Banff International Research Station

for Mathematical Innovation and Discovery

"Generalized complex and holomorphic Poisson geometry"
Sunday, 11-04-2010 to Friday, 16-04-2010

MEALS

◦ Breakfast (Buffet): 07:00–09:00, Sally Borden Building, Monday–Friday
◦ Lunch (Buffet): 11:30–13:30, Sally Borden Building, Monday–Friday
◦ Dinner (Buffet): 17:30–19:30, Sally Borden Building, Sunday–Thursday
◦ Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

∗ *Please remember to scan your meal card at the host/hostess station in the dining room for each meal.* ∗

MEETING ROOMS

All the prescheduled lectures will be held in Max Bell 159. The Max Bell Building is accessible by bridge on the 2nd floor of Corbett Hall. An LCD projector, overhead projectors and blackboards are available for presentations.
Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159.
Please respect that all other space has been contracted to other Banff Centre guests, including any food and beverages in those areas.

SCHEDULE

Sunday, 11-04-2010

16:00	Check-in begins (Front Desk – Professional Development Centre – open 24 hours)
	Lecture rooms are available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.

Monday, 12-04-2010

07:00–08:45	Breakfast
08:45–09:00	Introduction and Welcome to BIRS by the BIRS Station Manager, Max Bell 159
09:00–10:00	**Marco Gualtieri**
	"Gerbes and Poisson structures in generalized complex and Kähler geometry"
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Maxim Zabzine**
	"Why strings love generalized geometry"
11:30–13:00	Lunch
13:00–14:00	Guided Tour of the Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–15:00	**Ryushi Goto**
	"Unobstructed K-deformations of generalized complex structures and bihermitian structures"
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:30	**Justin Sawon**
	"Fourier-Mukai transforms and deformations in generalized complex geometry"
16:30–17:30	Informal discussions
17:30–19:30	Dinner

Tuesday, 13-04-2010

07:00–09:00	Breakfast
09:00–10:00	**Eckhard Meinrenken**
	"Twisted Spin^c *structures on conjugacy classes"*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Nigel Hitchin**
	"Generalized holomorphic bundles and the B-field action"
11:30–11:45	Group Photo; meet on the front steps of Corbett Hall
11:45–13:30	Lunch
14:00–15:00	**Sergey Arkhipov**
	"Conducting bundles of gerbes with connective structure and categorical symmetries"
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:30	**Frederik Witt**
	"Calibrations, D-branes and B-fields"
16:30–17:30	Informal discussions / Improptu talks / Recreation
17:30–19:30	Dinner
19:30–20:30	**Conan Leung**
	"Geometric Structures on Riemannian manifolds"

Wednesday, 14-04-2010

07:00–09:00	Breakfast
09:00–10:00	**Alberto Cattaneo**
	"Reduction via Graded Geometry"
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Ragnar-Olaf Buchweitz**
	"The super poisson structure on a Gerstenhaber algebra"
11:30–13:30	Lunch
13:30–17:30	Informal discussions / Improptu talks / Recreation
17:30–19:30	Dinner

Thursday, 15-04-2010

07:00–09:00	Breakfast
09:00–10:00	**Arlo Caine**
	"Poisson Structures on Toric Varieties"
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Susan Tolman**
	"Symplectic circle actions with minimal fixed points"
11:30–13:30	Lunch
14:00–15:00	**Georges Dloussky**
	"Normal singularities and holomorphic Poisson structures associated to a family of non-Kähler compact complex surfaces"
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:30	**Andrei Teleman**
	"Using moduli spaces of holomorphic bundles to prove existence of curves on class VII surfaces"
16:30–17:30	Informal discussions / Improptu talks / Recreation
17:30–19:30	Dinner
19:30–20:30	**Claudio Bartocci**
	"Geometric interpretation of the bi-hamiltonian structure of the Calogero-Moser system"

Friday, 16-04-2010

07:00–09:00	Breakfast
09:00–10:00	**Marco Aldi**
	"Twisted T-duality and Quantization"
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Tom Baird**
	"A Poisson structure on the moduli space of flat connections over a non-orientable surface"
11:30–13:30	Lunch
13:30–15:00	Informal discussions / Departures

Please remember to checkout by 12:00 noon.

You are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 15:00 on Friday, although participants are still required to checkout of the guest rooms by 12:00 noon.

