3D and 4D variational data assimilation

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1.

Historical perspective

- **Motivation for the development of 3D-Var**
  - Improve our capacity to use new types of observations particularly satellite radiances (Eyre, 1989; Thépaut and Moll, 1990)
  - New background-error statistics models without data selection
  - Extension to 4D-Var (Talagrand and Courtier, 1987)
- **NCEP (1992), ECMWF (1996), Météo-France and CMC (1997), MetOffice (1999)**

2.

Historical perspective (2)

- **Dual 3D-Var (Courtier, 1997)**
  - NASA's Global Modeling and Assimilation Office (GMAO) (Cohn et al., 1998)
  - Naval Research Laboratory (Daley and Barker, 2000)
- **4D-Var**
  - ECMWF (1997), Météo-France (2000), MetOffice (2004), JMA (2005), Meteorological Service of Canada (2005), NRL (2009)

3.

Plan of presentation

- **3D-Var**
  - Introduction of the incremental formulation
  - First-Guess at Appropriate Time (FGAT)
- **4D-Var**
  - Extension from 3D to 4D-Var
  - Incremental formulation
  - Evaluation of the impact of the first implementation of 4D-Var at the Meteorological Service of Canada
- **Current issues**
  - Comparaison of 4D-Var with the Ensemble Kalman filter
  - Hybrid formulation
  - Taking into account model error: the weak-constraint 4D-Var

4.

The variational problem

Bayes' Theorem: $$p(\mathbf{x}\,|\,\mathbf{y}) = \frac{p(\mathbf{y}\,|\,\mathbf{x})P(\mathbf{x})}{P(\mathbf{y})}$$

- **Example:**
  - Observation and background error with Gaussian distributions

$$p(\mathbf{y}\,|\,\mathbf{x}) = \frac{1}{C_3}\exp\left\{-\frac{1}{2}(\mathbf{y}-\mathbf{H}(\mathbf{x}))^T\mathbf{R}^{-1}(\mathbf{y}-\mathbf{H}(\mathbf{x}))\right\}$$

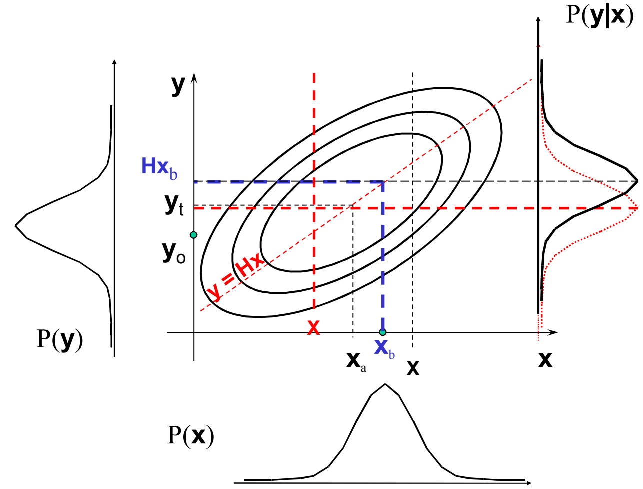
$$P(\mathbf{x}) = \frac{1}{C}\exp\left\{-\tfrac{1}{2}(\mathbf{x}-\mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b)\right\}$$

  - $p(\mathbf{y}|\mathbf{x})$ is Gaussian only if **H** is linear
  - Maximum likelihood estimate (mode of the distribution):

$$J(\mathbf{x}) = -\ln p(\mathbf{x}\,|\,\mathbf{y})$$
$$= \tfrac{1}{2}(\mathbf{x}-\mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b)+\tfrac{1}{2}(\mathbf{H}(\mathbf{x})-\mathbf{y})^T\mathbf{R}^{-1}(\mathbf{H}(\mathbf{x})-\mathbf{y})$$

- Reducing $J(\mathbf{x})$ implies an increase in the probability of **x** being the true value

Representation of related probability distributions



Incremental approach

Successive linearizations with respect to the full model state is obtained

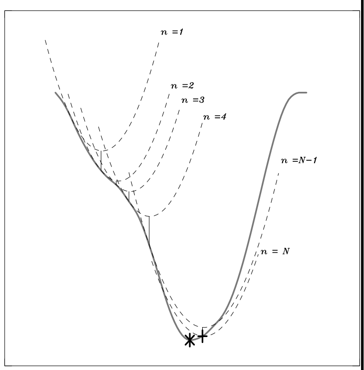
- Minimization of quadratic problems

$$J(\xi) = \tfrac{1}{2}\delta\mathbf{x}^T\mathbf{B}^{-1}\delta\mathbf{x}+\tfrac{1}{2}(\mathbf{H}'\delta\mathbf{x}-\mathbf{y}')^T\mathbf{R}^{-1}(\mathbf{H}'\delta\mathbf{x}-\mathbf{y}')$$
$$\equiv \tfrac{1}{2}\xi^T\xi+\tfrac{1}{2}(\mathbf{H}'\mathbf{B}^{1/2}\xi-\mathbf{y}')^T\mathbf{R}^{-1}(\mathbf{H}'\mathbf{B}^{1/2}\xi-\mathbf{y}')$$

where

$\delta\mathbf{x} = \mathbf{B}^{1/2}\xi$
$\delta\mathbf{x} = \mathbf{x} - \mathbf{x}_b$: increment
$\mathbf{H}' = \partial\mathbf{H}/\partial\mathbf{x}$: tangent-linear of the observation operator

$\mathbf{y}' = \mathbf{y} - \mathbf{H}(\mathbf{x}_b)$:
innovation vector (observation departure with respect to the high resolution background state)



From Laroche and Gauthier (1998)

3D-Var: variational formulation of the statistical estimation problem

Minimization of the cost function

$$J(\xi) = \frac{1}{2}\xi^T\xi+\frac{1}{2}\left(\mathbf{H}'\mathbf{B}^{1/2}\xi-\mathbf{y}'\right)^T\mathbf{R}^{-1}\left(\mathbf{H}'\mathbf{B}^{1/2}\xi-\mathbf{y}'\right)$$

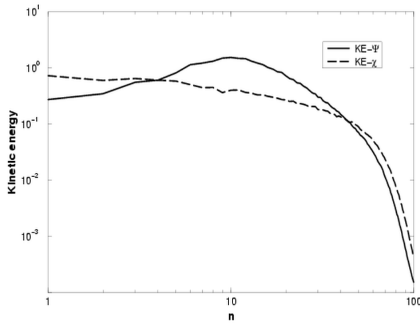
with $\delta\mathbf{x}_a = \mathbf{B}^{1/2}\xi^*$, ξ^* being min $(J(\xi))$

where
$\delta\mathbf{x} = \mathbf{x} - \mathbf{x}_b$: increment
$\mathbf{H}' = \partial\mathbf{H}/\partial\mathbf{x}$: tangent-linear of the observation operator
$\mathbf{y}' = \mathbf{y} - \mathbf{H}(\mathbf{x}_b)$: innovation vector (observation departure) (computed with respect to the high resolution background state)

8.

