Statistical Modelling by Geodesics of Critical Gait Event Motion

Stephan F. Huckemann[1], Michael Pierrynowski[2], Tara Kajaks[2], Peter Kim[3] and Ja-Yong Koo[4]

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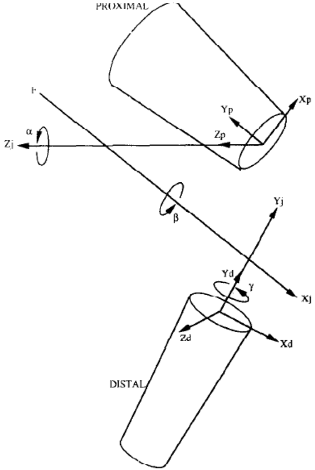
Geometry for Anatomy
Banff International Research Station
Aug. 29 – Sep. 2, 2011

supported by

DFG Grant HU 1575/2-1

Biomedical Gait Data



Discribing the knee frame, Wu and Cavanagh (1995).

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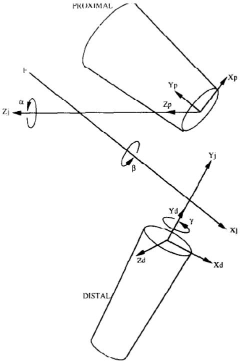

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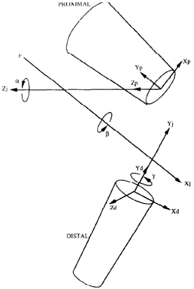

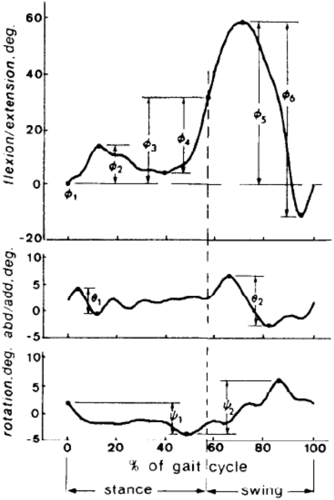

Angular motion of one gait cycle, Chao et al. (1983).

Biomedical Gait Analysis

Why?

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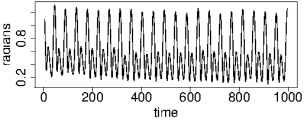

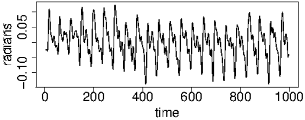

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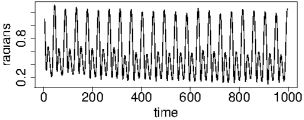
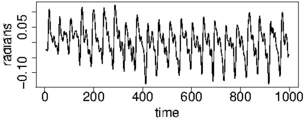
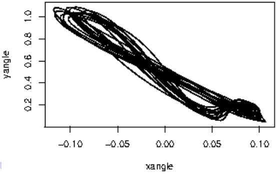




Today:

- Approximation by geodesics in $SO(3)$
- registration by critical events
- $\rightarrow$ statistics in the space of geodesics.



Further Plan

1. Biomedical Gait Data
2. Statistics on Manifold: Test for Common Means
3. Differential Geometry for the Space of Geodesics on $SO(3)$
4. Data Analysis: Evaluate an Intervention
5. Conclusions

Asymptotics for Manifold Means

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CLT : if is μ unique, if ρ^2 is $\mathcal{C}^2$ on supp(X) and if $\mu_n \in E_n$ is a measurable choice then there are matrices A_ϕ, Σ_ϕ such that

$$A_\phi \sqrt{n}(\phi(\mu_n) - \phi(\mu)) \overset{d}{\to} \mathcal{N}(0, \Sigma_\phi)$$

(Huckemann (2011)).

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- $[x,v]$ is a cut point of $[y,w] \in N/G$ in F/G, write $[x,v] \in C([y,w]) \Leftrightarrow$ the two great circles intersect orthogonally.

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- Project o.p. $\gamma_1, \dots, \gamma_n$ orthogonally to $T_{\mu^{(k)}} N \subset M(4,2)$.
- Under H_0 and $n_1/n_2 \stackrel{n\to\infty}{\to} 1$ the corresponding Hotelling T^2-statistic is asymptotically Hotelling T^2-distributed.