"Generalized complex and holomorphic Poisson geometry"
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ABSTRACTS
(in alphabetical order by speaker surname)

Speaker: **Marco Aldi** (UC Berkeley)
Title: "Twisted T-duality and Quantization"

Abstract: The goal of this talk is to describe some mathematical aspects of the so called "doubled formalism". Our main application is a detailed study, in the language of vertex algebras, of the (twisted) T-duality relating the Heisenberg nilmanifold and the H-twisted 3-torus. This is joint work with Reimundo Heluani.

Speaker: **Sergey Arkhipov** (University of Toronto)
Title: "Conducting bundles of gerbes with connective structure and categorical symmetries"

Abstract: following Severa and Bressler-Chervov, we recall the definition of the Courant algebroid playing the role of the Atiyah algebroid for a gerbe with connective structure and known under the name of the conducting bundle of the gerbe. We consider the category of gerbe-twisted coherent sheaves with connection and relate the conducting bundle with the homotopy Lie algebra of the categorical group of autoequivalences of this category. The material of the talk is a joint project with Xinwen Zhu.

Speaker: **Tom Baird** (Memorial University)
Title: "A Poisson structure on the moduli space of flat connections over a non-orientable surface"

Abstract: Let G be a simple Lie group and S a two dimensional manifold, and let $M(S,G)$ denote the moduli space of flat G-connections over S. When S is orientable, $M(S,G)$ is a (singular) Poisson manifold arising as the phase space of Chern-Simons theory and so has deep connections with low dimensional topology and mathematical physics.

In this lecture, we explain how to construct a Poisson structure on $M(S,G)$ when S is non-orientable. The construction will make use of the quasi-Hamiltonian reduction of Alekseev-Malkin-Meinrenken's, in the Dirac geometry framework developed by Bursztyn-Crainic-Weinstein-Zhu.

Speaker: **Claudio Bartocci** (Università degli Studi di Genova)
Title: "Geometric interpretation of the bi-hamiltonian structure of the Calogero-Moser system"

Abstract: We shall show that the bi-Hamiltonian structure of the rational n-particle (attractive) Calogero-Moser system can be obtained by means of a double projection from a very simple Poisson pair on the cotangent bundle of gl(n,R). The relation with the Lax formalism will be also discussed. Joint work with G. Falqui, I. Mencattini, G. Ortenzi and M. Pedroni.

Speaker: **Ragnar-Olaf Buchweitz** (University of Toronto)
Title: "The super poisson structure on a Gerstenhaber algebra"

Abstract: A Gerstenhaber algebra is a graded algebra G that carries both a graded commutative product and, on the shifted copy G[1], a super Lie algebra structure such that the bracket g,- from G to G becomes a graded derivation for every element of g. In this way, bracketing with elements from G induces a map from G into the graded derivations or vectorfields of G.

In analogy, as Drinfeld pointed out, one may view G as the algebra of functions on a Poisson super-space, and the question is: What else can we say about these spaces? So far, very little! However, the geometry should be rich, as there are often additional features, such as a Hodge decomposition on a "virtual" tangent cone.

We hope that insight from ordinary Poisson geometry might be transferable to this super context.

We will discuss in some detail two examples: Hochschild cohomology, concretely, say, for flat morphisms between smooth spaces and their fibres, on the one hand, and the cobar construction over a Lie algebra that underlies the the theory of Classical (CYBE) and Quantum Yang-Baxter Equations (QYBE) on the other. In general, these two classical examples, due originally to Gerstenhaber, are related and give rise to universal deformation formulas.

Speaker: **Arlo Caine** (University of Notre Dame)
Title: "Poisson Structures on Toric Varieties"

*Abstract:*Let $X(\Sigma)$ be a smooth projective toric variety for a complex torus $T_{\mathbb{C}}$. Through a GIT construction, $X(\Sigma)$ can be given a number of Poisson structures which are invariant under the action of the complex group $T_{\mathbb{C}}$. Examples include holomorphic Poisson structures on $X(\Sigma)$ as well as smooth real Poisson structures. Of particular interest will be the Poisson structure Π_Σ associated to the standard Poisson structure π on $\mathbb{C}^d$. The symplectic leaves of Π_Σ are the $T_{\mathbb{C}} - orbits$ in $X(\Sigma)$. Hence, Π_Σ is non-degenerate on an open dense set but is not symplectic. It will be shown that each leaf admits a Hamiltonian action by a sub-torus of the compact torus $T \subset T_{\mathbb{C}}$, but that the global action of $T_{\mathbb{C}}$ on $(X(\Sigma), \Pi_\Sigma)$ is Poisson but not Hamiltonian. I will discuss some work on understanding the Poisson cohomology of this structure, including a lower bound for the dimension of $H^1(X(\Sigma), \Pi_\Sigma)$, and conclude with a discussion of the curious geometry of modular vector field of Π_Σ with the Delzant Liouville form.

