma_a|_{\overline{\theta}^+ = \theta^- = 0} + \text{h.c.}, \\
S_W &= - \int d^2 y d^2\theta W(\Phi)|_{\overline{\theta}^+ = \overline{\theta}^- = 0} + \text{h.c.}.
\end{aligned}
$$

  - $U(1)^r$ abelian gauge theory ($a = 1,\ldots,r$)
  - Φ^i homogenous coordinates of target space ($i=1,\ldots,n$)
  - $q^a = e^{-2\pi r_a + i\theta_a}$ FI parameters ⇔ Kähler moduli
  - $W = 0$: target space is toric variety (*V-model*)
  - $W \neq 0$: superpotential induces a hypersurface (*M-model*)



Review of (0,2) GLSM

- Consider (0,2) theories with a (2,2) locus. Field content easily understood by decomposing (2,2) multiplets $\Phi_{2,2} = \underbrace{\phi + \theta^+\psi_+ + \ldots}_{\Phi_{0,2}} + \underbrace{\theta^-\gamma_- - \theta^-\theta^+ F + \ldots}_{\Gamma_{0,2}}$

- More generally

(2,2) Field	Bosons	Fermions
Matter fields	Φ^i	Γ^i
Vector multiplet	$V_{\pm,a}$	
Field Strength	Σ_a	Υ_a

$i = 1, \ldots, n$

$a = 1, \ldots, r$

$\pm$ left- or right-moving

Left-moving heterotic fermions (Γ^i)

- $\Phi^i \Leftrightarrow$ target space coordinates & $\Gamma^i \Leftrightarrow$ bundle $\mathcal{E}$
  - Bundle fermions Γ^i obey a constraint: $\overline{\mathcal{D}}_+\Gamma^i = E^i(\Phi, \Sigma)$ ← Holomorphic function
  - E^i determines the behavior of the Γ^i bundle $\mathcal{E}$
  - Gives rise to (0,2) deformations

(2,2)

$$E^i \sim \sum_a Q_i^a \Phi^i \Sigma_a$$

$$0 \longrightarrow \mathcal{O}^r \xrightarrow{Q_i^a \phi^i} \oplus_i \mathcal{O}(D_i) \longrightarrow T_V \longrightarrow 0$$

⇒

(0,2)

$$E^i \sim \sum_{a,j} M^i{}_j^a \Phi^j \Sigma_a$$

$$0 \longrightarrow \mathcal{O}^r \xrightarrow{E} \oplus_i \mathcal{O}(D_i) \longrightarrow \mathcal{E} \longrightarrow 0$$

Review of (0,2) GLSM

- Action for (0,2) GLSM:

$$
\begin{aligned}
S_{\rm kin} &= \int d^2y d^2\theta \left\{ -\frac{1}{8e_0^2}\overline{\Upsilon}_a\Upsilon_a - \frac{i}{2e_0^2}\overline{\Sigma}_a\partial_-\Sigma_a - \frac{i}{2}\overline{\Phi}^i(\partial_- + iQ_i^a V_{a,-})\Phi^i - \frac{1}{2}\overline{\Gamma}^i\Gamma^i \right\}, \\
S_{\rm F-I} &= \frac{1}{8\pi i}\int d^2y d\theta^+ \Upsilon_a \log(q_a)|_{\overline{\theta}^+=0} + {\rm h.c.}, \\
S_F &= \int d^2y d\theta^+ \Gamma^i J_i(\Phi)|_{\overline{\theta}^+=0} + {\rm h.c.}.
\end{aligned}
$$

⟵ Matter superpotential

where $q^a = \exp(-2\pi r_a + i\theta_a)$ and $J_i(\Phi)$ are polynomial in the Φ^i

- On the (2,2) locus: $J_i = \frac{\partial W}{\partial \Phi^i}$.
- More generally, for (0,2) supersymmetry we require $\sum_i E^i J_i = 0$
- Consider first massive theories V-model where $J_i = 0$ followed by M-model (CICYs) where superpotential defines hypersurface M in V.

