BIRS Workshop 10w5075

Rate-independent systems: Modeling, Analysis, and Computations

Banff, August 29 - September 3, 2010

General information

Checking-in

Check-in begins on Sunday at 16:00 (Front Desk - Professional Development Centre - open 24 hours).

Buffet Dinner (17:30-19:30), Sally Borden Building.

Lecture rooms available after 16:00 on Sunday.

Informal gathering in 2nd floor lounge after 20:00, Corbett Hall, where beverages and a small assortment of snacks will be available on a cash honor system.

Meals

- Breakfast (Buffet): 7:00 – 9:30, Sally Borden Building, Monday-Friday
- Lunch (Buffet): 11:30 – 13:30 , Sally Borden Building, Monday-Friday
- Dinner (Buffet): 17:30 – 19:30, Sally Borden Building, Sunday-Thursday
- Coffee breaks: as per daily schedule, 2nd floor lounge, Corbett Hall

Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

Wednesday

We might consider going for a short/medium hike on Wednesday. If interested, be prepared with appropriate equipment (shoes, rainwear etc.)

Checking-out

Check-out is by 12:00 on Friday. BIRS facilities (2nd floor lounge, Max Bell meeting rooms, reading room) are available until 15:00 on Friday.

The meeting will be over by Friday at 12:00. We shall try to organize group trasportation to Calgary Airport.

Meeting rooms

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155 - 159. There is also a 5-metre magnetic wall for posters. Please note that all other space has been contracted to other Banff Centre guests, including any food and beverages in those areas.

Library

The library at BIRS is in its infancy and has only a few books. Access to MathSciNet is provided. Participants who have online services at their home institutions are encouraged to establish proxy accounts that will allow them journal access from BIRS.

Files upload

Files related to the workshop, such as PDF or Powerpoint files for papers and presentations, can be published on the workshop website. Just let us have what you wish to share and we will put in online.

Mailing list

The e-mail address `10w5075@lists.birs.ca` forwards messages to all the participants to the workshop.

None of the above, but in this section

Participants of BIRS programmes are housed in Corbett Hall at the Banff Centre. There is a separate bedroom for each participant, though most rooms have a shared bathroom with one other participant. Each room is equipped with a computer terminal with a high-speed access to the Internet. There are wireless access points in some common areas. The Banff Centre has secure storage for luggage.

The Banff Centre, where BIRS is located, provides participants with many other facilities and activities, including fitness and recreation facilities (including a full-size swimming pool), music library, exhibitions and live performances.

For more information, please visit: `http://www.banffcentre.ca/`

Avarage weather for August-September in Banff:

- max temperature 19°C,
- min temperature 4°C,
- rain/month: 49 mm.

Schedule

Speakers please allow 5' for questions.

Monday, August 30:

8.45 – 9.00	Introduction and Welcome by BIRS Station Manager
9.00 – 9.50	Gilles Francfort *Non-associative elasto-plasticity*
9.50 – 10.20	Daya Reddy *Some models of strain gradient crystal plasticity and their variational characterisation*
10.20 – 10.40	Coffee break
10.50 – 11.40	Georg Dolzmann *The role of hardening in models in crystal plasticity*
11.40 – 12.10	Marco Veneroni *Periodic homogenization for a model of plasticity*
12.10 – 13.00	Lunch
13.00 – 13.55	Guided Tour of the Banff Centre: meet in the 2nd floor lounge, Corbett Hall
13.55 – 14.00	Group Photo: meet on the front steps of Corbett Hall
14.00 – 14.50	Ben Schweizer *Porous media and plasticity: homogenization for equations with hysteresis*
14.50 – 15.20	Coffee break
15.20 – 15.50	Tim Sullivan *Analysis of the effect of a heat bath on a rate-independent system*
15.50 – 16.40	Patrick Dondl *Pinning of interfaces in random media*

Tuesday, August 31:

8.40 – 9.30	Elena Bonetti *A variational model for adhesive contact with friction*
9.30 – 10.00	Marita Thomas *Rate-independent damage and delamination processes*
10.00 – 10.30	Coffee break
10.30 – 11.20	Alice Fiaschi *Young-measure quasistatic evolution for a damage model*
11.20 – 11.50	Jan Zeman *On rate-independent models of mixed-mode delamination phenomena*
11.50 – 13.30	Lunch
13.30 – 14.20	Tomáš Roubíček *Thermodynamics of rate-independent processes*
14.20 – 14.50	Dietmar Hömberg *On a thermomechanical milling model*

