“Generalized complex and holomorphic Poisson geometry”

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1 Overview of the Field

Generalized complex geometry is a relatively new subject in differential geometry, originating in 2001 with the work of Hitchin on geometries defined by differential forms of mixed degree. It has the particularly interesting feature that it interpolates between two very classical areas in geometry: complex algebraic geometry on the one hand, and symplectic geometry on the other hand. As such, it has bearing on some of the most intriguing geometrical problems of the last few decades, namely the suggestion by physicists that a duality of quantum field theories leads to a ”mirror symmetry” between complex and symplectic geometry.

Examples of generalized complex manifolds include complex and symplectic manifolds; these are at opposite extremes of the spectrum of possibilities. Because of this fact, there are many connections between the subject and existing work on complex and symplectic geometry. More intriguing is the fact that complex and symplectic methods often apply, with subtle modifications, to the study of the intermediate cases. Unlike symplectic or complex geometry, the local behaviour of a generalized complex manifold is not uniform. Indeed, its local structure is characterized by a Poisson bracket, whose rank at any given point characterizes the local geometry. For this reason, the study of Poisson structures is central to the understanding of generalized complex manifolds which are neither complex nor symplectic. Recently (Cavalcanti and Gualtieri 2007-8), the first examples were found of generalized complex 4-manifolds which admit neither complex nor symplectic structures; methods of Poisson geometry were central to the effort. This opens up the question of what obstructions there are to the existence of generalized complex structures.

While the local structure of generalized complex manifolds may be non-uniform, it is often describable as a deformation of a complex structure, where the deformation parameter is itself a holomorphic Poisson structure. This places strong constraints on the behaviour of the geometry, and allows the use of our knowledge of holomorphic Poisson manifolds coming from the study of integrable systems, mechanics, and algebraic geometry. Using this approach, we have gained a greater understanding of generalized complex manifolds, especially in dimension 4, in parallel with the recent completion of the classification of holomorphic Poisson surfaces (Bartocci-Macrì 2004).

Another surprising connection with holomorphic Poisson geometry arises from the study of a distinguished class of sub-objects of a generalized complex manifold, called D-branes because they correspond to Dirichlet boundary conditions on open strings with ends on a membrane in string theory (Kapustin-Li 2005). One finds that D-branes in a usual symplectic manifold correspond not only to Lagrangian submanifolds but also to new objects in symplectic geometry called co-isotropic A-branes (Kapustin-Orlov 2001). The presence of such branes may induce on the symplectic manifold the structure of a holomorphic symplectic structure. This was used to great effect in the recent work of Kapustin-Witten (2006) and Gukov-Witten (2008) on the geometric Langlands program, in which a D-brane on the Hitchin moduli space of Higgs bundles leads to the appearance of D-modules, a key step in establishing the Langlands correspondence.

Just as complex and symplectic geometry may be made compatible with a Riemannian metric, a generalized complex structure may be equipped with a compatible Riemannian metric to form a generalized Kähler structure. It can be shown (Gualtieri 2004) that generalized Kähler geometry is equivalent to the most general $N = (2, 2)$ supersymmetric sigma model target geometry, as described in 1984 by Gates-Hull-Roček. In fact, a generalized Kähler structure comprises not one but *two* integrable complex structures, each Hermitian with respect to the Riemannian metric. For this reason, the subject has great relevance to bi-Hermitian geometry, a subject which has been studied from the point of view of twistor theory for several decades (Apostolov-Gauduchon-Grantcharov 1988). In a generalized Kähler manifold, the difference between the two complex structures is, remarkably, measured by a holomorphic Poisson structure (Hitchin 2005). We see again the emergence of a Poisson structure in describing how far the generalized structure is from the usual one.

The construction of examples of generalized Kähler manifolds has proven surprisingly challenging. Without the standard tools of algebraic geometry for constructing usual Kähler manifolds, one must discover new methods of construction. Much effort has recently been expended in this aim. In (Hitchin 2005), examples are produced under homogeneity assumptions. In (Bursztyn-Cavalcanti-Gualtieri 2005) and (Lin-Tolman 2005), examples are produced via the development of a reduction procedure analogous to the Marsden-Weinstein symplectic reduction. In (Goto 2005), a large class of examples are produced by deformation of usual Kähler structures, and in (Hitchin 2006), new examples are produced via Hamiltonian deformation. There is much work under way on new methods of construction as well as better conceptual frameworks for understanding the presence of generalized Kähler geometry.