Autocorrelation spectra of rotational and divergent components of background-error

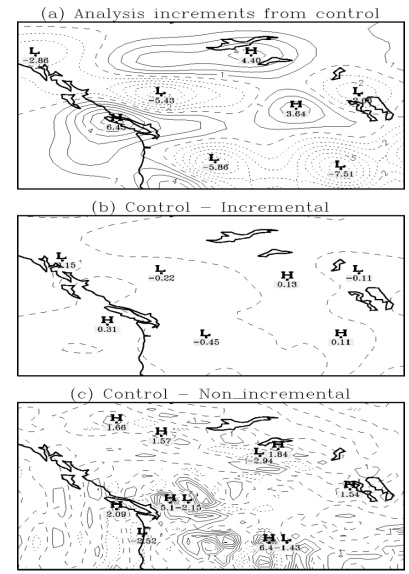
A triangular truncation $n \leq 108$ (T108) resolution is sufficient to represent the whole autocorrelation spectra



9.

Regional analysis increment

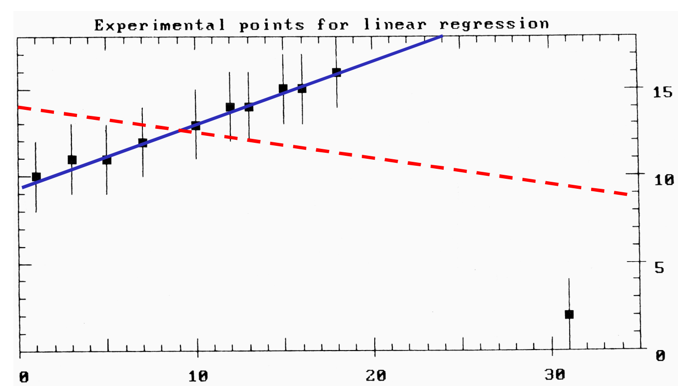
- **Analysis increment produced at full resolution (~ 50 km) (control)**
- **Control - Incremental (increment has a resolution of ~200 km)**
- **Control - Non-incremental (innovations produced with respect to the low resolution background state)**



Variational Quality control (QC-Var)

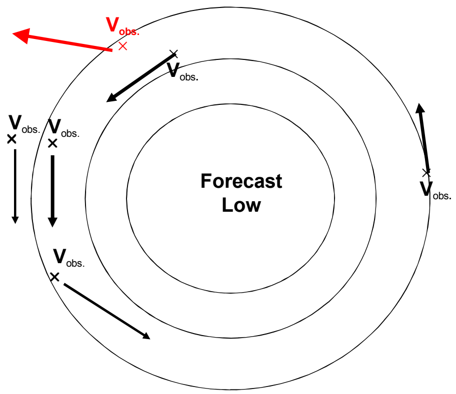
Example: least-square fit involving an erroneous datum (from Tarantola, 2005)

- **Least-square fit of data: y = ax + b**



12.

Background-check procedure



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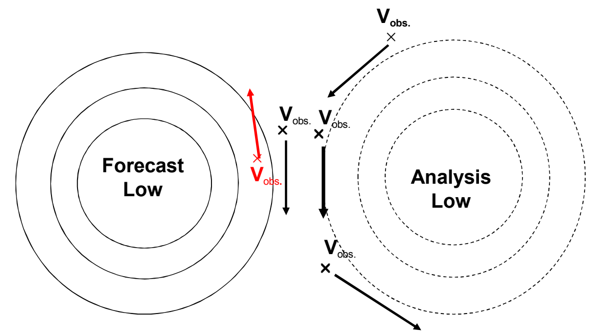
Difficulties that arise with the background-check procedure

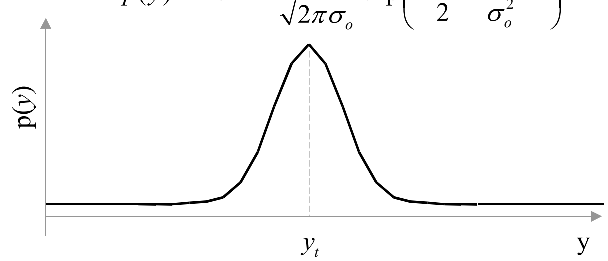


14.

Variational Quality Control (QC-Var)

- **Dharssi *et al.* (1992), Ingleby and Lorenc (1993), Andersson and Järvinen (1999)**
- **Probability of having a gross error**
  - consider that $y_t - D/2 \le y \le y_t + D/2$

$$p(y) = P/D + \frac{(1-P)}{\sqrt{2\pi}\sigma_o}\exp\left(-\frac{1}{2}\frac{(y-y_t)^2}{\sigma_o^2}\right)$$



15.

QC-Var

Definition of the cost function

$$J_o^{QC}(\mathbf{x}) \equiv J_o^{QC}(\hat{y}(\mathbf{x})) = -\ln p(y_o \mid H(\mathbf{x}))$$

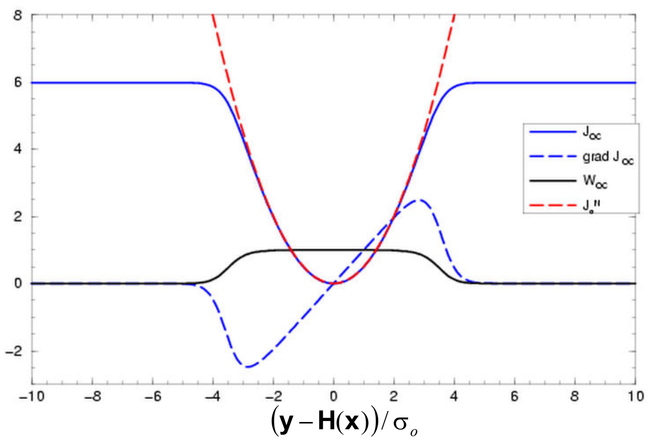
where

$$= -\ln\left(P/D + C\exp\left(-J^N(\hat{y})\right)\right)$$

$$J^N(\hat{y}(\mathbf{x})) = \frac{1}{2}\hat{y}^T\mathbf{R}^{-1}\hat{y} \equiv \frac{1}{2}\frac{(H(\mathbf{x})-y_o)^2}{\sigma_o^2}$$

16.