14.50 – 15.20 Coffee break

15.20 – 15.50 Petr Šittner
Microstructure evolution during heat treatment of shape memory alloys

15.50 – 16.20 Jana Kopfová
A model from thermoplasticity with temperature-dependent Preisach hysteresis

16.20 – 16.50 Johannes Zimmer
Rate-independent evolution for an elastic-plastic model of shape memory alloys and related function spaces

Wednesday, September 1:

8.40 – 9.30 Filip Rindler
Lower semicontinuity and Young measures in BD

9.30 – 10.20 Martin Kružík
Efficient numerical approach to rate-independent problems

10.20 – 10.50 Coffee break

Thursday, September 2:

8.40 – 9.30 Matteo Negri
Rate-independent evolutions in fracture mechanics

9.30 – 10.00 Christoph Ortner
Adaptive FEMs for quasistatic brittle fracture

10.00 – 10.30 Coffee break

10.30 – 11.20 Chris Larsen
Quasi-static fracture based on local minimality

11.20 – 12.10 Giuliano Lazzaroni
Crack growth with non-interpenetration: the case of Lipschitz data

12.10 – 13.30 Lunch

13.40 – 14.30 Rodica Toader
A model for crack propagation based on viscous approximation

14.30 – 15.00 Dorothee Knees
Numerical convergence analysis for a vanishing viscosity model in fracture mechanics

15.00 – 15.30 Coffee break

15.30 – 16.00 Adrien Petrov
On a 3D model for shape-memory alloys

16.00 – 16.30 Alessandro Reali
On the robustness and efficiency of integration algorithms for a 3D finite strain phenomenological SMA constitutive model

16.30 – 17.00 Edoardo Artioli
A nonlinear shell finite element formulation for shape memory alloy applications

Friday, September 3:

8.40 – 9.30	RICCARDA ROSSI *Some results on the vanishing viscosity approach to rate-independent modelling*
9.30 – 10.00	MATTHIAS LIERO *Rate-independent Kurzweil processes*
10.00 – 10.30	Coffee break
10.30 – 11.20	VINCENZO RECUPERO *A continuity method for sweeping processes*
11.30 – 13.30	Lunch

Abstracts

A nonlinear shell finite element formulation for shape memory alloy applications

Edoardo Artioli (Roma Tor Vergata)

In the last decades, the development of efficient computational models for the nonlinear analysis of structures made of shape memory alloys (SMA) has been one of the most important research activities. The shape memory alloys (SMA) represent one of the most interesting smart material for their ability to recover large strains during mechanical patterns, the pseudo elastic effect, and to recover residual deformations through mechanical-thermal cycles, the shape memory effect. In fact, under loading-unloading cycles, even up to 10-15% strains, the material shows distinct plateaux during the loading and unloading branches, hysteretic response and no permanent deformations. The present work presents a finite element model for the analysis of shell structures constituted of shape memory alloy material considering finite strains. A three dimensional constitutive model [1] for shape memory alloys in the framework of finite strains which is capable of describing the typical macroscopic effects of SMA, as the pseudo-elasticity and the shape memory effect is adopted. The structural model is formulated with a 2D shell theory where the midsurface and the covariant components of kinematic quantities are approximated element-wise with the standard isoparametric approach [2]. The displacement field assumption is based on the classical expansion in thickness direction in terms of increasing powers of the transverse coordinate and leads to an analogous form for the deformation gradient. The equilibrium statement is formulated considering the Virtual Work Principle in the total Lagrangian format. The proposed formulation is suitable for the simple derivation of high-order elements in a fully compatible fashion. The treatment of locking phenomena is then discussed. A set of numerical examples are presented, showing the accuracy and robustness of the proposed computational strategy and its capability of describing the structural response of shape memory alloy devices of technical interest.

[1] Evangelista V., Marfia S., Sacco E., A 3D SMA constitutive model in the framework of finite strain, International Journal for Numerical Methods in Engineering, DOI: 10.1002/nme, 2009.

[2] Arciniega R.A., Reddy J.N., Tensor-based finite element formulation for geometrically nonlinear analysis of shell structures, Computer Methods in Applied Mechanics and Engineering, 196, 1048-1073, 2007.