In the same way that Kähler geometry is generalized, other geometries of special Holonomy, such as Calabi-Yau and G2 manifolds, have extensions to generalized geometry. In fact, Generalized Calabi-Yau geometry was one of Hitchin's original motivations in the subject (Hitchin 2001), and such a structure was used shortly thereafter in (Huybrechts 2003) to establish an isomorphism between the moduli space of Generalized Calabi-Yau metrics on the K3 surface and its moduli space of $N = (2, 2)$ super-conformal field theories in the sense of (Aspinwall-Morrison 1997). For generalized G2 structures on 7-manifolds, (Witt 2004) shows that the presence of generalized G2 structure provides a solution to the supersymmetry equations on spinors in type IIA/B supergravity, and studies the resulting geometrical structure using Riemannian connections with skew torsion.

The major challenges in the emerging field of generalized geometry include the following three items. First, to achieve a greater understanding of the intrinsic structures involved, such as generalized complex or Kähler manifolds, and their relation to standard structures in Kähler or Poisson geometry. Second, to construct new examples of generalized complex and Kähler structures, hopefully with very general methods. Third, the development of an algebraic theory capturing a hypothesized categorical structure for generalized complex manifolds analogous to the Fukaya category of symplectic manifolds or the category of coherent sheaves on a complex manifold.

2 Objectives of the workshop

Before this BIRS workshop, a meeting gathering the growing number of researchers investigating the various forms of generalized geometry, such as generalized complex and Kähler geometry, had never been held. There was a general consensus among many of the researchers mentioned above that a meeting focused on the subject would be of great benefit to progress in the field.

The primary objective of the workshop was to gather together, in the secluded and stimulating environment of BIRS, the mathematicians who work on generalized complex geometry and closely related fields such as Poisson geometry, non-Kähler complex geometry, and integrable systems, so that they may combine their tools and approaches to further our understanding of these subjects. In particular, we worked to further the following goals.

- The myriad examples of holomorphic Poisson manifolds and dynamical systems studied in integrable systems should be better-known to the fields of Poisson geometry and generalized complex geometry and will hopefully provide new areas of investigation.

- Within Poisson geometry, much effort has been expended in recent years (Crainic-Fernandes 2001) to construct and study the symplectic groupoid, which is a kind of "resolution" of a Poisson manifold to a symplectic manifold with groupoid structure. Recently (Crainic 2004, Stiénon-Xu 2006) the notion of symplectic groupoid has been extended to generalized complex manifolds in an effort to resolve their complicated local structure. This project is still ongoing, as our understanding of the intrinsic structure of the groupoid is not complete. However it promises to shed enormous light on the problem of developing a generalized complex category in analogy to the symplectic and Poisson categories in the sense of Weinstein.
- The algebro-geometric approach to Poisson structures provides a wide set of tools for constructing objects such as Poisson modules on Poisson manifolds; by learning about and furthering these methods, we shall better understand the construction of D-branes on generalized complex manifolds.
- The known exotic examples of generalized complex 4-manifolds were constructed by surgery methods analogous to those used in complex algebraic geometry as well as Poisson geometry. A better understanding of these tools may lead to new constructions of generalized complex manifolds in dimension 4 and 6, especially.
- The well-developed theory of reduction in Poisson and symplectic geometry, including the theory of group-valued moment maps (Alekseev-Malkin-Meinrenken 1997, Alekseev-Bursztyn-Meinrenken 2007) has only begun to be extended and applied in a consistent fashion to generalized geometrical structures such as Dirac structures and generalized complex structures, and more effort in this direction will be helpful to clarify the study of geometrical structures admitting symmetries.

The workshop was intended as a 5-day workshop involving the main researchers in the fields above, both faculty and postdoctoral, together with the graduate students which have taken up the subject in their doctoral work.

3 Overview of the meeting

Here are the abstracts of the talks, in alphabetical order by speaker surname:

Speaker: **Marco Aldi** (UC Berkeley)
Title: "Twisted T-duality and Quantization"

Abstract: The goal of this talk is to describe some mathematical aspects of the so called "doubled formalism". Our main application is a detailed study, in the language of vertex algebras, of the (twisted) T-duality relating the Heisenberg nilmanifold and the H-twisted 3-torus. This is joint work with Reimundo Heluani.