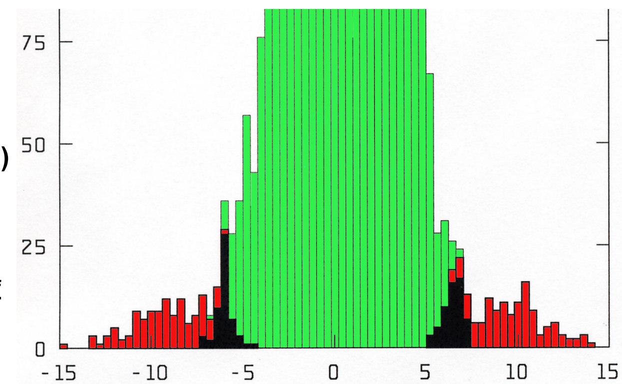
Representation of the QC-Var cost function (P = 0.01)



Observation - Forecast $(y - H(x_b))$
AIREP temperatures Period: March-April 2002

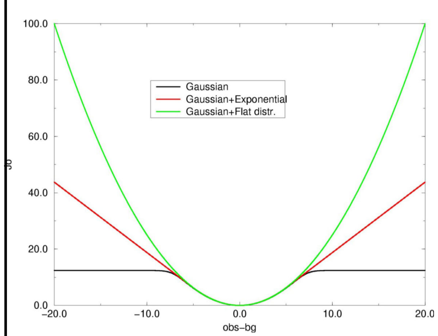
Rejected by background check (303)
Rejected by QC-Var (103)
Accepted (31,926)

Total Number of data 32239



18.

Recent developments in variational quality control
(Isaksen, L., 2010: presentation at the ECMWF training course)



Huber norm

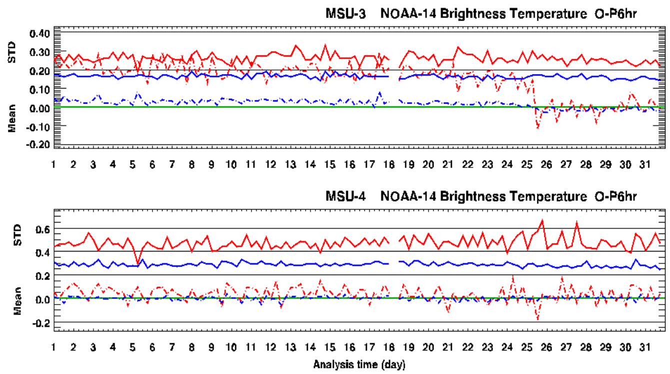
- Adds some weight on observations with large departures
- A set of observations with consistent large departures will influence the analysis

(Isaksen, 2010 ECMWF)

BIAS CORRECTION
(from Aulighé, McNally and Dee, QJ 2007)

Monitoring and quality control

Statistics based on innovations ($y - HX_b$): example from TOVS radiances



Static bias correction

- Consider innovations $\mathbf{d} = \mathbf{y} - H(\mathbf{x}_b)$ over a period of time (order of a month)

$$\langle \mathbf{y} - H(\mathbf{x}_b) \rangle \cong \langle \varepsilon_o - \mathbf{H}\varepsilon_b \rangle = \langle \varepsilon_o \rangle + \langle \mathbf{H}\varepsilon_b \rangle \approx \langle \varepsilon_o \rangle$$

- Based on the assumption that the background error itself is unbiased
- **Background error is constrained by *all* observations**
  - → Justified where unbiased observations are available (e.g., radiosondes)
  - → Only innovations of satellite data in the vicinity of radiosondes are considered

Static bias correction

- Modify the observation operator as

$$\tilde{H}(\mathbf{x}, \beta) = H(\mathbf{x}) + \sum_{i=0}^{N} \beta_i P_i(\mathbf{x})$$

- Find the coefficients β by minimising

$$J(\beta) = \frac{1}{2}\left(\mathbf{y} - \tilde{H}(\mathbf{x}_b, \beta)\right)^T \left(\mathbf{y} - \tilde{H}(\mathbf{x}_b, \beta)\right)$$

- The quantities $P_i(\mathbf{x})$ are the predictors which relate to the measurements

Predictors used for different satellite instruments

(Auligné, McNally and Dee, QJ 2007)

Instrument		Predictors		
AIRS	1000-300	200-50	10-1	50-5
ATOVS	1000-300	200-50	10-1	50-5
GEOS	1000-300	200-50	TCWV	
SSMI	V_s	T_s	TCWV	

- Geopotential thicknesses for the layers comprised between the pressures (in hPa)
- TCWV: total content in water vaport
- V_s: surface wind speed T_s: skin temperature

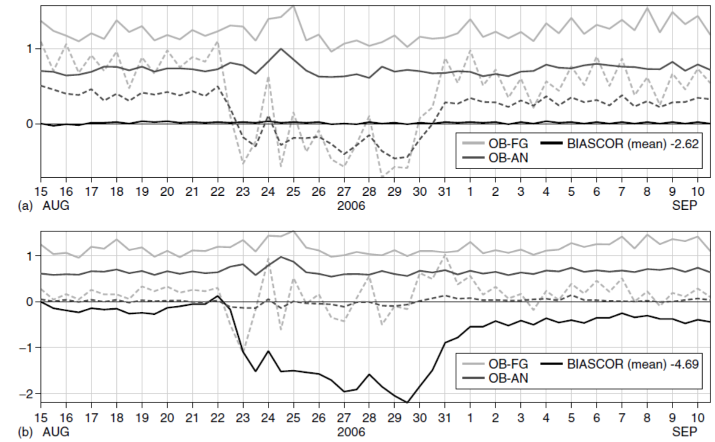
Adaptive online scheme: Var-BC

- **Bias correction is incorporated within the assimilation scheme itself**

$$J(\mathbf{x},\beta)=\frac{1}{2}\left(\mathbf{y}-\tilde{H}(\mathbf{x},\beta)\right)^T\mathbf{R}^{-1}\left(\mathbf{y}-\tilde{H}(\mathbf{x},\beta)\right)$$
$$+\frac{1}{2}(\beta-\beta_b)^T\mathbf{B}_\beta^{-1}(\beta-\beta_b)$$
$$+\frac{1}{2}(\mathbf{x}-\mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b)$$