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Toric Varieties – V-model

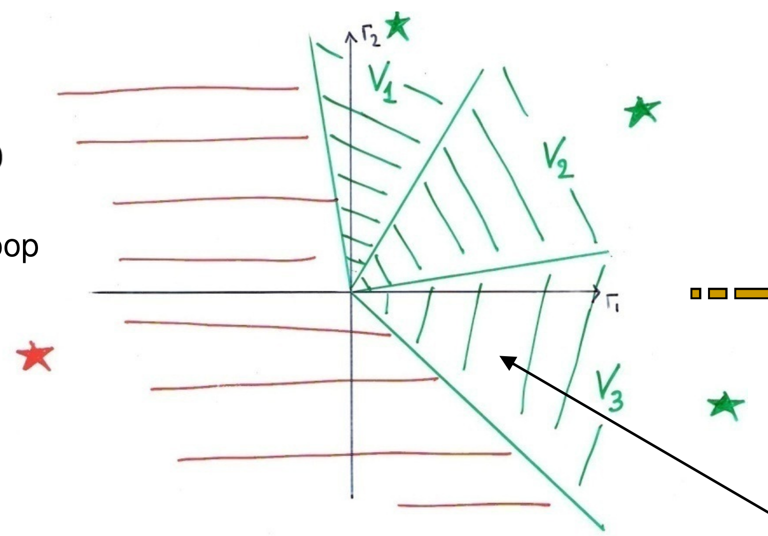
- First consider the (0,2) V-Model, W=0. Useful warm-up for M-modelAction splits as $S = S_{\mathrm{kin}} + S_{\mathrm{F-I}}$
  - Bosonic potential contains D-terms: $\sum_{a=1}^{r} \left(\sum_i Q_i^a |\phi_i|^2 - r^a \right)^2 = 0$
    r_a is FI parameter ~ Kähler modulus. Often write $q^a = e^{-2\pi r_a + i\theta_a}$
  - There exist many phases FI-parameter space (i.e. Kähler moduli space)

Higgs Branch $\langle \phi \rangle \neq 0$

D-terms have solution.
Geometric picture of toric variety V

Coulomb Branch $\langle \sigma \rangle \neq 0$

Dynamics captured by 1-loop Potential



Out here q -> 0
⇔
classical (supergravity) limit

Lots of quantum corrections here

What's an easy way to compute?

A/2-Twisted V-Model: An Easy Route to Correlators

- For (2,2)-theories, can do an A-twist
  - $Q_T = \overline{Q}_+ + Q_-$ BRST operator
  - Cohomology elements correspond to (1,1)-classes on V. Label them σ fields.
  - Stress Energy tensor is BRST exact => observables are RG invariant
- Correlators $\langle \sigma_1 \ldots \sigma_s \rangle$ may be computed by localization
  - Perturbative corrections cancel
  - Semi-classical analysis arbitrarily good
- Two methods:
  - ★ *Higgs Branch*: Summing gauge instantons
  - ★ *Coulomb branch*: 1-loop potential
- How does this change for (0,2)?

- For (0,2) theories, can do A/2-twist
  - $Q_T = \overline{Q}_+$ BRST operator
  - Cohomology elements are still σ
  - Theory not topological. Invariant under rescalings of the worldsheet metric => observables RG invariant
- Localization still applies
- ***Do the two methods still apply?***
  - ★ Higgs Branch
  - ★ Coulomb Branch
- ***If so, some more questions:***
  - ***Where are correlators singular?***
  - ***What is their moduli dependence?***

Review: Summing Gauge Instantons on (2,2)