A variational model for adhesive contact with friction

Elena Bonetti (Pavia)

We discuss a rate independent frictional law combined with unilateral conditions for a model describing contact with adhesion. In the framework of continuum mechanics, we derive the constitutive laws by a generalization of the priciple of virtual powers in which internal constraints, as well as the unilateral and the rate independent frictional conditions, are included in the balance of the energy and dissipation. The resulting PDE system is highly nonlinear: the main analytical diculties are related to the presence of multivalued operators rendering the Signorini conditions, the Coulomb law (accounting for friction) and the physical constraints on the variables, and to the nonlinear coupling between the equations themselves. We obtain a global-in-time existence result. These results have been obtained in a joint work with Riccarda Rossi and Giovanna Bonfanti.

The role of hardening in models in crystal plasticity

Georg Dolzmann (Regensburg)

We investigate Gamma limits of elastoplastic models in the limit of large elastic constants. The surprising result is that models with dissipation only are predicted to have a very soft behaviour while models with hardening converge in the sense of Gamma convergence to models with rigid elasticity. This is joint work with Sergio Conti (Bonn) and Carolin Kreisbeck (Regensburg)

Pinning of interfaces in random media
Patrick Dondl (Bonn)

We consider the evolution of an interface, modeled by a parabolic equation, in a random environment. The randomness is given by a distribution of smooth obstacles of random strength. To provide a barrier for the moving interface, we construct a positive, steady state supersolution. This construction depends on the existence, after rescaling, of a Lipschitz hypersurface separating the domain into a top and a bottom part, consisting of boxes that contain at least one obstacle of sufficient strength. We prove this percolation result. This work shows the emergence of a rate independent hysteresis in systems subject to a viscous microscopic evolution law through the interaction with a random environment. Joint work with N. Dirr (Bath University) and M. Scheutzow (TU Berlin).

Young-measure quasistatic evolution for a damage model
Alice Fiaschi (IMATI Pavia)

An existence result for the quasistatic evolution of incomplete damage in elastic materials will be presented. The absence of gradient terms in the damage variable causes a critical lack of compactness. Therefore, the analysis is developed in the framework of Young measures, where a notion of solution is defined, presenting some improvements with respect to previous contributions.

Non-associative elasto-plasticity
Gilles Francfort (Paris)

In this joint work with Jean-Franois Babadjian and Maria-Giovanna Mora, I propose to discuss a model of non-associative elasto-plasticity which is commonly used in rock and soil mechanics. I will show how to impart a variational structure on the model which is commonly thought not to possess one, and will then describe the existence of a quasi-static rate independent evoultion.

On a thermomechanical milling model
Dietmar Hömberg (TU and WIAS Berlin)

In my talk I will discuss a mathematical model that characterizes the interaction between machine, work-piece, and process dynamics for a complex milling system. While the machine dynamics is modeled in terms of a standard multi-body system, the work-piece is described as a linear thermo-elastic continuum. The coupling of both parts is realized by an empirical process model permitting an estimate of heat and coupling forces occurring during milling. I will briefy describe the governing equations emphasizing the coupling, then a first analytical result will be outlined concerning the well-posedness of the system. I will conclude with some numerical results showing the dynamics of this complex thermo-mechanical system. (joint work with Krzysztof Chelminski (Warsaw University of Technology) and Oliver Rott, (WIAS)).

Numerical convergence analysis for a vanishing viscosity model in fracture mechanics
Dorothee Knees (WIAS Berlin)

The Griffith criterion is an energetic fracture criterion which is frequently applied to decide whether a preexisting crack in an elastic body is stationary for given external forces. In this lecture we model the evolution of a single crack as a rate-independent process based on the Griffith criterion. The focus lies on the numerical analysis of a model which appears as the limit of evolution models with a viscous regularization term. We discuss the convergence of fully discretized models (i.e. with respect to time and space) to the vanishing viscosity model. The convergence proof relies on regularity estimates for the elastic fields close to the crack tip taking into account non-penetration conditions on the crack faces. The research is joint work with A. Schröder and C. Zanini.