- **More apt to distinguish between model bias and observation biases.**

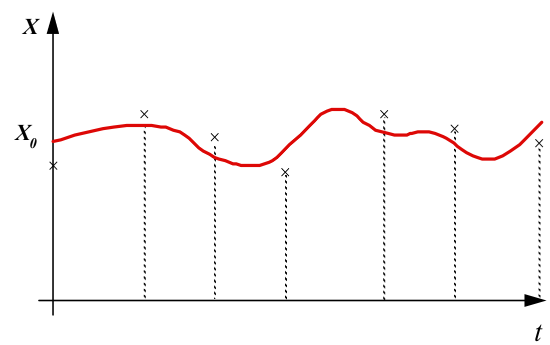
Comparison between VarBC and static bias correction (Auligné *et al.*, 2007)



Summary up to now

- **Variational assimilation made it possible to assimilate raw measurements, particularly those from satellite instruments**
- **Derived from a Bayesian perspective, the variational form is not restricted to Gaussian probability distributions**
  - Non-Gaussian observation error distributions are used to perform implicitly the quality control of observations
- **Online variational bias corrections is also a very convenient method to detect and correct systematic errors in the observations**
- **3D-Var can be naturally extended to 4D-Var**

4D variational data assimilation (4D-Var)



- **Observation operator now involves a model integration that carry the initial conditions up to the time of the observations**

32.

Extension of 3D-Var to 4D-Var

Cost function

$$J(\xi)=\frac{1}{2}\xi^T\xi+\frac{1}{2}\left(\mathbf{H}'\mathbf{L}(t_o,t)\mathbf{B}^{1/2}\xi-\mathbf{y}'\right)^T\mathbf{R}^{-1}\left(\mathbf{H}'\mathbf{L}(t_o,t)\mathbf{B}^{1/2}\xi-\mathbf{y}'\right)$$

- Representation of the covariances contained within the change of variables $\delta\mathbf{x}_0=\mathbf{B}^{1/2}\xi$
- Each iteration of the minimization involves approximately 2-3 model integrations over the assimilation window (0 < t < T)
- Incremental formulation allows to reduce the cost of 4D-Var by using a simplified model, the tangent linear model linearized around the current model trajectory (Courtier *et al.*, 1994)

33.

Tangent Linear model and Adjoint Model (LeDimet and Talagrand, 1986)

- **Direct Model** : $\frac{d\mathbf{X}}{dt}=\mathbf{F}(\mathbf{X})$
- **Tangent Linear Model** : $\frac{d\delta\mathbf{X}}{dt}=\left(\frac{\partial\mathbf{F}}{\partial\mathbf{X}}(\mathbf{X}_R(t))\right)\delta\mathbf{X}$ $\equiv L\delta\mathbf{X}$
- **Adjoint Model** : $\frac{d\delta^*\mathbf{X}}{dt}=-\left(\frac{\partial\mathbf{F}}{\partial\mathbf{X}}(\mathbf{X}_R(t))\right)^*\delta^*\mathbf{X}$ $\equiv -L^*\delta^*\mathbf{X}$

Example: the Lorenz (1963) model

- Direct Model

$$\frac{dX}{dt}=\sigma(-X+Y),\quad \frac{dY}{dt}=-XZ+rX-Y,\quad \frac{dZ}{dt}=XY-bZ,$$

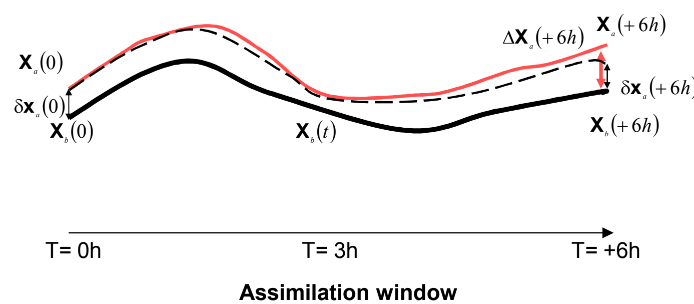
- Tangent Linear Model (TLM)

$$\frac{d}{dt}\begin{pmatrix}\delta X\\ \delta Y\\ \delta Z\end{pmatrix}=\begin{pmatrix}-\sigma & +\sigma & 0\\ -Z_R(t)+r & -1 & -X_R(t)\\ Y_R(t) & X_R(t) & -b\end{pmatrix}\begin{pmatrix}\delta X\\ \delta Y\\ \delta Z\end{pmatrix}$$