Speaker: **Alberto Cattaneo** (Universität Zürich)
Title: "Reduction via Graded Geometry"

Abstract: Various geometric (e.g. Poisson, Courant, generalized complex) structures may be rephrased in terms of graded symplectic manifolds endowed with functions satisfying certain equations. From the latter point of view the most general reduction is just that of graded presymplectic submanifolds compatible with the given functions. By translating this back to the language of ordinary differential geometry, we recover all the known reduction procedures plus new ones. For example, in the Poisson world we get various generalizations of the Marsden-Ratiu reduction. This is based on joint work with Bursztyn, Mehta, and Zambon.

Speaker: **Georges Dloussky** (Université de Provence)
Title: "Normal singularities and holomorphic Poisson structures associated to a family of non-Kähler compact complex surfaces"

Abstract: Complex surfaces S containing global spherical shells (GSS) with Betti numbers $b_1(S) = 1$ and $n = b_2(S) > 0$ contain n rational curves. When the intersection matrix of the rational curves $M(S)$ is

negative definite a Grauert theorem insures that the maximal divisor can be contracted to one or two normal isolated singularities. We show that the genus of the singularities are one or two, may be Gorenstein. This local property is equivalent to the existence of a holomorphic Poisson structure on S. The topological invariant $k(S) = \sqrt{detM(S)} + 1$ plays an important role in the study of surfaces with GSS, non-invertible contracting germs of mappings and some birationnal mappings of $P^2(C)$. We explain how to compute the integer $k(S)$ using a family of polynomials.

Speaker: **Ryushi Goto** (Osaka University)
Title: "Unobstructed K-deformations of generalized complex structures and bihermitian structures"

*Abstract:*We survey our results on deformations of generalized complex and Kähler structures. At first, we introduce K-deformations of generalized complex structures on a compact Kähler manifold with effective anti-canonical line bundle. It turns out that K-deformations are always unobstructed. This is a generalization of unobstructedness theorem of deformations of Calabi-Yaus by Bogomolov-Tian-Todorov to two directions: from ordinary complex structures to generalized complex structures, and from trivial canonical line bundle to effective anti-canonical line bundle.

Next we explain the stability theorem of generalized Kähler structures with one pure spinor, which shows that generalized Kähler structures are stable under small deformations of generalized complex structures. This is regarded as a generalization of the stability theorem of ordinary Kähler structures by Kodaira-Spencer. Then a non-zero holomorphic Poisson structure gives rise to deformations of generalized Kähler manifolds starting with ordinary Kähler manifolds.

As an application, we construct bihermitian structures on compact Kähler manifolds with holomorphic Poisson structures by using K-deformations and the stability theorem together. In particular, we show that a compact Kähler surface S admits a non-trivial bihermitian structure if and only if S has a non-zero holomorphic Poisson structure. Examples of bihermitian structures on del Pezzo surfaces, degenerate del Pezzo surfaces, Hirzebruch surfaces and some ruled surfaces are discussed.

[1] Unobstructed K-deformations of Generalized Complex Structures and Bihermitian Structures, arXiv:1002.0391.

[2] Poisson structures and generalized Kähler structures, arXiv:0712.2685, J. Math. Soc. Japan, Vol. 61, No. 1 (2009) pp.107-132.

[3] Deformations of generalized complex and generalized Kähler structures, arXiv:0705.2495.

[4] On deformations of generalized Calabi-Yau, hyperKähler, G_2 and Spin(7) structures I, arXiv:math/0512211.

Speaker: **Marco Gualtieri** (University of Toronto)
Title: "Gerbes and Poisson structures in generalized complex and Kähler geometry"

Abstract: I will review and explain the mechanism by which Poisson geometry enters in generalized complex geometry and generalized Kähler geometry, as a sort of introduction to the topic of the conference. I will then use this to deduce some algebraic relationships between the two possibly non-isomorphic complex structures occurring in a generalized Kähler structure.

Speaker: **Nigel Hitchin** (University of Oxford)
Title: "Generalized holomorphic bundles and the B-field action"

Abstract: Gualtieri introduced the notion of a generalized holomorphic vector bundle over a generalized complex manifold. If a closed 2-form preserves the generalized complex structure then it transforms as a B-field one generalized holomorphic vector bundle to another and acts on the moduli space. We look at some examples of this and also describe twisted structures which involves a cocycle of B-field actions, spectral covers and holomorphic gerbes.