A model from thermoplasticity with temperature-dependent Preisach hysteresis

Jana Kopfová (Opava)

Classical models for shape memory materials [1,2,5] can be alternatively described by a constitutive equation involving hysteresis operators. For example, the stress-strain law in the Souza-Auricchio model [1,5], which is very popular in the engineering community for its simplicity and a small number of model parameters which can easily be identified, can be equivalently reformulated, in the 1D pure tension stress-controlled or strain-controlled cases, as

$$\varepsilon = \frac{\sigma}{E} + \varepsilon_L Q\left(\frac{1}{E_h \varepsilon_L}\mathfrak{p}_r[\sigma - f(\theta)]\right) \iff \sigma = E\varepsilon - E\varepsilon_L Q\left(\frac{E}{(E_h+E)\varepsilon_L}\mathfrak{p}_{r/E}\left[\varepsilon - \frac{f(\theta)}{E}\right]\right), \tag{1}$$

see [3], where σ is the stress, ε is the strain, θ is the absolute temperature, $E > 0$ is the elasticity modulus, $E_h > 0$ is the hardening modulus, $\varepsilon_L > 0$ is the reorientation strain, $r > 0$ is the yield stress, $Q : \mathbb{R} \to [0,1]$ is the projection of $\mathbb{R}$ onto $[0,1]$, and f is the piecewise affine function $f(\theta) = b(\theta - \theta_M)$, where b and θ_M are positive parameters. The symbol $\mathfrak{p}_r[v]$ for a given function v of time $t \in [t_1, t_2]$ denotes the play operator, which is defined as the solution operator $\xi_r(t) = \mathfrak{p}_r[v](t)$ of the variational inequality

$$\begin{aligned} &|v(t) - \xi_r(t)| \le r \quad \forall t \in [t_1, t_2];\\ &\dot{\xi}_r(t)(v(t) - \xi_r(t) - y) \ge 0 \quad \text{a.e.} \ \ \forall |y| \le r\,. \end{aligned} \tag{2}$$

The ATL model of [2] is more complex and involves a nonlinear modification of the play. A mathematical analysis of the Souza-Auricchio model carried out in [3] shows that even in the quasistatic case, the energy balance is ill-posed and a regularization of the function f is necessary for the thermodynamic consistency. The problem of well-posedness in the fully dynamical case (momentum and energy balance) is completely open. To account for the memory erasure during the austenite-martensite phase transition, we propose here a different regularization inspired by the Preisach model. We replace the variational inequality (2) by

$$\begin{aligned} &|\varepsilon(t) - \eta_r(t)| \le r \quad \forall t \in [t_1, t_2];\\ &(\mu_1(\theta)\dot{\eta}_r(t) + \mu_2(\theta)(\eta_r(t) - \varepsilon(t)))(\varepsilon(t) - \eta_r(t) - y) \ge 0 \quad \text{a.e.} \ \ \forall |y| \le r\,, \end{aligned} \tag{3}$$

with given functions μ_1, μ_2 such that μ_1 vanishes for high temperatures (i.e. no memory) and μ_2 vanishes for low temperatures (i.e. the standard play operator). We propose to replace (1) by a Preisach-like formula

$$\sigma = E\varepsilon - \int_0^\infty g(r, \eta_r, \theta)\,\mathrm{d}r \tag{4}$$

with a given constitutive function g. The obvious drawback of this definition is its relative complexity in comparison with the Souza-Auricchio model. On the other hand, we show in [4] that the model is thermodynamically consistent and that the full system of dynamical balance equations is well posed.

[1] F. Auricchio and L. Petrini, "A three-dimensional model describing stress-temperature induced solid phase transformations. Part II: thermomechanical coupling and hybrid composite applications," *Internat. J. Numer. Methods Engrg.*, vol. 61, pp. 716–737, 2004.

[2] F. Auricchio, R.L. Taylor, and J. Lubliner, "Shape-memory alloys: macromodelling and numerical simulations of the superelastic behavior," *Comput. Methods Appl. Mech. Engrg.*, vol. 146, pp. 281–312, 1997.

[3] J. Kopfová and P. Krejčí, "A thermodynamically consistent temperature-dependent Preisach hysteresis model", in preparation.

[4] P. Krejčí and U. Stefanelli, "Existence and nonexistence for the full thermomechanical Souza-Auricchio model of shape memory wires", submitted, 2010.

[5] A. C. Souza, E. N. Mamiya, and N. Zouain, "Three-dimensional model for solids undergoing stress-induced phase transformations," *European J. Mech. A Solids*, vol. 17, pp. 789–806, 1998.