- **First technique: ★ Higgs phase** $\langle\phi\rangle \neq 0$
- General considerations imply correlator given by sum over gauge instantons

$$\langle\sigma_1 \ldots \sigma_s\rangle = \sum_{\vec{n}} \langle\sigma_1 \ldots \sigma_s\rangle_{\vec{n}}\ q^{\vec{n}}$$ ← Kähler parameters

- Compute term-by-term in the instanton expansion. Correlators reduce to integration over zero modes

$$\langle\sigma_1 \ldots \sigma_s\rangle_{\vec{n}} = \int_{\mathcal{M}_{\vec{n}}} (\sigma_1 \ldots \sigma_s \chi_n)$$ ← Straightforward to compute using toric geometry

- σ_a map to (1,1)-classes on $\mathcal{M}_{\vec{n}}$, the space of zero modes
- Matter fields $\phi^i : \Sigma \to V$ are holomorphic maps of degree $d_i = \sum_a Q_i^a n_a$
- Moduli space of maps is a toric variety: $\mathcal{M}_{\vec{n}} = \frac{\mathbb{C}^N - F}{[\mathbb{C}^*]^r}$
- Euler class for obstruction bundle $\chi_{\vec{n}} = \prod_{i|d_i<0} \det(\sigma_a Q_i^a)^{-1-d_i}$

A/2 V-Model: Summing Gauge Instantons on (0,2)

- For (0,2) theories story is much the same
- Sum over instanton sectors, and answer reduces to an integral over zero modes. In instanton sector n:

$$\langle \sigma_1 \dots \sigma_s \rangle_{\vec{n}} = \int_{\mathcal{M}_{\vec{n}}} (\widetilde{\sigma}_1 \dots \widetilde{\sigma}_s \widetilde{\chi}_{\vec{n}})$$

← Now "sheafy" type objects. Hard?

  - For (2,2) theories, operators mapped to forms on the moduli space. Moduli space is toric & correlators reduce to toric intersection computations
  - For (0,2) theories, moduli space is unchanged. Operators now map to 1-forms valued in the bundle. What is the analogue of intersection theory in $H^*(V, \mathcal{E}^*)$?
- GLSM naturally generates toric like structures. Are there toric-like methods to compute this integral?

(0,2) Toric Intersection Theory

- Inspired by the (0,2) GLSM, conjecture "toric" methods for (0,2) theories
- Define some objects familiar to (2,2)/toric intersection theory:
  - π_i – Grassmannian object with bundle indices
  - $\tilde{\eta}_a$ – basis for $H^1(V, \mathcal{E}^*)$
  - $\tilde{\xi}_i = \pi_j \tilde{\eta}_a E_i^{aj}$ (analogous to $\xi_i = Q_i^a \eta_a$ in (2,2) models)
- Analogue of Stanley-Resiner relations $\prod_{i \in F} \tilde{\xi}_i = 0$ hold if $\tilde{\eta}_a = \eta_a$
- Normalisation of cup product: [(2,2) theories $\#(\widetilde{\xi}_{i_1} \cdots \widetilde{\xi}_{i_d}) = \int_V \widetilde{\xi}_{i_1} \wedge \cdots \wedge \widetilde{\xi}_{i_d}$]

$$\#(\widetilde{\xi}_{i_1} \cdots \widetilde{\xi}_{i_d}) \;=\; \#(\widetilde{\eta}_{a_1} \cdots \widetilde{\eta}_{a_d}) \, \#(\pi_{j_1} \cdots \pi_{j_d}) \; E_{i_1}^{a_1 j_1} \cdots E_{i_d}^{a_d j_d}$$

where

$$\begin{aligned} \#(\widetilde{\xi}_{i_1} \cdots \widetilde{\xi}_{i_d}) &= \det{}_p Q \\ \#(\pi_{j_1} \cdots \pi_{j_d})|_p &= \left|\det{}_p Q\right| \, \epsilon_{j_1 \cdots j_d j_{d+1} \cdots j_n} \, \left[\epsilon_{i_1 \cdots i_d i_{d+1} \cdots i_n}\right]^2 \; E_{i_{d+1}}^{1, j_{d+1}} \cdots E_{i_n}^{r, j_n} \end{aligned}$$