Speaker: **Conan Leung** (Institute of Mathematical Sciences and Chinese University of Hong Kong)
Title: "Geometric Structures on Riemannian manifolds"

Abstract: In this talk, I will describe various geometric structures from a unified approach using normed division algebras. By doubling the geometry, this method also give a nice description of all Riemannian symmetric spaces as Grassmannians.

Speaker: **Eckhard Meinrenken** (University of Toronto)
Title: "Twisted Spin^c structures on conjugacy classes"

Abstract: Let G be a compact, connected Lie group. An essential ingredient in the Borel-Weil construction of irreducible G-representations is the fact that the co-adjoint orbits $\mathcal{O} \subset \mathfrak{g}^*$ carry distinguished Kähler structures. More generally, Hamiltonian G-spaces in symplectic geometry carry distinguished Spin-c-structures, and the associated Dirac operators are used in their quantization.

By contrast, conjugacy classes $\mathcal{C} \subset G$ need not admit complex structures in general, or even Spin-c structures. I will explain in this talk that the conjugacy classes, and more generally all quasi-Hamiltonian G-spaces, carry canonical *twisted* Spin^c structures, with twisting by a distinguished background 'gerbe' over G. The twisted Spin-c structures appear in a quantization procedure for quasi-Hamiltonian spaces.

This talk is based on joint work with Anton Alekseev.

Speaker: **Justin Sawon** (University of North Carolina)
Title: "Fourier-Mukai transforms and deformations in generalized complex geometry"

Abstract: In this talk I will describe Toda's results on deformations of the category Coh(X) of coherent sheaves on a complex manifold X. They come from deformations of X as a complex manifold, non-commutative deformations, and gerby deformations (which can all be interpreted as deformations of X as a generalized complex manifold). Toda also described how to deform Fourier-Mukai equivalences, and I will present some examples coming from mirror SYZ fibrations.

Speaker: **Andrei Teleman** (Université de Provence)
Title: "Using moduli spaces of holomorphic bundles to prove existence of curves on class VII surfaces"

Abstract: The classification of complex surfaces is not finished yet. The most important gap in the Kodaira-Enriques classification table concerns the Kodaira class VII, e.g. the class of surfaces X having kod(X) $= -\infty$, $b_1(X) = 1$. These surfaces are interesting from a differential topological point of view, because they are non-simply connected 4-manifolds with definite intersection form. Class VII surfaces with $b_2 = 0$ are completely classified, but the methods used for this subclass do not extend to the general case. In the case $b_2 > 0$ important progress has been obtained by Kato, Nakamura, Dloussky and later by Dloussky-Oeljeklaus-Toma, but the complete classification has been considered since many years to be a hopeless goal. The difficulty is to show that any minimal class VII surface with $b_2 > 0$ admits sufficiently many curves. I will explain my program (based on ideas from Donaldson theory) to prove existence of curves on minimal class VII surfaces with $b_2 > 0$ and the first effective results obtained using this program: the classification up to biholomorphism for $b_2 = 1$ and up to deformation equivalence for $b_2 = 2$. Finally I will discuss the challenges to overcome (but also the expectations) for extending these methods to the case $b_2 > 2$.

Speaker: **Susan Tolman** (University of Illinois at Urbana-Champaign)
Title: "Symplectic circle actions with minimal fixed points"

Abstract: The purpose of this talk is to show that there are very few "extremely simple" symplectic manifolds with sympliectic actions. For example, consider a Hamiltonian circle action on a compact symplectic manifold (M, ω). It is easy to check that the sum of $\dim(F)+2$ over all fixed components is greater than or equal to $\dim(M)+2$. We show that, in certain cases, equality implies that the manifold "looks like" one of a handful of standard examples. This can be viewed as a symplectic analog of the Petrie conjecture. We will also discuss related results for non-Hamiltonian actions. Based on joint work with Hui Li and Alvaro Pelayo.

Speaker: **Frederik Witt** (Universität München)
Title: "Calibrations, D-branes and B-fields"

Abstract: In their quest for minimal submanifolds, Harvey and Lawson introduced the notion of a calibrated submanifold. In this talk, I shall present a natural extension of this concept to the generalised geometry framework and explain how this relates to D-branes in type II string theory.

Speaker: **Maxim Zabzine** (Uppsala Universitet)
Title: "Why strings love generalized geometry"

Abstract: I will review the relation between sigma models, Poisson vertex algebras and vertex algebras. The generalized geometry plays a central role in this relation. I will discuss the recent results on the connection between the sheaves of susy vertex algebras and the different aspects of generalized geometry.