Efficient numerical approach to rate-independent problems

Martin Kružík (Prague)

Incremental minimization problems approximating the rate-independent evolution are typically non(quasi)convex and a suitable relaxation is necessary. A standard numerical approach is to relax the problem partially using sequential lamination. This problem is still nonconvex and difficult to solve numerically. We propose to use polyconvexification instead, which leads to a linear minimization task for which the first order optimality conditions are known. They can be used in an effective algorithm. Our recent numerical experiments prove very good performance of the algorithm. This is a joint work with Soeren Bartels (Bonn).

Quasi-static fracture based on local minimality

Chris Larsen (Worcester)

I will describe some issues in trying to prove existence for quasi-static fracture based on local minimization. In particular, why, if one does discrete-time minimization and takes the continuous-time limit, the ability of trajectories to overcome (arbitrarily small) energy barriers seems necessary. To show this, I will explain in some detail something called Jump Transfer (which is based on a quite natural view of jump sets of BV functions) and how it can be adapted for locally-minimizing evolutions.

Crack growth with non-interpenetration: the case of Lipschitz data

Giuliano Lazzaroni (Paris)

This talk concerns an existence result for quasistatic evolutions of cracks in finite elasticity, obtained in collaboration with Gianni Dal Maso. The issue is to take into account the local non-interpenetration condition, i.e., we assume that the bulk energy diverges as the deformation gradient vanishes. In this context, one of the main tools is the "multiplicative splitting" method introduced by Francfort and Mielke.

In the seminar I will discuss in particular the extension to boundary conditions with discontinuous first derivatives. The aim is to consider a class of data that is invariant under Lipschitz reparametrizations of time.

G. Dal Maso, G. Lazzaroni: *Quasistatic crack growth in finite elasticity with non-interpenetration.* Ann. Inst. H. Poincaré Anal. Non Linéaire.

G. Lazzaroni: *Quasistatic crack growth in finite elasticity with Lipschitz data.* Ann. Mat. Pura Appl.

Rate-independent Kurzweil processes

Matthias Liero (Humboldt Berlin)

The Kurzweil integral was introduced in [1] as a framework for solving ODEs with singular right-hand sides. In this talk we demonstrate how the Kurzweil integral technique can be applied to a class of rate-independent processes that may formally be described by the inclusion

$$0 \in \partial_\xi \mathcal{E}(t, \xi(t)) + \partial \mathcal{M}_{K(t)}(\dot{\xi}(t)),$$

where $\mathcal{E}$ is a convex energy functional and $\mathcal{M}_{K(t)}$ is a dissipation potential represented by the Minkowski functional of a moving convex set $K(t)$. We show that the Kurzweil integral setting allows to solve a more general problem in the space of left continuous functions of bounded variation. It is true, however, that our technique does not cover the whole range of problems treated in [3], in particular, further constraints on the state space or nonstrictly convex energies. If we reformulate the problem in the setting of energetic solutions of [2,3], it turns out that the dissipation is no longer homogeneous of degree one as in the continuous case, but additional dissipation terms related to the discontinuities occur. For a quadratic energy $\mathcal{E}$, this dissipation is quadratic and can be obtained as the limit of the viscous dissipation as the viscosity parameter tends to zero.

[1] *J. Kurzweil:* Generalized ordinary differential equations and continuous dependence on a parameter. Czechoslovak Math. J. 7 (82) (1957), 418–449.

[2] *A. Mielke, R. Rossi:* Existence and uniqueness results for a class of rate-independent hysteresis problems. Math. Models Methods Appl. Sci. 17 (2007), 81–123.

[3] *A. Mielke, F. Theil:* On rate-independent hysteresis models. NoDEA, Nonlinear Differ. Equ. Appl. 11 (2004), 151–189.

Rate-independent evolutions in fracture mechanics

Matteo Negri (Pavia)

The talk is schematically divided into two parts. The first part deals with the quasi-static evolution of a pre-existing crack in a brittle material, considering the special case of rectilinear propagation. It will be presented a characterization of the solutions in terms of Kuhn-Tucker conditions for BV functions. We will see how to find such an evolution by two methods: incremental gradient flows [1] and viscosity solutions [2]. The second part deals with the propagation of a pre-existing crack in the general case, when the path of the crack is not known a priori. We will present a preliminary existence result and show that the two methods employed above for the rectilinear case seem not suitable here.

