Geometric properties of solutions of non–linear PDE's and their applications

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: Baojun Bian
Title: Convexity and partial convexity for solutions of partial differential equations
Abstract: In this talk, we will discuss the convexity and partial convexity for solution of partial differential equations. We establish the microscopic (partial) convexity principle for (partially) convex solution of nonlinear elliptic and parabolic equations. As application, we discuss the (partial) convexity preserving of solution for parabolic equations. This talk is based on the joint works with Pengfei Guan.

Speaker: Barbara Brandolini
Title: Symmetrization for singular elliptic equations
Abstract: We prove some comparison results for the solution to a Dirichlet problem associated to a singular elliptic equation and we study how the summability of such a solution varies depending on the summability of the datum.

Speaker: Xavier Cabr´e
Title: Uniqueness and stability of saddle–shaped solutions to the Allen–Cahn equation
Abstract: We establish the uniqueness of a saddle-shaped solu- tion to the diffusion equation $-\Delta u = f(u)$ in all of R_{2m}, where f is of bistable type, in every even dimension $2m \geq 2$. In addition, we prove its stability whenever $2m \geq 14$. Saddle-shaped solutions are odd with respect to the Simons cone $C = \{(x_1; x_2) \in R_m$ Å~ $R_m : |x_1| = |x_2|\}$ and exist in all even dimensions. Their uniqueness was only known when $2m = 2$. On the other hand, they are known to be unstable in dimensions 2, 4, and 6. Their stability in dimensions 8, 10, and 12 remains an open question. In addition, since the Simons cone minimizes area when $2m \geq 8$, saddle-shaped solutions are expected to be global minimizers when $2m \geq 8$, or at least in higher dimensions. This is a property stronger than stability which is not yet established in any dimension.

Speaker: Andrea Cianchi
Title: A sharp iteration principle for higher-order Sobolev-type embeddings
Abstract: A general principle yielding sharp higher-order Sobolev type embeddings via first-order ones is described. This principle is applied to Euclidean Sobolev embeddings in (possibly) irregular domains, to trace Sobolev embeddings, and to Sobolev embeddings in product probability spaces, of which the Gauss space is a classical instance. As a consequence, the validity of Sobolev inequalities of arbitrary order for rearrangement invariant norms is reduced to inequalities for one-dimensional integral operators involving the same norms. This is a joint work with Lubos Pick.

Speaker: Jean Dolbeault
Title: Improved Sobolev inequalities, relative entropy and fast diffusion equations
Abstract: The difference of the two terms in Sobolev's inequality (with optimal constant) is known measure a distance to the manifold of the optimal functions. We give a precise expression of the remainder term, that is, we establish an improved inequality, with explicit norms and fully detailed constants. Our approach is based on nonlinear evolution equations and improved entropy - entropy production estimates along the associated flow. Optimizing a relative entropy functional

with respect to a scaling parameter, or handling properly second moment estimates, turns out to be the central technical issue, although it is by no mean trivial in nonlinear evolution equations. Our method also applies to other interpolation inequalities of Gagliardo-Nirenberg-Sobolev type. This is a joint work with G. Toscani.

Speaker: Filippo Gazzola
Title: Radial solutions to the Emden–Fowler equation on the Hyperbolic space
Abstract: We study the Emden-Fowler equation $-\Delta u = |u|^{p-1}u$ on the hyperbolic space. We are interested in radial solutions, namely solutions depending only on the geodesic distance from a given point. The critical exponent for such equation is the same as in the Euclidean setting, but the properties of the solutions show striking differences with the Euclidean case. While previous papers consider finite energy solutions, we deal with infinite energy solutions and we determine the exact asymptotic behavior of wide classes of finite and infinite energy solutions.

Speaker: Cristian Gutierrez
Title: Geometric optics and the Monge–Amp`ere type equations
Abstract: The physical phenomena of refraction and reflection occur simultaneously: if a light ray strikes a boundary separating to media with different refractive indices, then the ray splits into an internally reflected ray and a refracted (or transmitted) ray, each one having certain intensity. A precise description these intensities is given by the Fresnel formulas, a consequence of Maxwell's equations. In this talk, I will give some physical background to understand this phenomena, next present a new model taking into account the splitting of energy, and then show existence of surfaces separating two homogeneous materials transmitting radiation in a prescribed way. The problem gives rise to a Monge-Amp`ere type equation. This is joint work with Henok Mawi.

