

- I. Build a ring R_P from a poset P
- II. Geometric/topological motivation
- III. Open problems

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I. Ring R_P : We build a ring R_P from a labeled poset P

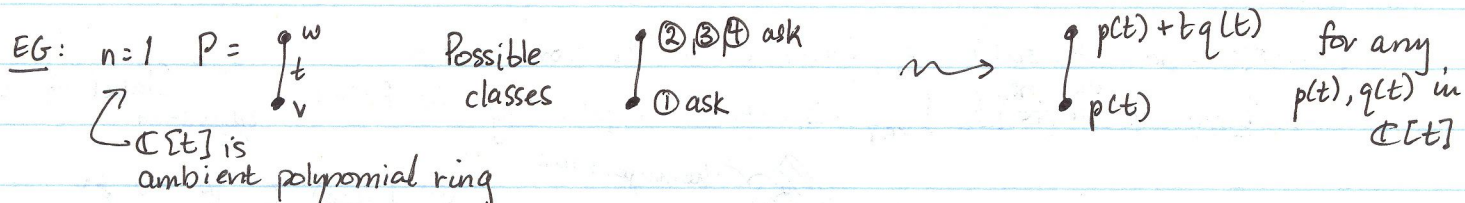
SET-UP: P is a poset with vertices V and (directed) edges E

- Each edge e is labeled by a linear form α_e (= homogeneous deg-1 poly)
- Additional conditions: we won't worry about them (see Guillemin-Zara, or open qstns below).
- n a positive integer

RING: $R_P \subseteq \bigoplus_{v \in V} \mathbb{C}[t_1, \dots, t_n]$ is defined by

$$R_P = \left\{ f: V \rightarrow \mathbb{C}[t_1, \dots, t_n] \text{ such that } \left. \begin{array}{l} \text{for each edge } e = vw \\ f(v) - f(w) \text{ is divisible by } \alpha_e \end{array} \right\}$$

- Each $f(v)$ is a polynomial. Can equivalently write $f(v) - f(w) \in (\alpha_e)$ ^{ideal}
- Can represent elements f pictorially by drawing poset & labeling vertex v by $f(v)$
- Then R_P is polynomial-labeled graphs so that polynomials on either side of edge have difference divisible by label on edge



Ring structure: add/multiply vertex-wise. $\mathbb{C}[t_1, \dots, t_n]$ -module structure: ditto.

$$\begin{array}{c} p(t) + tq(t) \\ | \\ p(t) \end{array} = \begin{array}{c} p(t) \\ | \\ p(t) \end{array} + \begin{array}{c} tq(t) \\ | \\ 0 \end{array} = \begin{array}{c} p(t) \\ | \\ 1 \end{array} + \begin{array}{c} tq(t) \\ | \\ 0 \end{array}$$

ie $\begin{array}{c} | \\ 1 \end{array}, \begin{array}{c} t \\ | \\ 0 \end{array}$ are free (!) $\mathbb{C}[t]$ -module basis for R_P

II. Why bother? Geometry/topology

Given a "good" algebraic variety X with action of torus T , can build poset P_X

GKM: $R_{P_X} \cong H_T^*(X)$ is the equivariant cohomology ring of X

PS: $H^*(X) \cong \frac{H_T^*(X)}{\langle t_1, \dots, t_n \rangle H_T^*(X)}$
 \uparrow ring \cong

"Can obtain ordinary cohomology straightforwardly from equivariant cohomology"