[1] M. Negri, C. Ortner: Quasi-static crack propagation by Griffith's criterion. *Math. Models Methods Appl. Sci.* **18** (2008) 1895-1925

[2] M. Negri: A comparative analysis on variational models for quasi-static brittle crack propagation. *Adv. Calc. Var.* **3** (2010) 149-212

[3] M. Negri: From rate-dependent to rate-independent brittle crack propagation. *J. Elasticity* **98** (2010) 159-178

Adaptive FEMs for quasistatic brittle fracture

Christoph Ortner (Oxford)

We formulate and analyze an adaptive finite element method for the approximation of various flavours of the Ambrosio–Tortorelli approximation of the Griffith functional. The adaptive mesh refinement is based on residual estimates. By driving the residual to zero, we are able to prove convergence of numerical solutions to a critical point. We also investigate different flavours of the AT approximation, which lead to interesting properties of the solutions. Our numerical experiments show various pitfalls, e.g., a very sensitive dependence on the regularization parameter and the refinement tolerance.

Joint work with Siobhan Burke and Endre Süli.

On a 3D model for shape-memory alloys

Adrien Petrov (WIAS Berlin)

We investigate a three-dimensional model for isothermal stress-induced transformations in shape-memory alloys. The problem is formulated within the energetic framework of rate-independent processes. Existence and uniqueness results are recalled, a fully-discrete approximation is presented and an explicit space-time convergence rate is provided. Finally, some numerical examples are reported and analyzed. This research has been developed jointly with A. Mielke, L. Paoli and U. Stefanelli.

On the robustness and efficiency of integration algorithms for a 3D finite strain phenomenological SMA constitutive model

Alessandro Reali (Pavia)

Most devices based on shape memory alloys experience large rotations and moderate or finite strains. This motivates the development of finite-strain constitutive models together with the appropriate computational counterparts. To this end, in the present paper a three-dimensional finite-strain phenomenological constitutive model is investigated and a robust and efficient integration algorithm is proposed. Properly defining the variables, extensively used regularization schemes are avoided and a nucleationcompletion criterion is defined. Moreover, introducing a logarithmic mapping, a new form of time-discrete equations is proposed. The solution algorithm as well as a suitable initial guess for the resultant nonlinear equations are also deeply discussed. Extensive numerical tests are performed to show robustness as well as efficiency of the proposed integration algorithm. Implementation of the integration algorithm within a user-defined subroutine UMAT in the commercial nonlinear finite element software ABAQUS/Standard makes also possible the solution of a variety of boundary value problems. The obtained results show the efficiency and robustness of the proposed approach and confirm the improved efficiency (in terms of solution CPU time) when a nucleationcompletion criterion is used instead of regularization schemes, as well as when a logarithmic mapping is used for the time-discrete evolution equation instead of an exponential mapping.

A continuity method for sweeping processes

Vincenzo Recupero (Torino Politecnico)

We set the sweeping processes in the framework of rate independent operators acting between curves in metric spaces. We prove some abstract results and show their applications.

Some models of strain gradient crystal plasticity and their variational characterisation

Daya Reddy (Cape Town)

In this talk an overview is presented of some models for strain gradient single crystal plasticity, with particular reference to the modelling of dislocation densities and their incorporation in the defect energy. The variational structure of these problems is presented and aspects of their well-posedness discussed.

Lower semicontinuity and Young measures in BD

Filip Rindler (Oxford)

We establish a general weak* lower semicontinuity result in the space $\mathrm{BD}(\Omega)$ of functions of bounded deformation for functionals of the form

$$\begin{aligned}\mathcal{F}(u) := \int_\Omega f\big(x, \mathcal{E}u\big)\,\mathrm{d}x + \int_\Omega f^\infty\Big(x, \frac{\mathrm{d}E^s u}{\mathrm{d}|E^s u|}\Big)\,\mathrm{d}|E^s u| \\ + \int_{\partial\Omega} f^\infty\big(x, u|_{\partial\Omega} \odot n_\Omega\big)\,\mathrm{d}\mathcal{H}^{d-1}, \qquad u \in \mathrm{BD}(\Omega).\end{aligned}$$

The main novelty is that we allow for non-vanishing Cantor-parts in the symmetrized derivative Eu. The proof is accomplished via Jensen-type inequalities for generalized Young measures and a construction of good blow-ups, which is based on local rigidity arguments for some differential inclusions involving symmetrized gradients. This strategy allows us to prove the lower semicontinuity result without an Alberti-type theorem in $\mathrm{BD}(\Omega)$, which is not available at present. A similar strategy also allows to prove lower semicontinuity in BV without the use of Alberti's Rank One Theorem.