Speaker: Antoine Henrot
Title: About the two first eigenvalues of the Laplacian
Abstract: Let a, b be two positive numbers, does there exist an open set Ω with given volume such that a and b are the two first eigenvalues of the Laplacian with Dirichlet boundary conditions? Does there exist a convex one? What happen if we replace Dirichlet boundary conditions by Neumann ones? In this talk, we will discuss these questions and give some open problems.

Speaker: Ritva M. Hurri-Syrjanen
Title: On the (1, p)-Poincar´e inequality
Abstract: I explore the (1, p)-Poincar´e inequality in irregular domains. My talk is based on my joint work with P. Harjulehto and A. V. Vahakangas, in which we show that s- John domains support the (1, p)-Poincare inequality for all finite $p > p_0$ where p_0 is sharp.

Speaker: Bernd Kawohl
Title: Variations on the p-Laplacian
Abstract: I address several issues involving Dirichlet problems for the classical p-Laplacian operator for $p \in (1,\infty)$. First I look at p-harmonic functions as $p \to \infty$ and $p \to 1$. Then I compare the p-Laplacian with its normalized version Δ_{Np}
$:= \frac{1}{p} |\nabla u|^{2-p}\Delta_p u$ and study equations like $-\Delta_p = 1$ and $-\Delta_{Np} u = 1$.
Finally I present results and open problems on the eigenvalue problem $-\Delta_p u = \lambda|u|^{p-2}u$.

Speaker: Kazuhiro Ishige
Title: L_p norms of non–negative Schr¨odinger heat semigroup and the large time behavior of hot spots
Abstract: This talk is concerned with the Cauchy problem for the heat equation with a potential

$\partial_t u = \Delta u - V(|x|)u$ in R_N Å~ $(0,\infty)$,
$u(x, 0) = \varphi(x)$ in R_N,
(P) where $\partial_t = \partial/\partial t$, $N \geq 3$, $\varphi \in L_2(R_N)$, and $V = V(|x|)$ is a smooth, nonpositive, and radially symmetric function having quadratic decay at the space infinity. We assume that the Schr¨odinger operator $H = -\Delta + V$ is nonnegative on $L_2(R_N)$, and give the exact power decay rates of L_q-norm ($q \geq 2$) of the solution $e_{-tH}\varphi$ of (P) as $t \to \infty$. Furthermore we study the large time behavior of the solution of (P) and its hot spots.

Speaker: Chang-Shou Lin
Title: The counting topological degree formulas for a class of generalized Liouville systems
Abstract: In this talk, I will consider a generalized Liouville system with exponential nonlinear terms. We prove a uniform bound for non-critical parameters, and obtain a degree counting formulas. We also prove for that system, any bubbling solutions should blow up fully, i.e. all components of solutions should simultaneously blow up at any blow-up points.

Speaker: Xinan Ma
Title: The convexity of the solutions of heat equation and its geometry applications
Abstract: We study the spacetime convexity of the solution for parabolic equations, the key tool is the constant rank theorem for the spacetime Hessian of the solution. Then we study possible applications to other geometrical flow.

Speaker: Rolando Magnanini
Title: The location of the hot spot in a grounded convex conductor
Abstract: As a grounded heat conductor we mean one whose boundary is constantly kept at zero temperature. We also suppose that the conductor's initial temperature is constant (and positive) and we are interested in the evolution of the points where temperature takes its maximum — the hot spots. If the conductor is convex, there is only one hot spot, that starts from the set where the distance from the boundary takes its maximum and, as times grows, approches the (unique) maximum point of the first Dirichlet eigenfunction of the Laplace operator. In this talk, I present two methods to estimate the location of the hot spot in a convex conductor. The first is based on Aleksandrov's reflection principle and leads to a numerical algorithm to approximately locate the hot spot. The second is inspired by Aleksandrov-Bakelman-Pucci maximum principle and, by employing techniques of convex geometry, gives a lower bound for the distance of the hot spot from the boundary. This bound gives an answer to a problem raised in a paper by Gidas, Ni and Nirenberg. This is a joint work with L. Brasco and P. Salani.

Speaker: Lei Ni
Title: Estimates on the modulus of expansion for vector fields solving non–linear equations
Abstract: By adapting methods of [?] we prove a sharp estimate on the expansion modulus of the gradient of the parabolic kernel to the Schr¨odinger operator with convex potential, which improves an earlier work of Brascamp-Lieb. We also include alternate proofs to the improved log-concavity estimate, and to the fundamental gap theorem of Andrews-Clutterbuck via the

elliptic maximum principle. Some applications of the estimates are also obtained, including a sharp lower bound on the first eigenvalue.