Speaker: **Sergey Arkhipov** (University of Toronto)
Title: "Conducting bundles of gerbes with connective structure and categorical symmetries"

Abstract: following Severa and Bressler-Chervov, we recall the definition of the Courant algebroid playing the role of the Atiyah algebroid for a gerbe with connective structure and known under the name of the conducting bundle of the gerbe. We consider the category of gerbe-twisted coherent sheaves with connection and relate the conducting bundle with the homotopy Lie algebra of the categorical group of autoequivalences of this category. The material of the talk is a joint project with Xinwen Zhu.

Speaker: **Tom Baird** (Memorial University)
Title: "A Poisson structure on the moduli space of flat connections over a non-orientable surface"

Abstract: Let G be a simple Lie group and S a two dimensional manifold, and let $M(S,G)$ denote the moduli space of flat G-connections over S. When S is orientable, $M(S,G)$ is a (singular) Poisson manifold arising as the phase space of Chern-Simons theory and so has deep connections with low dimensional topology and mathematical physics.

In this lecture, we explain how to construct a Poisson structure on $M(S,G)$ when S is non-orientable. The construction will make use of the quasi-Hamiltonian reduction of Alekseev-Malkin-Meinrenken's, in the Dirac geometry framework developed by Bursztyn-Crainic-Weinstein-Zhu.

Speaker: **Claudio Bartocci** (Università degli Studi di Genova)
Title: "Geometric interpretation of the bi-hamiltonian structure of the Calogero-Moser system"

Abstract: We shall show that the bi-Hamiltonian structure of the rational n-particle (attractive) Calogero-Moser system can be obtained by means of a double projection from a very simple Poisson pair on the cotangent bundle of gl(n,R). The relation with the Lax formalism will be also discussed. Joint work with G. Falqui, I. Mencattini, G. Ortenzi and M. Pedroni.

Speaker: **Ragnar-Olaf Buchweitz** (University of Toronto)
Title: "The super poisson structure on a Gerstenhaber algebra"

Abstract: A Gerstenhaber algebra is a graded algebra G that carries both a graded commutative product and, on the shifted copy G[1], a super Lie algebra structure such that the bracket g,- from G to G becomes a graded derivation for every element of g. In this way, bracketing with elements from G induces a map from G into the graded derivations or vectorfields of G.

In analogy, as Drinfeld pointed out, one may view G as the algebra of functions on a Poisson super-space, and the question is: What else can we say about these spaces? So far, very little! However, the geometry should be rich, as there are often additional features, such as a Hodge decomposition on a "virtual" tangent cone.

We hope that insight from ordinary Poisson geometry might be transferable to this super context.

We will discuss in some detail two examples: Hochschild cohomology, concretely, say, for flat morphisms between smooth spaces and their fibres, on the one hand, and the cobar construction over a Lie algebra that underlies the the theory of Classical (CYBE) and Quantum Yang-Baxter Equations (QYBE) on the other. In general, these two classical examples, due originally to Gerstenhaber, are related and give rise to universal deformation formulas.

Speaker: **Arlo Caine** (University of Notre Dame)
Title: "Poisson Structures on Toric Varieties"

*Abstract:*Let $X(\Sigma)$ be a smooth projective toric variety for a complex torus $T_{\mathbb{C}}$. Through a GIT construction, $X(\Sigma)$ can be given a number of Poisson structures which are invariant under the action of the complex group $T_{\mathbb{C}}$. Examples include holomorphic Poisson structures on $X(\Sigma)$ as well as smooth real Poisson structures. Of particular interest will be the Poisson structure Π_Σ associated to the standard Poisson structure π on $\mathbb{C}^d$. The symplectic leaves of Π_Σ are the $T_{\mathbb{C}} - orbits$ in $X(\Sigma)$. Hence, Π_Σ is non-degenerate on an open dense set but is not symplectic. It will be shown that each leaf admits a Hamiltonian action by a sub-torus of the compact torus $T \subset T_{\mathbb{C}}$, but that the global action of $T_{\mathbb{C}}$ on $(X(\Sigma), \Pi_\Sigma)$ is Poisson but not Hamiltonian. I will discuss some work on understanding the Poisson cohomology of this structure, including a lower bound for the dimension of $H^1(X(\Sigma), \Pi_\Sigma)$, and conclude with a discussion of the curious geometry of modular vector field of Π_Σ with the Delzant Liouville form.

