Outline

Definitions, unit

Climatology of PV and PV anomalies

Main characteristics: conservation, invertibility, partitioning

*Cloud condensational PV production / destruction*

Case study 1: North Atlantic cyclogenesis

Case study 2: extratropical transition of hurricane *Hanna* (2008)

Further reading
Note the gradation in definitions of potential vorticity

**Full (Ertel) PV**  \[ PV \propto \tilde{\eta} \cdot \tilde{\nabla} \theta \]  with  \[ \tilde{\eta} = \tilde{\nabla} \wedge \vec{u} + 2\vec{Q} \]

**Isentropic PV**  \[ IPV \propto (\zeta + f) \cdot \theta_z \]

**Quasi-geostrophic PV**  \[ q \propto \zeta + \theta_z \]

and for each definition there is a conservation principle, i.e.,

\[ \frac{D\{PV\}}{Dt} = 0 \]  for inviscid & adiabatic flow

where \( D/Dt \) denotes the appropriate material derivative
Definition of Ertel PV and unit

Definition

\[ Q = \frac{1}{\rho} \vec{\eta} \cdot \vec{\nabla} \theta \]

good approximation for synoptic scales

Unit

\[ Q \approx \frac{1}{\rho} f \frac{\partial \theta}{\partial z} \approx 1 \text{ kg}^{-1} \text{ m}^3 \cdot 10^{-4} \text{ s}^{-1} \cdot 100 \text{ K/10}^4 \text{ m} \]

\[ = 10^{-6} \text{ m}^2 \text{ s}^{-1} \text{ K kg}^{-1} \]

1 pvu = 10^{-6} m^2 s^{-1} K kg^{-1}
PV characteristics

Conservation for adiabatic, frictionless flows

\[
\frac{D}{Dt} Q = 0,
\]

where

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}
\]

is the total or material derivative

Invertibility

Q-distribution in atmosphere & potential temperature at the surface can be “inverted” to derive \( u, v, T, p \)

math: solve elliptic PDE

Partitioning

Q-distribution can (often) be partitioned in distinct “anomalies”, which interact with each other
winter climatology of PV and $\theta$
winter climatology of PV and θ
summer climatology of PV and $\theta$
PV inversion of positive upper-level PV anomaly
negative upper-level PV anomaly
negative surface $\theta$-anomaly
positive surface $\theta$-anomaly
PV non-conservation

PV non-conservation in the presence of frictional and diabatic processes:

\[
\frac{D}{Dt} Q = -g \mathbf{\eta}_p \cdot \nabla_p \dot{\theta} - g \nabla_p \theta \cdot (\nabla_p \wedge \mathbf{F}).
\]

- gradient of diabatic heating rate
- non conservative forces (e.g., friction)

Good approximation on synoptic scales:

\[
\frac{D}{Dt} Q \simeq -g (f + \zeta) \frac{\partial \theta}{\partial p}.
\]
A case study of North Atlantic cyclogenesis

06 UTC 22 Nov

potential temperature on 850 hPa sea level pressure (SLP)
potential vorticity on 315 K sea level pressure (SLP)
18 UTC 22 Nov
IR satellite image at 13 UTC 23 Nov
The PV tower
at 18 UTC 23 Nov (PV and RH)

S-N cross section

W-E cross section
The PV tower
at 18 UTC 23 Nov (PV and backward trajectories)
Recommended further reading

Kleinschmidt (1950, Meteorol. Zeitschrift): A historical application of PV concepts

Hoskins et al. (1985, QJRMS): The key paper that launched „PV thinking“

Bishop and Thorpe (1994, QJRMS): analogy between PV and electrostatics


Schär (1990, JAS): PV flux and the Bernoulli function

Davis and Emanuel (1991, MWR): PV analysis of cyclogenesis

Rossa et al. (2000, MAP): cyclones and PV towers

... and many others!!
Kleinschmidt (1950)

Upper-level „Höhenkörper“ (= positive PV anomaly) associated with surface cyclogenesis
Basic schematic of upper-level induced extratropical cyclogenesis
PV can be expressed as divergence of a vector field – exactly as the relationship in electrostatics between charge and the electric displacement field.

Atmosphere is analogous to anisotropic dielectric material, which implies the existence of "bound PV charges" at the boundaries.
On the Evolution of Vorticity and Potential Vorticity in the Presence of Diabatic Heating and Frictional or Other Forces

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\[
\frac{\partial (\sigma Q)}{\partial t} + \nabla \cdot \mathbf{J} = 0,
\]

where (in isentropic coordinates)
\[
\mathbf{J} = (u, v, 0)\sigma Q + \mathbf{J}_\theta + \mathbf{J}_F,
\]
\[
\mathbf{J}_\theta = \{\partial u/\partial \theta, -\partial v/\partial \theta, 0\},
\]
\[
\mathbf{J}_F = \{-G, F, 0\}
\]

There is no cross-isentropic flux of "PV substance" (= \(\sigma Q\)) also in the presence of diabatic effects and/or frictional forces.

For instance, diabatically produced PV anomalies emerge through dilution and concentration of "PV substance".
For statistical steady state conditions (but in the presence of diabatic heating and/or frictional forces), the flux is given by

\[ \frac{\partial}{\partial t} (\rho Q) + \nabla \cdot \mathbf{J} = 0, \]

For non-steady conditions, the flux is given by

\[ \mathbf{J} = \nabla \theta \times \nabla B. \]

where \( B \) denotes the Bernoulli function

\[ B = c_p T + \frac{1}{2} u^2 + gz. \]

For non-steady conditions, the flux is given by

\[ \mathbf{J} = \nabla \theta \times \left( \nabla B + \frac{\partial \mathbf{u}}{\partial t} \right) - \mathbf{\omega} \frac{\partial \theta}{\partial t} \]
Summary

PV is key variable of (large & synoptic-scale) atmospheric dynamics

Positive (negative) PV anomalies induce cyclonic (anticyclonic) wind field

PV is materially conserved in frictionless flows outside of clouds

PV can be produced / destroyed near regions of cloud condensational heating

Cyclogenesis can be regarded as formation of vertically coherent columns of positive PV anomalies