Evaluate an Intervention

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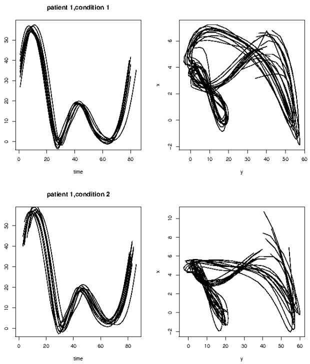

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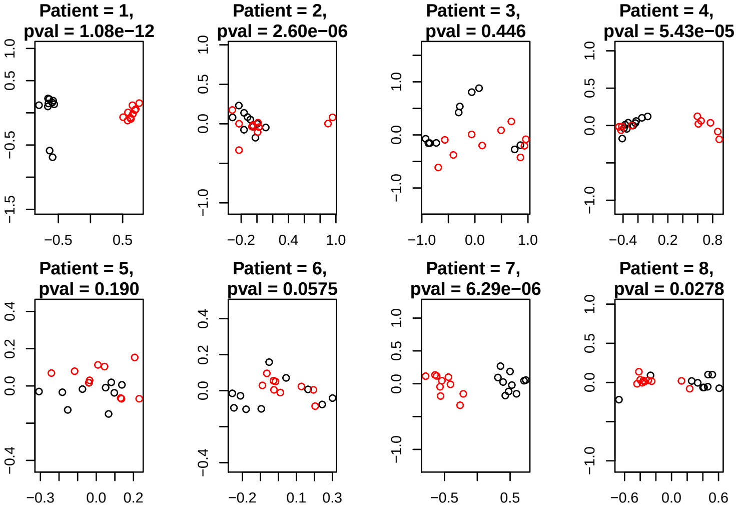

Evaluation into beginning of stance.
Black: before, red: after intervention.

Evaluate an Intervention

subjects	1	2	3	4
in: begin stance	$1.08e-12$	$2.60e-06$	0.446	$5.43e-05$
out: begin stance	$4.64e-08$	$3.63e-07$	0.013	$8.45e-05$
in: mid stance	$1.54e-06$	$7.09e-05$	$2.52e-03$	0.032
out: mid stance	$1.70e-08$	$1.14e-06$	0.800	0.029
in: end stance	$3.35e-08$	$1.23e-07$	$4.62e-03$	0.873
out: end stance	$1.29e-09$	$4.98e-07$	0.010	0.226

subjects	5	6	7	8
in: begin stance	0.190	0.058	$6.29e-06$	0.028
out: begin stance	0.057	0.459	0.015	0.024
in: mid stance	0.180	0.865	$9.20e-03$	$4.69e-03$
out: mid stance	0.033	0.255	0.060	$5.42e-04$
in: end stance	$7.27e-03$	0.016	$7.05e-03$	0.978
out: end stance	$1.39e-03$	$5.70e-04$	$6.87e-03$	0.039

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Bonferoni correction yields:

All subjects feature (all but subject 3: highly) significant gait change after intervention.

(high significance level 0.01 $\to 0.01/48 \approx 2e-03$)

Add on: Discriminate Subjects

Into beginning of stance, before intervention:

subjects	2	3	4
1	$2.52e-06$	$4.78e-03$	$2.86e-10$
2		$5.45e-05$	$1.65e-08$
3			$2.46e-03$

subjects	5	6	7	8
1	$4.27e-10$	$3.08e-08$	$4.76e-08$	$2.43e-05$
2	$4.89e-06$	$4.49e-04$	$4.00e-08$	0.034
3	0.024	$6.62e-04$	$1.22e-03$	0.011
4	$4.34e-04$	$2.08e-08$	$2.72e-13$	$5.12e-06$
5		$8.76e-06$	$1.26e-11$	$5.74e-05$
6			$2.13e-10$	$9.48e-03$
7				$1.37e-09$

Add on: Discriminate Subjects

Into beginning of stance, before intervention:

subjects	2	3	4
1	$2.52e-06$	$4.78e-03$	$2.86e-10$
2		$5.45e-05$	$1.65e-08$
3			$2.46e-03$

subjects	5	6	7	8
1	$4.27e-10$	$3.08e-08$	$4.76e-08$	$2.43e-05$
2	$4.89e-06$	$4.49e-04$	$4.00e-08$	0.034
3	0.024	$6.62e-04$	$1.22e-03$	0.011
4	$4.34e-04$	$2.08e-08$	$2.72e-13$	$5.12e-06$
5		$8.76e-06$	$1.26e-11$	$5.74e-05$
6			$2.13e-10$	$9.48e-03$
7				$1.37e-09$

Bonferoni ($0.01 \to 0.01/28 \approx 3.6e-03$) yields here that

All subjects discriminate highly significantly
(except 8 from $2, 3, 6$ and 3 from 5).

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- N.B.: Loosening the knee? No! No significant change of variances after intervention. But: motor control alteration.

Advertisment

- Grant for a PhD student at Univ. of Göttingen available
  starting in fall 2011 or later:
  statistics of geometry and differential equations in biomechanics

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