Speaker: **Alberto Cattaneo** (Universität Zürich)
Title: "Reduction via Graded Geometry"

Abstract: Various geometric (e.g. Poisson, Courant, generalized complex) structures may be rephrased in terms of graded symplectic manifolds endowed with functions satisfying certain equations. From the latter point of view the most general reduction is just that of graded presymplectic submanifolds compatible with the given functions. By translating this back to the language of ordinary differential geometry, we recover all the known reduction procedures plus new ones. For example, in the Poisson world we get various generalizations of the Marsden-Ratiu reduction. This is based on joint work with Bursztyn, Mehta, and Zambon.

Speaker: **Georges Dloussky** (Université de Provence)
Title: "Normal singularities and holomorphic Poisson structures associated to a family of non-Kähler compact complex surfaces"

Abstract: Complex surfaces S containing global spherical shells (GSS) with Betti numbers $b_1(S) = 1$ and $n = b_2(S) > 0$ contain n rational curves. When the intersection matrix of the rational curves $M(S)$ is negative definite a Grauert theorem insures that the maximal divisor can be contracted to one or two normal isolated singularities. We show that the genus of the singularities are one or two, may be Gorenstein. This local property is equivalent to the existence of a holomorphic Poisson structure on S. The topological invariant $k(S) = \sqrt{detM(S)} + 1$ plays an important role in the study of surfaces with GSS, non-invertible contracting germs of mappings and some birationnal mappings of $P^2(C)$. We explain how to compute the integer $k(S)$ using a family of polynomials.

Speaker: **Ryushi Goto** (Osaka University)
Title: "Unobstructed K-deformations of generalized complex structures and bihermitian structures"

*Abstract:*We survey our results on deformations of generalized complex and Kähler structures. At first, we introduce K-deformations of generalized complex structures on a compact Kähler manifold with effective anti-canonical line bundle. It turns out that K-deformations are always unobstructed. This is a generalization of unobstructedness theorem of deformations of Calabi-Yaus by Bogomolov-Tian-Todorov to two directions: from ordinary complex structures to generalized complex structures, and from trivial canonical line bundle to effective anti-canonical line bundle.

Next we explain the stability theorem of generalized Kähler structures with one pure spinor, which shows that generalized Kähler structures are stable under small deformations of generalized complex structures. This is regarded as a generalization of the stability theorem of ordinary Kähler structures by Kodaira-Spencer. Then a non-zero holomorphic Poisson structure gives rise to deformations of generalized Kähler manifolds starting with ordinary Kähler manifolds.

As an application, we construct bihermitian structures on compact Kähler manifolds with holomorphic Poisson structures by using K-deformations and the stability theorem together. In particular, we show that a compact Kähler surface S admits a non-trivial bihermitian structure if and only if S has a non-zero holomorphic Poisson structure. Examples of bihermitian structures on del Pezzo surfaces, degenerate del Pezzo surfaces, Hirzebruch surfaces and some ruled surfaces are discussed.

[1] Unobstructed K-deformations of Generalized Complex Structures and Bihermitian Structures, arXiv:1002.0391.

[2] Poisson structures and generalized Kähler structures, arXiv:0712.2685, J. Math. Soc. Japan, Vol. 61, No. 1 (2009) pp.107-132.

[3] Deformations of generalized complex and generalized Kähler structures, arXiv:0705.2495.

[4] On deformations of generalized Calabi-Yau, hyperKähler, G_2 and Spin(7) structures I, arXiv:math/0512211.

Speaker: **Marco Gualtieri** (University of Toronto)
Title: "Gerbes and Poisson structures in generalized complex and Kähler geometry"

Abstract: I will review and explain the mechanism by which Poisson geometry enters in generalized complex geometry and generalized Kähler geometry, as a sort of introduction to the topic of the conference. I will then use this to deduce some algebraic relationships between the two possibly non-isomorphic complex structures occurring in a generalized Kähler structure.

Speaker: **Nigel Hitchin** (University of Oxford)
Title: "Generalized holomorphic bundles and the B-field action"

Abstract: Gualtieri introduced the notion of a generalized holomorphic vector bundle over a generalized complex manifold. If a closed 2-form preserves the generalized complex structure then it transforms as a B-field one generalized holomorphic vector bundle to another and acts on the moduli space. We look at some examples of this and also describe twisted structures which involves a cocycle of B-field actions, spectral covers and holomorphic gerbes.

Speaker: **Conan Leung** (Institute of Mathematical Sciences and Chinese University of Hong Kong)
Title: "Geometric Structures on Riemannian manifolds"

Abstract: In this talk, I will describe various geometric structures from a unified approach using normed division algebras. By doubling the geometry, this method also give a nice description of all Riemannian symmetric spaces as Grassmannians.