Some results on the vanishing viscosity approach to rate-independent modelling

Riccarda Rossi (Brescia)

The vanishing viscosity approximation of rate-independent problems has recently attracted a good deal of attention, especially in connection with the problem of describing the system behaviour at jump points, in the case of a nonconvex driving energy functional. In this talk, based on an ongoing collaboration with Alexander Mielke and Giuseppe Savare', we illustrate a technique for taking the vanishing viscosity limit, which is based on arc-length reparametrization. In this way, in the limit we obtain a novel formulation for rate-independent problems, which highlights the interplay of viscous and rate-independent effects in the jump regime. We then introduce and analyze the related concept of BV rate-independent evolutions.

Thermodynamics of rate-independent processes

Tomáš Roubíček (Prague)

Some thermodynamical systems may host activated processes with faster time scale than the rest of the system, and that might be thus considered rate independent. Coupling of such rate-independent sub-systems with the (naturally rate-dependent) thermodynamical system is usually delicate but possible in some cases. Energetic-solution concept is essential for facilitating convergence of nonlinear terms in the heat equation needed for mere existence of solutions. Specific examples will be presented.

Porous media and plasticity: homogenization for equations with hysteresis

Ben Schweizer (Dortmund)

We present two applications in which hysteresis phenomena are crucial features of the problem. One is porous media flow where hysteresis appears through the bottle-neck effect. Another is plasticity where the stress-strain relation remembers the history of the process. In both applications, homogenization questions appear naturally, namely in the following form: Given a volume that is occupied by different materials with different hysteresis properties, what is the averaged hysteresis property of the heterogeneous medium? We present an approach that adopts the method of oscillating test-functions and that gives strong theorems and simple proofs even in the context of stochastic homogenization. By this spatial averaging procedure, a play-type hysteresis relation is averaged to a Prandtl-Ishlinskii relation.

Microstructure evolution during heat treatment of shape memory alloys

Petr Šittner (Prague)

Thermomechanical properties of metals depend, in addition to their basic characteristics as elastic constants, crystal structure, phase transformations also on the microstructure which is given to the metal by the final thermomechanical processing. Typically, this includes hot/cold working passes followed by final heat treatment. The microstructure characteristics include mainly grain size, texture, density of dislocation defects, grain boundary misorientations etc.. In particular case of NiTi shape memory alloy, the final heat treatment affects in a significant extent its functional properties as superelasticity or shape memory effect. This is because during the final heat treatment, the heavily deformed microstructure of the NiTi alloy resulting from the final cold work is rebuilt by the thermally driven recovery processes into the new microstructure in which the alloy exhibits functional properties. If we can control these processes we can prepare NiTi wires with desired microstructures and functional properties. We have used a nonconventional method of heat treatment by short pulses of DC electric current to heat treat thin NiTi filaments. The cold worked NiTi filament is heated by short DC electric current pulses to temperatures as high as 1000C under stresses as high as 600MPa. We have obtained first ever experimental evidence on the recovery processes responsible for the ultrafast evolution of microstructures from the in-situ mechanical, electric resistance and synchrotron X-ray studies during the fast heat treatment and ex-situ TEM and X-ray studies. It is concluded that the recovery processes are extremely fast although probably not rate independent. These processes will be introduced, discussed and modeling approaches and problems will be outlined.

Analysis of the effect of a heat bath on a rate-independent system

Tim Sullivan (Pasadena)

The rate-independent models that describe many physical systems such as plastic evolutions usually neglect thermal effects. We develop a Markov chain model that incorporates the effect of a heat bath upon a dissipative system and assigns probabilities to system trajectories. The theory is made mathematically rigorous and leads to precise predictions regarding the behaviour of dissipative systems at positive temperature. In particular, we derive a non-linear transformation of the dissipation potential that describes the effective dissipation potential at positive temperature. Predictions derived from this theory include the effect of temperature on yield phenomena and rheological time exponents; for example, the Andrade $t^{1/3}$ creep law follows as a corollary of our results.

Timothy J. Sullivan, Marisol Koslowski, Florian Theil & Michael Ortiz (2009): *On the behavior of dissipative systems in contact with a heat bath: Application to Andrade creep,* Journal of the Mechanics and Physics of Solids 57 (7) 1059-1077.