- Extra fermion zero modes can result in a factor of

$$\chi_{\vec{n}} = \prod_{i | d_i < 0} \det(\widetilde{\eta}_a Q_i^a)^{-1-d_i}$$

(0,2) Toric Intersection Theory

- End result:

$$\langle \sigma_{a_1} \cdots \sigma_{a_k} \rangle = \sum_{n \in \mathcal{K}^\vee} \#(\widetilde{\eta}_{a_1} \cdots \widetilde{\eta}_{a_k} \chi_n)_{\mathcal{M}_n} \prod_{a=1}^{r} q_a^{n_a}$$

- Checks:
  - Recover (2,2) result
  - $\#(\widetilde{\eta}_{a_1} \cdots \widetilde{\eta}_{a_d})$ match the $q_a \to 0$ (classical) limit of Coulomb branch analysis
  - Works in a number of non-trivial examples
- *Thus, we have conjectured generalisation of toric interesection theory.*
  - *Is there a mathematical proof?*
  - *Mathematical consequences?*

A/2 V-Model: Coulomb Branch

- **Second technique: ★ Coulomb Branch**
- Simple algebraic technique. Instantons are summed automatically
- Φ fields get massive and can be integrated out
- Dynamics completely determined by 1-loop superpotential

$$\mathcal{L}_{\text{eff}} = \int d\theta^+ \Upsilon_a \widetilde{J}^a + \text{h.c.} \quad \text{with} \quad \widetilde{J}^a = \log\left[\Pi_\alpha \left(\det M_{(\alpha)}\right)^{Q^a_{(\alpha)}} / q^a\right]$$

't Hooft anomaly matching and holomorphy implies 1-loop result is exact

- Vacua are discrete and located at points where

$$\Pi_\alpha \left(\det M_{(\alpha)}\right)^{Q^a_{(\alpha)}} = q_a$$

- Correlators may be evaluated by localization

$$\langle \sigma_1 \dots \sigma_s \rangle = \sum_{\sigma^*} \sigma_1 \dots \sigma_s \left[\det(\widetilde{J}_{a,b}) \Pi_\alpha \det M_{(\alpha)}\right]^{-1}$$

sum over Coulomb vacua (0,2) parameters

- Reproduces answer computed on Higgs branch

Example: Resolved $\mathbb{P}^4_{1,1,2,2,2}$

- Compute by Coulomb branch technique and gauge instanton sum
- For example:

$$\begin{aligned} \langle \sigma_1^3 \sigma_2 \rangle &= \frac{1}{D_2}, \\ \langle \sigma_1 \sigma_2^3 \rangle &= \frac{\epsilon_1^2 + \epsilon_2\epsilon_3(1-2\epsilon_1) + (6\epsilon_1 - 12\epsilon_2\epsilon_3 + 1)q_2 + 4q_2^2}{D_1^2 D_2}, \end{aligned}$$

Get instanton expansion by expanding in powers of q^a

ϵ_i are E-deformation parameters

$$\begin{aligned} D_1 &= 4q_2 - 1 \\ D_2 &= 1 + 2\epsilon_1 - 4\epsilon_2\epsilon_3 \end{aligned}$$

$$q_2 = e^{-2\pi r_2 + i\theta_2}$$

- Interesting singularity structure:
  - $D_1 = 0$ Kähler singularity. Familiar from (2,2)
  - $D_2 = 0$ Bundle singularity. Visible even when q -> 0 (large radius limit)
- In (0,2) parameter space -> find a new branch (mixed Coulomb-Higgs)
- Example of new structures present in the Heterotic bundle moduli space