Speaker: **Eckhard Meinrenken** (University of Toronto)
Title: "Twisted Spin^c structures on conjugacy classes"

Abstract: Let G be a compact, connected Lie group. An essential ingredient in the Borel-Weil construction of irreducible G-representations is the fact that the co-adjoint orbits $\mathcal{O} \subset \mathfrak{g}^*$ carry distinguished Kähler structures. More generally, Hamiltonian G-spaces in symplectic geometry carry distinguished Spin^c structures, and the associated Dirac operators are used in their quantization.

By contrast, conjugacy classes $\mathcal{C} \subset G$ need not admit complex structures in general, or even Spin^c structures. I will explain in this talk that the conjugacy classes, and more generally all quasi-Hamiltonian G-spaces, carry canonical *twisted* Spin^c structures, with twisting by a distinguished background 'gerbe' over G. The twisted Spin^c structures appear in a quantization procedure for quasi-Hamiltonian spaces.

This talk is based on joint work with Anton Alekseev.

Speaker: **Justin Sawon** (University of North Carolina)
Title: "Fourier-Mukai transforms and deformations in generalized complex geometry"

Abstract: In this talk I will describe Toda's results on deformations of the category Coh(X) of coherent sheaves on a complex manifold X. They come from deformations of X as a complex manifold, non-commutative deformations, and gerby deformations (which can all be interpreted as deformations of X as a generalized complex manifold). Toda also described how to deform Fourier-Mukai equivalences, and I will present some examples coming from mirror SYZ fibrations.

Speaker: **Andrei Teleman** (Université de Provence)
Title: "Using moduli spaces of holomorphic bundles to prove existence of curves on class VII surfaces"

Abstract: The classification of complex surfaces is not finished yet. The most important gap in the Kodaira-Enriques classification table concerns the Kodaira class VII, e.g. the class of surfaces X having $\mathrm{kod(X)} = -\infty, b_1(X) = 1$. These surfaces are interesting from a differential topological point of view, because they are non-simply connected 4-manifolds with definite intersection form. Class VII surfaces with $b_2 = 0$ are completely classified, but the methods used for this subclass do not extend to the general case. In the case $b_2 > 0$ important progress has been obtained by Kato, Nakamura, Dloussky and later by Dloussky-Oeljeklaus-Toma, but the complete classification has been considered since many years to be a hopeless goal. The difficulty is to show that any minimal class VII surface with $b_2 > 0$ admits sufficiently many curves. I will explain my program (based on ideas from Donaldson theory) to prove existence of curves on minimal class VII surfaces with $b_2 > 0$ and the first effective results obtained using this program: the classification up to biholomorphism for $b_2 = 1$ and up to deformation equivalence for $b_2 = 2$. Finally I will discuss the challenges to overcome (but also the expectations) for extending these methods to the case $b_2 > 2$.

Speaker: **Susan Tolman** (University of Illinois at Urbana-Champaign)
Title: "Symplectic circle actions with minimal fixed points"

Abstract: The purpose of this talk is to show that there are very few "extremely simple" symplectic manifolds with sympliectic actions. For example, consider a Hamiltonian circle action on a compact symplectic manifold (M, ω). It is easy to check that the sum of $\dim(F) + 2$ over all fixed components is greater than or equal to $\dim(M) + 2$. We show that, in certain cases, equality implies that the manifold "looks like" one of a handful of standard examples. This can be viewed as a symplectic analog of the Petrie conjecture. We will also discuss related results for non-Hamiltonian actions. Based on joint work with Hui Li and Alvaro Pelayo.

Speaker: **Frederik Witt** (Universität München)
Title: "Calibrations, D-branes and B-fields"

Abstract: In their quest for minimal submanifolds, Harvey and Lawson introduced the notion of a calibrated submanifold. In this talk, I shall present a natural extension of this concept to the generalised geometry framework and explain how this relates to D-branes in type II string theory.

Speaker: **Maxim Zabzine** (Uppsala Universitet)
Title: "Why strings love generalized geometry"

Abstract: I will review the relation between sigma models, Poisson vertex algebras and vertex algebras. The generalized geometry plays a central role in this relation. I will discuss the recent results on the connection between the sheaves of susy vertex algebras and the different aspects of generalized geometry.