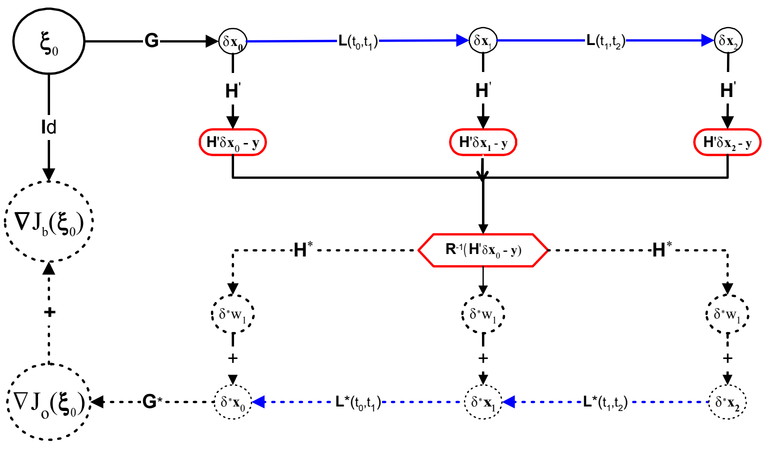
- Adjoint Model

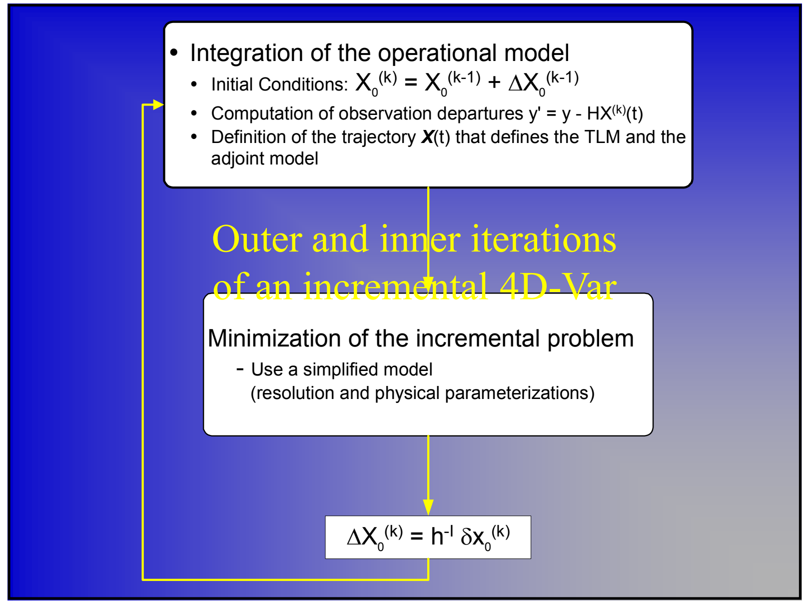
$$\frac{d}{dt}\begin{pmatrix}\delta^* X\\ \delta^* Y\\ \delta^* Z\end{pmatrix}=-\begin{pmatrix}-\sigma & -Z_R(t)+r & Y_R(t)\\ +\sigma & -1 & X_R(t)\\ 0 & -X_R(t) & -b\end{pmatrix}\begin{pmatrix}\delta X\\ \delta Y\\ \delta Z\end{pmatrix}$$

Schematic of the incremental 4D-Var



Operations involved in a single iteration of 4D-Var

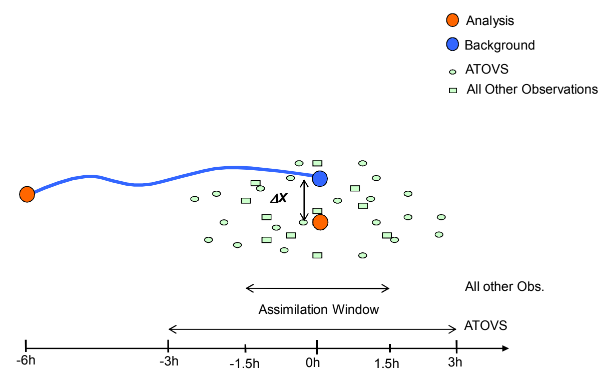




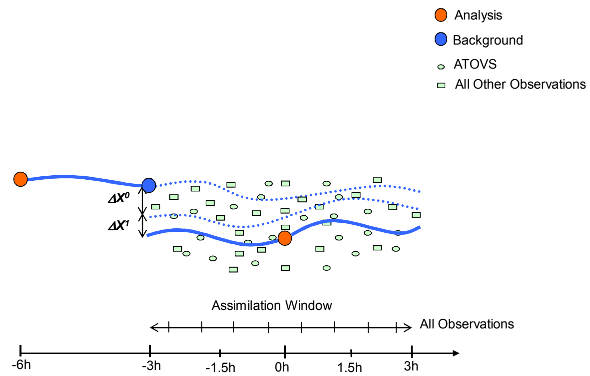
Impact of 4D-Var in the Canadian operational assimilation and forecasting system

Results from Laroche et al. (2007), Environment Canada

3D-Var



4D-Var



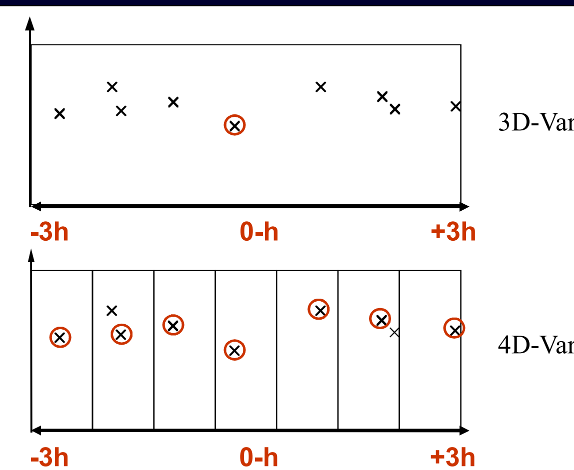
Configurations

	Outer loop	Number of inner loops	Simplified physics	Low-resolution Analysis increments	High-resolution trajectory
Regional					
3D-Var	1	~ 90	-	1.5º (T108) L58	~15 km L58
Global					
4D-Var	1	30	-PBL	1.5º (T108) L58	(0.3º x 0.45º) L58
	2	25	-PBL -SGO -Stratiform precip.	1.5º (T108) L58	(0.3º x 0.45º) L58

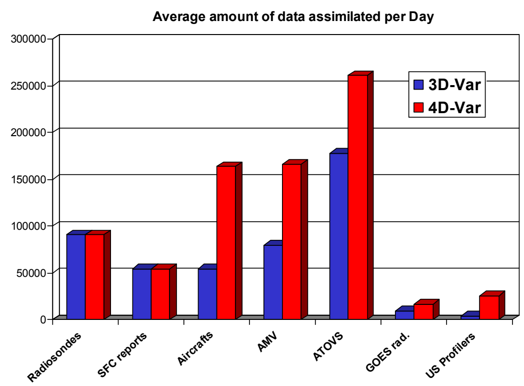
Observations assimilated at the CMC

Type	Variables	Thinning
radiosonde/dropsonde	U, V, T, (T-T$_d$), p$_s$	28 levels
Surface report	T, (T-T$_d$), p$_s$, (U, V over water)	1 report/6h
Aircraft (BUFR, AIREP, AMDAR, ADS)	U, V, T	1º x 1º x 50 hPa
ATOVS NOAA , AQUA	Ocean / Land AMSU-A: 3-10 / 6-10 AMSU-B: 2-5 / 3-4	250 km x 250 km
Water vapor channel GOES	IM3 (6.7 μ)	2º x 2º
AMV (Meteosat, GOES, MTSAT)	U,V (IR, WV, VI channels)	1.5º x 1.5º
MODIS (Aqua, Terra)	U,V	1.5º x 1.5º
Profiler (NOAA Network)	U,V	(750 m) Vertical