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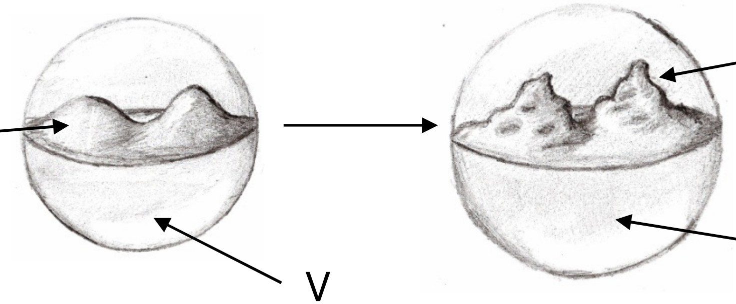
M-Model: Hypersurfaces & Calabi-Yau's

- To construct a Calabi-Yau, we add two additional multiple (Φ^0, Γ^0) . Then turn on a superpotential term:

$$S_J = \int d^2y d\theta^+ \Gamma^i J_i(\Phi)|_{\bar{\theta}^+=0} + \text{h.c.}.$$

$$J_i \sim \frac{\partial W}{\partial \phi^i} \quad \text{for (2,2)}$$



- Vacua:
  - D-terms => matter fields Φ^i parameterize V
  - F-terms => Imply constraints (e.g. P = 0). Defines a hypersurface $M \subset V$
- J functions give second type of (0,2)-deformations:

$$J_i = \frac{\partial W}{\partial \phi^i} \quad \rightarrow J_i = \frac{\partial W}{\partial \phi^i} + \sum a_{ijkl}\phi^i \phi^j \phi^k \phi^l$$

(2,2) (0,2) J-deformations (quintic)

  - *J-deformations* correspond geometrically to wiggling the hypersurface bundle
- To summarize (0,2)-deformations in M-model:
  - E-deformations from V
  - J-deformations from hypersurface

A-Twist of M-Model (CY Hypersurface)

- With (2,2)-supersymmetry the M-model admits an A-twist
- Similar to V-Model (toric variety):
  - Q_T cohomology given by $\sigma \leftrightarrow H^{1,1}(M)$ pullbacks of $H^{1,1}(V)$
  - Localization still works: correlators reduce to an integration over moduli space
- Some important twists:
  - Selection rule implies compute 3-point functions which are $\overline{27}^3$ Yukawa's
  - Vacuum equations are those of the V-model with additional constraints e.g. P=0
  - Defines a locus $\mathcal{M}_{n;P} \subset \mathcal{M}_n$. Tricky to compute gauge instantons on $\mathcal{M}_{n;P}$ (as opposed to $\mathcal{M}_n$ which is toric)
  - Looks hard to compute correlators in conformal models…
- All is not lost! Superpotential is Q_T exact. Correlators independent of details of the hypersurface (i.e. complex structure moduli)
- Implies M-model correlators (hard) may be related to V-model correlators (easy). Made precise by the *Quantum Restriction Formula:*

$$\langle\langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \rangle\rangle_M = \langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \frac{-K}{1-K} \rangle_V \quad -K = \sum_{i=1}^n Q_i^a \sigma_a$$

- Computations now simple! Does this work for (0,2) theories?

M-Model: Quantum Restriction Formula for (0,2)

- Some *a priori* considerations:
  - (0,2) Supersymmetry => only $\overline{J}_i$ BRST exact.
    Are correlators independent of all J-parameters? (e.g. may be holomorphic J dependence?)
  - Does the Quantum Restriction Formula still apply?
    (M-model correlators reduce to V-model correlators?)
- We show it does work for (0,2)
- By integrating out (Φ^0, Γ^0) fields

$$\langle\langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \rangle\rangle_{\vec{n}} = -\int D[\text{fields}]_{V;\mathcal{M}_n} e^{-S_V} e^{-[P\overline{P}]_0} [(-K)^{1-d_0} + g(J,\overline{P})] \sigma_{a_1} \cdots \sigma_{a_{d-1}},$$