Speaker: Carlo Nitsch
Title: The longest–shortest fence and sharp Poincar´e–Sobolev inequalities
Abstract: The shortest curve bisecting the area of a given planar set provides a sharp estimate of its best constant in Poincar-Sobolev type inequalities. We discuss two fencing conjectures raised more than fifty years ago. In particular we will show that among all the planar convex sets of given measure the disk, and only the disk, has the longest shortest bisecting curve.

Speaker: Gerard Philippin
Title: Blow–up phenomena for solutions of some non–linear parabolic problems
Abstract: A class of initial boundary value problems for the semilinear heat equation with time dependent coefficients is considered. Using a first order differential inequality technique, the influence of the data on the behaviour of the solutions (blow-up in finite or infinite time, global existence) is investigated. Lower and upper bounds are derived for the blow-up time when blow-up occurs.

Speaker: Wolfgang Reichel
Title: Electrostatic characterization of spheres
Abstract: TBA
Speaker: Shigeru Sakaguchi
Title: Reaction-diffusion with similar level sets
Abstract: We consider the bounded nonnegative solution of the Cauchy problem for Fisher's equation with initial data having compact support, and we introduce some property such that the solution has a sequence of similar level sets as time tends to infinity. It is shown that this property characterizes the spatially radial solutions in some class of initial data. We also consider the case where the support of initial data is unbounded, and give a similar characterization of the spatially one-dimensional solutions. Similar problems for the heat equation are dealt with. This is a joint work with Tatsuki Kawakami.

Speaker: Joel Spruck
Title: Boundary value problems at infinity and their curvature flows in Hyperbolic space
Abstract: Let H_{n+1} be the n + 1 dimensional hyperbolic space and $\partial_\infty H_{n+1}$ its boundary at infinity. By a boundary value problem at infinity (or an asymptotic Plateau problem) we understand a solution to the following problem: Let $\omega \subset \partial_\infty H_{n+1}$ be a smooth bounded domain, $\Gamma = \partial\Omega$ and a smooth symmetric function f of n variables be given. We seek a complete hypersurface Σ of constant curvature" in H_{n+1} satisfying (0.1) $f(\kappa[\Sigma]) = \sigma$ (0.2) $\partial\Sigma = \Gamma$
where $\kappa[\Sigma] = (\kappa_1, \ldots, \kappa_n)$ denotes the hyperbolic principal curvatures of Σ and $\sigma \in (0, 1)$ is a constant. Classical choices for the curvature function $f(\kappa[\Sigma])$ are the mean curvature H, the Gauss curvature K and the curvature quotient K/H. Corresponding to this "stationary" problem with prescribed boundary, it is natural to study a corresponding curvature ow (oriented by the outward hyperbolic unit normal N to the complete hypersurface X(t)), starting from a complete hypersurface Σ_0 with asymptotic boundary $\partial\Sigma_0 = \Gamma$: (0.3) $\dot{X} = (f(\kappa[X(t)])N$, $\partial X(t) = \Gamma$, $X(0) = \Sigma_0$.
In this talk we will survey our work on existence and uniqueness of solutions of the stationary problem and time permitting that of our student Ling Xiao on the convergence of the corresponding curvature ow to the stationary solution.

Speaker: Cristina Trombetti
Title: Sharp estimates for a non–local eigenvalue problem
Abstract: We study a Dirichlet eigenvalue problem for a linear operator with a non-local term. Varying the coefficient of the non-local term we look at those domains which, for a given measure, achieve the least first eigenvalue. It turns out that there exists a threshold value of such a coefficient above which the geometry of the optimal domains changes radically.

Speaker: Deane Yang
Title: The Orlicz Minkowski problem
Abstract: TBA

Speaker: Jie Xiao
Title: Singularities for Morrey potentials with applications to some non–linear elliptic systems
Abstract: In this paper joint with D. R. Adams, we will use the newly discovered facts about the embedding of a Morrey space into the Zorko spaces and the Morrey-Hausdorff iso-capacitary inequality to control the Hausdorff dimension of the singular set of a Morrey potential, and consequently, to produce some surprising good Hausdorff dimension estimates for the singular sets of: weak solutions of nonlinear elliptic systems of Meyers-Elcrat's type; weak solutions of the second order quasilinear elliptic systems with quadratic gradient growth; stationary harmonic maps; weak W-harmonic maps; stationary bi-harmonic maps; stationary admissible Yang-Mills connections.