Temporal thinning



Average amount of data assimilated per Day



Impact of the various components of 4D-Var

Type	Outer loops	Simplified Physics	Temporal thinning
3D-Var	1	-	3D
3D-Var (FGAT)	1	-	3D
4D-Var (1 loop)	1	(simpler)	4D
4D-Var (simpler)	2	(simpler, simpler)	4D
4D-Var (3D-thin)	2	(simpler, better)	3D
4D-Var	2	(simpler, better)	4D

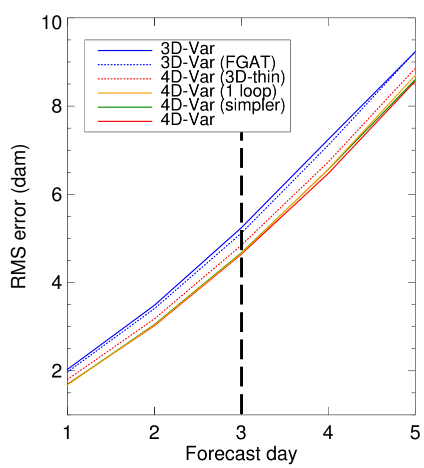
Impact of the various components of 4D-Var

August 2004

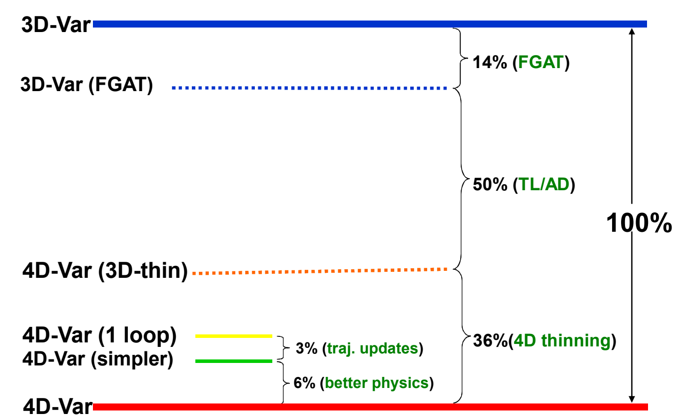
RMS error

GZ 500 hPa

Southern Hemisphere



Impact of the various components of 4D-Var



Assimilated radiances: major input in Strato-2b from new data and increased thinning

Number of radiance observations assimilated February 1st, 2009 (4 analyses):

Instrument	Platform	Strato 2a	Strato 2b	% Change
AIRS	AQUA	392 554	659 751	+ 68%
IASI	Metop-2	0	500 783	New
AMSU-A	NOAA-15	121 875	338 194	+ 178%
	NOAA-18	170 773	472 474	+ 177%
	AQUA	119 805	331 557	+ 177%
AMSUB	NOAA-15	14 762	41 350	+ 180%
	NOAA-16	30 082	84 341	+ 180%
	NOAA-17	32 965	92 609	+ 181%
MHS	NOAA-18	34 671	96 025	+ 177%
SSMI	DMSP-13	37 965	60 761	+ 60%
SSMIS	DMSP-16	0	39 330	New
GOES Imager	GOES-11	11 813	34 967	+ 196%
	GOES-12	10 024	41 919	+ 318%
SEVERI	MSG-2	0	69 183	New
MVIRI	Meteosat-7	0	41 882	New
GMS MTSAT	MTSAT-1	0	20 612	New
All Radiances:		**977 289**	**2 925 788**	**+ 199%**

Environment Canada Environnement Canada — Canada

4D-Var – EnKF intercomparison

Acknowledgments: Mark Buehner, Peter Houtekamer, Herschel Mitchell
Environment Canada,

Experimental Systems (Buehner et al., 2010,a-b)

Modifications to configurations operational during summer 2008

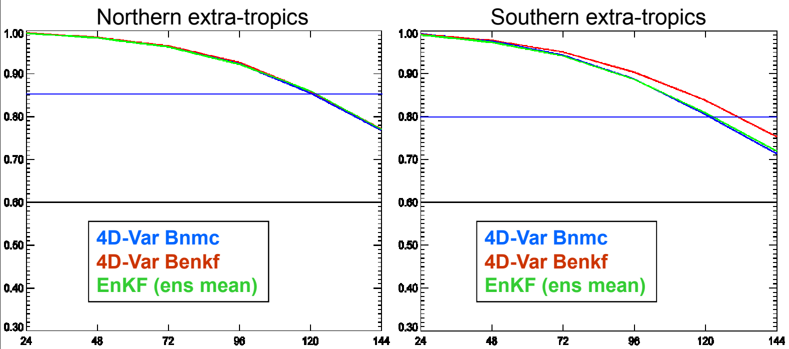
- **4D-Var**
  - incremental approach: ~35km/150km grid spacing, 58 levels, 10hPa top → Increased horizontal resolution of inner loop to 100km to match EnKF
- **EnKF**
  - 96 ensemble members: ~100km grid spacing, 28 levels, 10hPa top → Increased number of levels to 58 to match 4D-Var
- **Same observations assimilated in all experiments:**
  - radiosondes, aircraft observations, AMVs, US wind profilers, QuikSCAT, AMSU-A/B, surface observations
  - eliminated AIRS, SSM/I, GOES radiances from 4D-Var
  - quality control decisions and bias corrections extracted from an independent 4D-Var experiment

Experimental Configurations

- **Variational data assimilation system:**
  - 3D-FGAT and 4D-Var with **B** matrix nearly like operational system: NMC method
  - 3D-FGAT and 4D-Var with flow-dependent **B** matrix from EnKF at middle or beginning of assimilation window (same localization parameters as in EnKF)
  - Ensemble-4D-Var (En-4D-Var): use 4D ensemble covariances to produce 4D analysis increment without TL/AD models (most similar to EnKF approach)
- **EnKF:**
  - Deterministic forecasts initialized with EnKF ensemble mean analysis (requires interpolation from ~100km to ~35km grid)