Rate-independent damage and delamination processes

Marita Thomas (WIAS Berlin)

In [1] the existence of energetic solutions is shown for a model describing partial, isotropic damage in nonlinearly elastic materials. The model involves a regularization for the damage variable $z : \Omega \to [0,1]$, $\Omega \subset \mathbb{R}^d$, of the form $\mathcal{G}(z) = \int_\Omega |\nabla z|^r \,\mathrm{d}x$ with $r \in (1, \infty)$.

Based on this result, brittle Griffith-type delamination of compounds is deduced in [2] by means of Γ-convergence from partial, isotropic damage of sandwich-structures consisting of three constituents by flattening the middle component to thickness 0. Both processes are assumed to be rate-independent and they are treated in their so-called energetic formulation. This approach relies on a stability condition and an energy balance for an energy functional and a dissipation potential.

The limit passage is performed via a double limit: first,we gain a delamination model involving the gradient of the delamination variable, which is essential to overcome the lack of a uniform coercivity arising from the passage from partial damage to (complete) delamination. In a second limit the delamination-gradient is suppressed. Both limit models contain noninterpenetration and transmission conditions along the interface.

In this talk we discuss extentions of the results from [1,2], which feature additional properties of damage and delamination.

[1] M. Thomas and A. Mielke, *damage of nonlinearly elastic materials at small strains – existence and regularity results*, ZAMM Z. Angew. Math. Mech. 90 (2010), no. 2.

[2] A. Mielke, T. Roubíček and M. Thomas, *From Damage to Delamination in Nonlinearly Elastic Materials*, WIAS Preprint 2010.

A model for crack propagation based on viscous approximation

Rodica Toader (Udine)

In the setting of antiplane linearized elasticity, we show the existence of quasistatic evolutions of cracks in brittle materials by using a vanishing viscosity approach, thus taking into account local minimization. The main feature of our model is that the path followed by the crack needs not be prescribed a priori: indeed, it is found as the limit (in the sense of Hausdorff convergence) of curves obtained by an incremental procedure. The result is based on a continuity property for the energy release rate in a suitable class of admissible cracks.

Joint work with Giuliano Lazzaroni, UPMC Univ. Paris 06.

Periodic homogenization for a model of plasticity
MARCO VENERONI (DORTMUND)

We study the n-dimensional wave equation coupled with an elasto-plastic nonlinear stress-strain relation. In the framework of periodic homogenization, we derive an effective system of equations in the limit of small-period oscillations. We prove well-posedness for the original and homogenized systems and we build a sequence of correctors. The proof relies on Tartar's method of the oscillating test function and on an adapted version of div-curl Lemma.

On rate-independent models of mixed-mode delamination phenomena
JAN ZEMAN (PRAGUE)

In this contribution, we introduce an energy-based rate-independent formulation of decohesion processes in adhesively bonded assemblies such as laminated composites. The model itself is based on a variational formulation of isotropic, possibly non-associated approaches of interfacial damage mechanics. Existence of the energetic solution is shown by extending the existing framework to systems with state-dependent dissipation to treat non-convex energetic functionals. Next, following the alternate minimization strategy introduced recently by Bourdin, a fully discrete version of the model is presented and efficiently resolved using duality-based solvers. The numerical efficiency and robustness of the resulting algorithm is demonstrated by analyzing representative tests of mixed-mode delamination. This is a joint work with M. Kružík, T. Roubíček and P. Gruber (Prague).

Rate-independent evolution for an elastic-plastic model of shape memory alloys and related function spaces
JOHANNES ZIMMER (BATH)

We describe a (meso/-macroscopic) rate-independent model that combines plasticity with the shape memory effect. Thus, the evolution is a rate-independent elastic-plastic process, with the dissipation having elastic and plastic contributions. We present the model, describe the analysis and show two-dimensional simulation results. Rate-independent evolution processes in plasticity (as well as their static counterparts) often lead to energies with linear growth, and the talk will recall the reason for this. For rate-independent evolution, one would then often like to prescribe boundary data, possibly time dependent. This cannot be formulated easily in the space BD of functions of bounded deformations, since compactness properties *as well as* continuity of the trace need to hold simultaneously for simple arguments. We present a new function space which is a fine extension of the space of bounded deformations, building on (in my opinion) very useful but little-know work by Souček. This is joint work Martin Kružík (Czech Academy of Sciences, Prague).

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