4 Presentation Highlights/Scientific Progress Made

Examples of recent breakthroughs:

1. Prof. Hitchin described new work in progress concerning generalized holomorphic bundles over generalized complex manifolds. These are generalizations of usual holomorphic bundles and they may be approached with the same questions that have been applied to holomorphic bundles. For example, do they have moduli spaces? How can we construct examples? He focused on a particularly intriguing case of viewing a complex manifold as a generalized complex manifold. In this case, the notion of a generalized holomorphic bundle coincides with the usual notion of a holomorphic bundle, except that it is equipped with a co-Higgs field. Hitchin explained how the co-Higgs field leads to a spectral description of the bundle, as in the case of Higgs bundles. Even more interesting was the observation that the complex manifold may support a nontrivial holomorphic gerbe, in which case the generalized holomorphic bundles are related to twisted coherent sheaves. This provides a novel differential-geometric interpretation of these objects.

2. Prof. Goto described his recent project of studying deformation theory of generalized complex and Kähler manifolds, which has led to many successes in the search for examples of bi-Hermitian manifolds. A particularly striking result which he emphasized in his talk was that his theory has something new to say even about deformations of usual complex manifolds. That is, if we consider complex deformations of a Kähler manifold which fix an anti-canonical divisor, then the deformation theory

is *unobstructed*. In other words, the celebrated Tian-Todorov lemma which describes deformations of Calabi-Yau manifolds actually applies to any Kahler manifold once we fix an anti-canonical divisor. Goto's more general theorem is a statement about unobstructedness of certain deformations of generalized Kähler manifolds, and it guarantees the existence of many bi-Hermitian structures with non-isomorphic constituent complex structures.

3. The classification of complex surfaces is not yet complete. Prof. Dloussky and Prof. Teleman gave an overview of the state of the art in the classification of surfaces, focusing on the topic of global spherical shells, which come up in the study of class VII surfaces, which are those that remain to be completely classified. Interestingly, there is a relationship between these surfaces and the presence of nontrivial holomorphic gerbes, making an intriguing connection to Hitchin's recent work. In the case of the Hopf surface, for example, which has a global spherical shell, the study of nontrivial holomorphic gerbes is essential in understanding the generalized geometry of the space. We also saw how gauge theory in the guise of Donaldson theory can be used to study the still-open question of classifying these manifolds.

4. Dr. Aldi's talk introduced a mysterious and quite intriguing generalization of the vast topic of vertex operator algebras. His generalization (joint with Heluani) is motivated from the study of T-duality, an operation which forms part of the formalism of generalized geometry. By T-dualizing a torus, he obtains an exotic version of the usual algebra of operators in the conformal field theory associated to the torus, which contains terms in its operator product expansion which are disallowed in the usual theory of vertex operator algebras. These di-logarithmic terms represent a new direction in the subject, motivated from dualities in quantum field theory.

5 Outcome of the Meeting

We had 41 participants. It was important to us that the participants not only be main researchers in the area, both faculty and postdoctoral, but also graduate students who have taken up the subject in their doctoral work. We were very successful in that respect, with the participation of 8 graduate students from the Universities of British Columbia, Toronto, McGill, California at Berkeley, and Stony Brook. The topics of the conference were directly relevant to the thesis topics of several of those graduate students present, and they were highly motivated by the opportunity to talk with experts in the field. There were also 5 postdoctoral researchers, from the Universities of Toronto, Waterloo, California at Berkeley, and Notre Dame.

We also had the participation of physicists. The origins of Poisson and generalized complex geometry can be traced back to physics, and many of the questions studied in the field are motivated by physics. In fact, two of the talks were given by physicists, Prof. Zabzine and Dr. Aldi. There were many interactions between the mathematicians and the physicists, and at least one collaboration emerged from them, between Prof. Zabzine and Dr. Cabrera.

The workshop, as well as the extended time at BIRS due to the intervening volcano, stimulated a number of collaborations. Those we are aware of are as follows: One began between Profs. Dancer and Tolman after her talk about the symplectic Petrie conjecture. The collaboration between Profs. Karigiannis and Leung was also nourished by the conference. A new collaboration between Profs. Apostolov and Gauduchon concerning generalized Kähler metrics was also begun. Furthermore, collaborations began between Profs. Gualtieri and Bursztyn as well as Profs. Gualtieri and Moraru.

The participants were very enthusiastic about the scientific content of the workshop, as well as the facilities and breathtaking natural setting of BIRS. Moreover, the warm hospitality and professionalism of the staff were very much appreciated, and we would in particular like to thank the Scientific Director and Scientific Coordinator of BIRS for being so helpful and accommodating to European participants stranded in Banff due to the eruptions of Eyjafjallajökull.