  - $\overline{P}$ is BRST exact => does not formally affect correlators. Can take the limit $\overline{P} \to 0$ which implies $g(J,\overline{P}) \to 0$
  - As the moduli space & worldsheet are *compact*, this will not affect large field asymptotics
- Summing over instantons gives (0,2) Quantum Restriction Formula

$$\langle\langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \rangle\rangle_M = \langle \sigma_{a_1} \cdots \sigma_{a_{d-1}} \frac{-K}{1-K} \rangle_V \quad -K = \sum_{i=1}^n Q_i^a \sigma_a$$

- Important feature: J dropped out => A/2-twisted theory is *independent of complex structure and J-deformations*

M-Model: Quantum Restriction Formula

- Additional comments:
  - Related a M-model correlator (hard) to a V-model correlator (easy)
  - This gives rise to unnormalized Yukawa couplings in the SCFT
  - Can be extended to Complete Intersection Calabi-Yau's (CICY)
  - Independence of J-deformations important for any mirror symmetry considerations
- Let's compute an example….

M-Model Example: CY Hypersurface in resolved $\mathbb{P}^4_{1,1,2,2,2}$

- Same example consider previously. Hypersurface defined using a superpotential W. On the (2,2) locus W is:

$$W = \Phi_0 P(\Phi_1, \ldots, \Phi_6), \quad P = (\Phi_1^8 + \Phi_2^8)\Phi_6^4 + \Phi_3^4 + \Phi_4^4 + \Phi_5^4$$

- Applying our V-model techniques and Quantum Restriction we get $\overline{27}^3$ Yukawas:

$$\langle\langle \sigma_1^3 \rangle\rangle = \frac{8}{D_\epsilon}, \qquad \langle\langle \sigma_1^2 \sigma_2 \rangle\rangle = \frac{4(1-2^8 q_1)}{D_\epsilon},$$

$$\langle\langle \sigma_1 \sigma_2^2 \rangle\rangle = \frac{4(2^{10} q_1 q_2 - 2q_2 + 2^8 \epsilon_1 q_1 + 2\epsilon_2\epsilon_3 - \epsilon_1)}{(1-4q_2)D_\epsilon},$$

$$\text{where } D_\epsilon = (1-2^8 q_1)^2 - 2^{18} q_1^2 q_2 + 2\epsilon_1(1-2^8 q_1) - 4\epsilon_2\epsilon_3 = 0$$

- Interesting features:
  - Kähler and bundle moduli mixing -> treated on the same footing
  - Large volume limit q -> 0 -- still can get bundle moduli singularities
  - Easy to parameterize locus of points where SCFT is singular:

$$(1-2^8 q_1)^2 - 2^{18} q_1^2 q_2 = 0 \longrightarrow (1-2^8 q_1)^2 - 2^{18} q_1^2 q_2 + 2\epsilon_1(1-2^8 q_1) - 4\epsilon_2\epsilon_3 = 0$$

(2,2) (0,2)

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B/2-Twisted M-Model (CY Hypersurface)

- M-Model admits a B/2-twist
- On (2,2)-locus the B-Model has the following features:
  - BRST invariance => independent of Kähler parameters & no quantum corrections
  - Correlators depend holomorphically on complex structure moduli
  - Observables correspond to monomials in the superpotential e.g. $\mathcal{O} = \phi^0(\phi^i)^5$
  - Correlators compute 27^3 Yukawa couplings
- We show these features persist for a large class of (0,2)-models:
  - Fermion zero mode analysis => most models have no quantum corrections
  - In addition, if there is a Landau-Ginzburg phase (eg. quintic and $\mathbb{P}^4_{1,1,2,2,2}$):
    - Correlators do not depend E-deformations
    - Reduce to a Landau-Ginzberg computation, exactly as on the (2,2)-locus
  - Some models can not be ruled out from having instanton corrections

B/2-Twisted Model: Quantum Corrections?