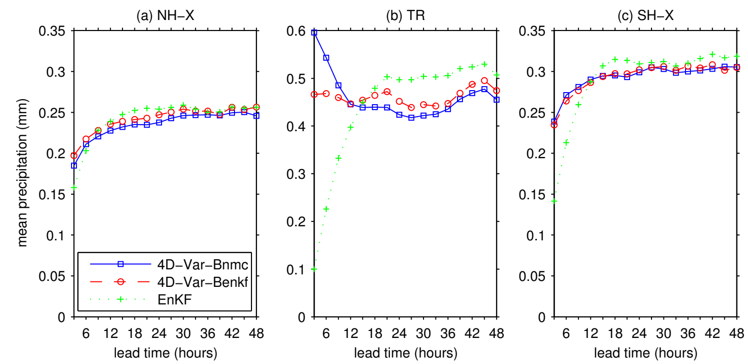
Results – 500hPa GZ anomaly correlation

Large improvement from using flow-dependent covariances in 4D-Var



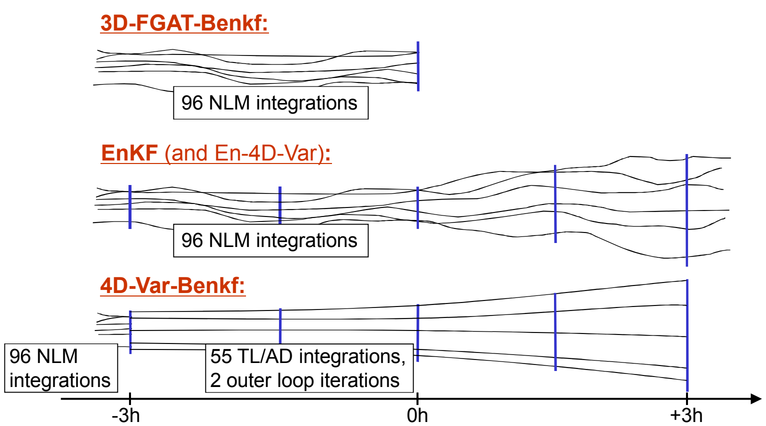
Forecast Results – Precipitation

Evolution of mean 3-hour accumulated precipitation



4D error covariances

Temporal covariance evolution (explicit vs. implicit evolution)

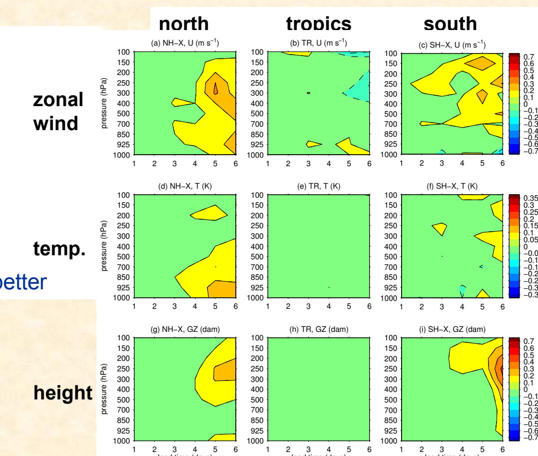


Forecast Results:
En-4D-Var vs. 3D-FGAT-Benkf

Difference in stddev relative to radiosondes:

Positive → En-4D-Var better

Negative → 3D-FGAT-Benkf better



Forecast Results:
En-4D-Var vs. 4D-Var-Benkf

Difference in stddev relative to radiosondes:

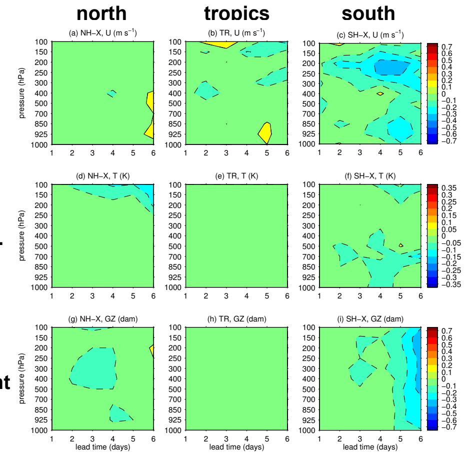
Positive → En-4D-Var better

Negative → 4D-Var-Benkf better

zonal wind

temp.

height



Summary

- **Major future improvements of 4D-Var would require significant effort:**
  - optimization/reformulation of GEM TL/AD and development of linearized physics
  - improved background-error covariances by using EnKF ensemble → requires synchronized development of 4D-Var and EnKF
  - significant redesign of variational code to facilitate major future changes to model (vertical co-ord, yin-yang, icosahedral etc.)
- **Use of En-4D-Var (without GEM TL/AD):**
  - advantages of a variational analysis could be preserved by using a variational solver within EnKF
  - allows use of some alternative approaches for modelling covariances: e.g. averaged covariances
  - allows use of var QC and Var-BC
  - requires further research to determine if it can be made sufficiently computationally efficient (in progress)

Ensemble Kalman filter

- **Basic equations of a Kalman filter**

$$\mathbf{X}_n^a = \mathbf{X}_n^f + \mathbf{K}_n\left(\mathbf{y}_n - \mathbf{H}_n\mathbf{X}_n^b\right) \qquad \mathbf{P}_n^a = \left(\mathbf{I} - \mathbf{K}_n\mathbf{H}_n\right)\mathbf{B}_n$$

$$\mathbf{K}_n = \mathbf{B}_n\mathbf{H}_n^T\left(\mathbf{R}_n + \mathbf{H}_n\mathbf{B}_n\mathbf{H}_n^T\right)^{-1} \qquad \mathbf{X}_{n+1}^f = F(\mathbf{X}_n^a)$$

$$\mathbf{B}_{n+1} = R_n\mathbf{P}_n^a R_n^T + \mathbf{Q}$$

- **Ensemble Kalman filter**

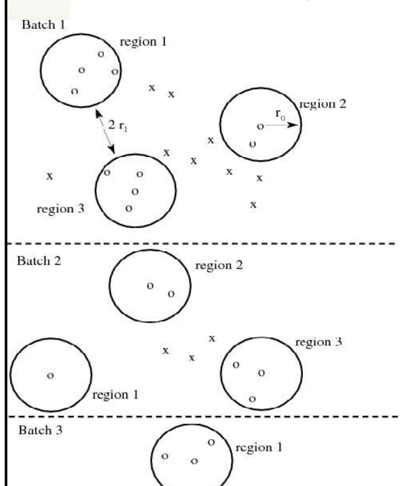
$$\delta\mathbf{x}_f^{(i)}(t_F) = N\left(\mathbf{X}_a^{(i)}\right) - N(\mathbf{X}_a)$$

$$\mathbf{B}(t_F) \cong \left\langle \left(\delta\mathbf{x}_f^{(i)}\right)\left(\delta\mathbf{x}_f^{(i)}\right)^T \right\rangle = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(\delta\mathbf{x}_f^{(i)}\right)\left(\delta\mathbf{x}_f^{(i)}\right)^T$$

- **The MSC EnKF solves explicitly**

$$\left(\mathbf{R} + \mathbf{HBH}^T\right)^{-1}\left(\mathbf{y} - \mathbf{H}\left(\mathbf{x}_b^{(i)}\right)\right) = \mathbf{w}^{(i)}$$

Sequential algorithm



Schematic illustration of the strategy used to form batches of observations.

$$\left(\mathbf{R} + \mathbf{HB}^{(k)}\mathbf{H}^T\right)^{-1}\left(\mathbf{y} - \mathbf{H}\left(\mathbf{x}_b^{(k)}\right)\right) = \mathbf{w}^{(k)}$$

At each assimilation step,

- the circles represent the observations to be assimilated at this step, while
- the x's denote observations that have not yet been assimilated.

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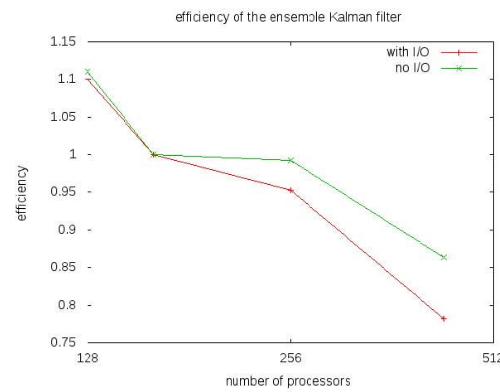
Sequential algorithm
(Houtekamer and Mitchell)

- In the EnKF, batches of p_{max} (~1000) neighbouring observations are assimilated using a sequential algorithm.
- Allows use of a direct solution method (Cholesky decomposition) for solving the analysis equation.
- Computational cost increases as p_{max}^3 and approximately linearly with number of batches.
- In practice, then, more observations implies more batches.

Efficiency of the ensemble Kalman filter

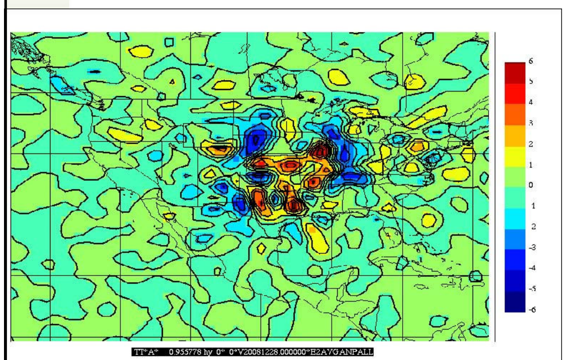


EnKF uses a sequential algorithm to solve

$$(\mathbf{R}+\mathbf{HBH}^T)^{-1}\left(\mathbf{y}-\mathbf{H}(\mathbf{x}_b^{(k)})\right)=\mathbf{w}^{(k)}$$

This approach would have to be changed if the volume of data is to be doubled

Impact of altering the order of observations in the processing



Where there are lots of observations, changing the order of the observation processing can significantly alter the result

Results from one extreme case

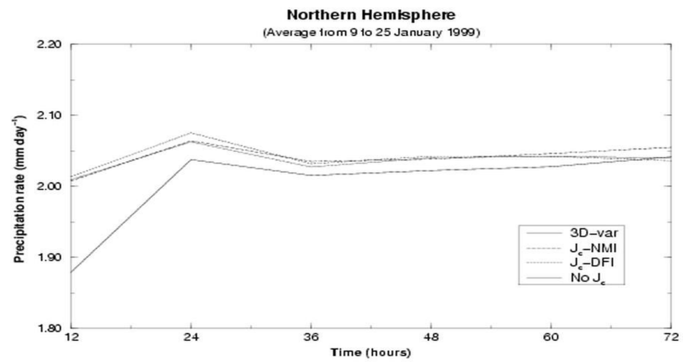
Impact of having larger volumes of data

- The EnKF algorithm behaves poorly when the number of observations exceeds the number of degrees of freedom of the model state
- The sequential algorithm then shows a large dependence to the order in the observation processing and the ensemble then lacks dispersion
- To allow for small scale structures, with the current algorithm, it would be necessary to localize even more (at the expense of the larger scales) or increase the number of members.
- High resolution reference member should be used instead of the ensemble mean (incremental EnKF)

Conclusion

- **Variational formulation of the statistical estimation problem allows**
  - Take into account non-Gaussian error distributions
  - Quality control of observations can be embedded within the variational problem
  - Online bias correction of observations is very useful to detect and correct faulty observations and prevent them from altering the analysis
- **Extension to 4D-Var**
  - All forms of variational assimilation can be expressed in a similar form (e.g., 3D/4D-Var, strong and weak constraint 4D-Var) (Courtier, 1997)
  - Additional penalty term can be added to enforce balance constraints (Gauthier and Thépaut, 2001)

Impact of having a J_c-DFI penalty term



Gauthier and Thépaut (2001)

- **Intercomparison of EnKF and 4D-Var and the impact of flow dependent background-error covariances**
  - 4D-Var with operational **B** and EnKF ensemble mean analyses have comparable quality
    - → 4D-Var better in extra-tropics at short-range, EnKF better in the medium range and tropics
  - Largest impact (~9h gain at day 5) in southern extra-tropics for 4D-Var with flow-dependent EnKF **B** vs. 4D-Var with operational **B** and also better in tropics
- **Use of 4D ensemble B in variational system (i.e. En-4D-Var):**
  - improves on 3D-FGAT, but inferior to 4D-Var (both with 3D ensemble **B**), least sensitive to covariance evolution in tropics
  - comparable with EnKF

Conclusion

- **Weak-constraint 4D-Var offers both promises and challenges**
  - Provides information about model error that can be used to diagnose and correct deficiencies in the model
  - Improving the model is a requirement to improve the forecasts
  - Computational issues need to be addressed before a full fledge weak-constraint 4D-Var can be envisioned
  - … but already implemented at ECMWF