- An example of a smooth M-model that is not ruled out by the zero-mode analysis. Charge matrix

$$Q = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{pmatrix}$$

  with polynomial $P = \phi_1^4 + \phi_2^4 + (\phi_3^4 + \phi_4^4 + \phi_3^2\phi_4^2)\phi_5^4 + (\phi_3^4 + \phi_4^4)\phi_6^4$

- *Further work is needed.*
  - *Possible resolution (inspired by E. Sharpe 2006): zero mode analysis not good enough; but path integral reduces to an exact form on a compact moduli space*

B/2-Twisted M-Model: Hypersurface in Resolved $\mathbb{P}^4_{1,1,2,2,2}$

- Do an example. This will be illustrative of how things work in general
- M-model for Resolved $\mathbb{P}^4_{1,1,2,2,2}$. Is independent of quantum corrections. Landau-Ginzburg phase: $r_2 < 0 \text{ and } 2r_1 + r_2 < 0$
- Consider $\langle\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle\rangle_M$ where $\mathcal{O}_a = \phi^0 f_a$, e.g. $\mathcal{O} = \phi^0 \phi_3^4$ ⇔ 27^3 Yukawa couplings
- Take $r_2 \sim -M^2 \text{ and } 2r_1 + r_2 \sim -M^2$ ⇔ Expanding $\mathcal{L}_{GLSM}$ deep in the LG phase
- Performing some field redefinitions, we show

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle_{GLSM} \quad = \quad \langle f_1 f_2 f_3 \rangle_{LG-Orb}$$

- In particular, the E-parameters drop out of the correlator!
- Thus, the B/2- theory depends only on *complex structure and J-deformations*
- *Further worked needed:*
  - *When is there a LG phase? Reformulation of this condition, as well as selection rules in terms of combinatorial data i.e. polytopes would be a good, useful start.*
  - *Better understanding of analogue of residue techniques for B/2-twisted models*

Outline

- ☑ 1. Motivation: How much do we know about the Heterotic String?
- ☑ 2. (0,2) GLSMs
- ☑ 3. A/2-Twist V-Model (toric varieties – a good warm-up)
- ☑ 4. A/2-Twist M-Model (Calabi-Yau's – Yukawa couplings)
- ☑ 5. B/2-Twist M-Model (LG theories)
- **6. Summary & Conclusion**

A/2-Twisted and B/2-Twisted Models: Mirrors?

- On the (2,2)-locus there is a well-developed notion of mirror symmetry. In the language of the GLSM it is quite pretty:

A-twisted M-model ⟺ B-twisted W-model

  - M and W are mirror Calabi-Yaus. Can be easily constructed via toric geometry (Batyrev, 1993, Borisov 1994)

Kähler moduli of M ⟺ *complex structure moduli* of W

  - In the GLSM this is the *'monomial-divisor mirror-map'*
- The results we've obtained here are suggestive of a natural generalization to (0,2) theories:

A/2-twisted M-model ⟺ ***B/2-twisted W-model***

Kähler + E-deformations ⟺ ***Complex structure + J-deformations***

- Is there a mirror map? For plain reflexive polytopes, this looks to be the case

Summary and Future Work

- We've explored some aspects of (0,2)-theories using half-twists

- Compute Yukawa couplings in a range of models via:
  - $\overline{27}^3$ Quantum Restriction Formula via A/2-twist
  - 27^3 Classical Intersection Theory via B/2-twist

- We find the moduli space splits in a nice way:
  - (Kähler + E-deformations) ⇔ (Complex Structure + J-deformations)
  - Interesting bundle singularities

- Many future directions
  - Understanding GLSM mirror map? How do cohomology rings map?
  - Kähler potential for the matter and moduli fields (normalize couplings). Is there a generalization of special geometry?
  - The most phenomenologically interesting vacua are rank 4 and rank 5 bundles. Does our analysis extend to these theories?

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